

## Winning Strategies

- What happens if you get really good at playing a game?
  - A simple two-player game like tic-tac-toe
  - With experienced players, the same result always happens.
- What about a more complicated game?
  - Played by two perfect players, who could look ahead to every possible sequence of moves

## Winning Strategies (contd)

- It could be that one player has a **winning strategy**: a way to always win, no matter what the other player does.
  - A strategy isn't just a sequence of moves, because you have to be prepared for every response by your opponent.
  - Instead, a strategy is a complete plan—a move you'll make for every possible state of the game.
- If a game always ends in a finite number of moves and there is always a winner at the end, one player has a winning strategy.
  - This is true for checkers, chess, etc.

## Analyzing Winning Strategies

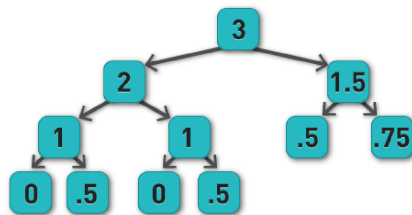
- Call a state of the game hot if the current player has a winning strategy from that state.
  - That is, the current player has a way to win, no matter what the opponent does.
- Call a state of the game cold if it's not hot.
  - That means that the opponent has a winning strategy from that state.
- If it's your move, each of your choices will put the game in a different state for your opponent.

## Analyzing Winning Strategies (contd)

- You should play not to lose:
  - If you put the game in a hot state, your opponent has a way to win.
  - If you put the game in a cold state for your opponent, you don't give your opponent a guaranteed way to win.
- Based on this, you can write the recursive rule to tell if a state is hot or cold.

## The Decreasing Number Game

- The decreasing number game begins with a float  $n$ .
- In each turn, there are two possible moves: subtract 1 from the number, or divide it by 2.
- The two players take turns making moves.
- The player who decreases the number below 1 loses.
- For a starting value  $n$ , which player has a winning strategy?



Starting from 3,  
there are 6 paths  
through the game

### Example

