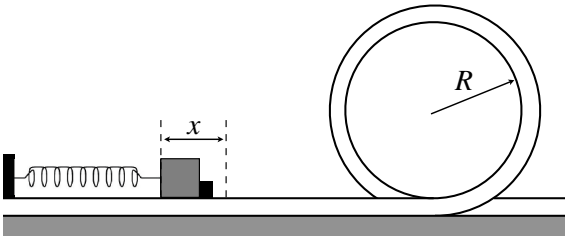
Problem Set 9

Potential energy, conservation of energy

PHYS-101(en)

1. Spring-propelled block going through a loop

A small block of mass m is pushed against a spring with spring constant k and is held in place with a latch. The spring is compressed an unknown distance x. When the latch is released, the block leaves the spring and slides along a frictionless track with a circular loop of radius R. When the block reaches the top of the loop, the force of the loop on the block (i.e. the normal force) is equal to twice the gravitational force on the mass. You may neglect air drag.



1. Using conservation of energy, find the kinetic energy of the block at the top of the loop K_t.
2. Using Newton's second law, find the speed of the block at the top of the loop v_t in terms of the gravitational constant g and the loop radius R.
3. What was the original distance x that the spring was compressed by?

2. Two-body interaction

The force of interaction between a particle of mass m_1 and a second particle of mass m_2 separated by a distance r is given by an attractive gravitational force and a repulsive force that is proportional to r^{-3} with a proportionality constant C,

$$\vec{F}(r) = \left(-\frac{Gm_1m_2}{r^2} + \frac{C}{r^3}\right)\hat{r}.$$

Choose your zero point for potential energy at infinity and note that the differential line element in spherical coordinates is $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$.

1. If the masses start off an infinite distance apart and are then moved until they are a distance R apart, what is the potential energy difference $\Delta U = U(R) - U(\infty)$?
2. What is the distance R_0 between the two masses when they are in equilibrium? What is the value of the potential energy $U(R_0)$? Is this equilibrium stable or unstable?

3. A particle in Gaussian potential

A particle of mass m moves in one dimension. Its potential energy is given by

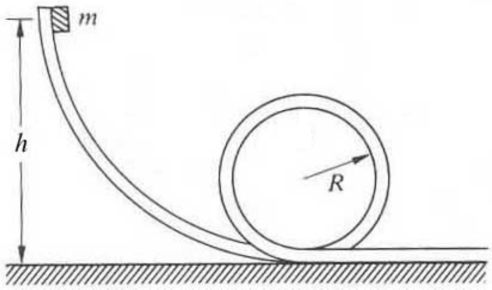
$$U(x) = -U_0 e^{-x^2/a^2},$$

where U_0 and a are positive constants.

1. Draw an energy diagram showing the potential energy $U(x)$, the kinetic energy $K(x)$, and the total energy $E < 0$ for the motion of a particle that is trapped between two turning points at $x = \pm a$.
2. Find the force $F(x)$ on the particle as a function of position x.
3. Find the particle's speed at the origin $x = 0$ such that, when it reaches either of the turning points at $x = \pm a$, it will reverse its motion.

4. Circular loop

An object of mass m is released from rest at a height h above the surface of a table. The object slides along the inside of a loop-the-loop track, consisting of a ramp and a circular loop of radius R (as shown in the figure). If the mass is just barely able to complete the loop without ever losing contact with the track, what height h did the object start at? Assume that the track is frictionless and neglect air resistance.



5. Review: A smooth rope and a rough block

A uniform inextensible horizontal rope with a mass M and length L is attached to a block of mass m_b and is lying on a table. The opposite end of the rope is pulled horizontally with a force $\vec{F}_a$. The coefficient of kinetic friction between the block and table is μ_k, while friction between the rope and table can be neglected. Find the tension in the rope as a function of the distance from the block.