Problem Set 6

Momentum, impulse, center of mass

PHYS-101(en)

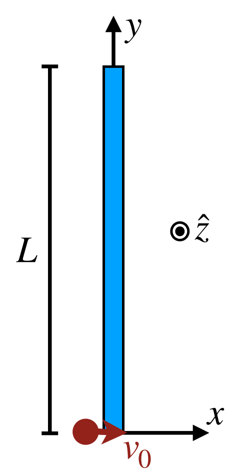
1. Center of mass of a rod

A thin rod of length L and total mass M lies along the x-axis.

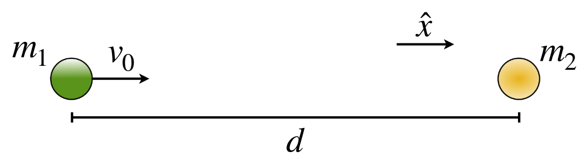
1. Suppose the rod is uniform. Find the position of the center of mass with respect to the left end of the rod in terms of L, M, and $\hat{x}$.
2. Now suppose the rod is not uniform but has a mass density that varies linearly with the distance x from the left end according to
$$\lambda(x) = \lambda_0 \frac{x}{L},$$
where λ_0 is a constant that has units of kg/m. The total mass of the rod is still M. Find λ_0 and the position of the center of mass with respect to the left end of the rod. Express your answer in terms of L, M, and $\hat{x}$.

2. Center of mass of a particle-rod system

A slender uniform rod of length L and mass M rests along the y-axis on a frictionless, *horizontal* table. A particle of equal mass M is moving along the x-axis at a speed V_0. At $t = 0$, the particle strikes the end of the rod and sticks to it. Find the position $\vec{R}_{CM}(t)$ and velocity $\vec{V}_{CM}(t)$ of the center of mass of the system as a function of time. Note that gravity would be acting in the $\hat{z}$ direction, but does not need to be considered.

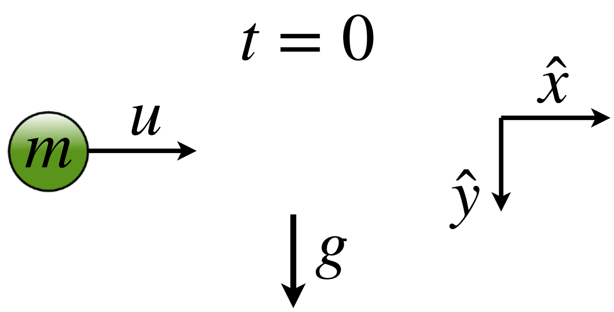


3. Two particles colliding



Two small particles of mass m_1 and mass m_2 attract each other with a force that varies inversely with the cube of their separation. At time t_0, m_1 has a velocity of magnitude v_0 directed towards m_2, which is at rest a distance d away. At time t_1, the particles collide. Calculate L, the distance traveled by particle 1 during the time interval $t_1 - t_0$. Express your answer using some or all of the following variables: m_1, m_2, t_0, t_1, v_0, and d.

4. Drag force at low speeds



At low speeds (especially in liquids rather than gases), the fluid flow around an object is laminar and the drag force is proportional to the velocity, i.e. $\vec{F} = -\beta\vec{v}$ where β is a constant. At time $t = 0$, a small ball of mass m is injected into a liquid such that it has an initial horizontal speed u in the $\hat{x}$ direction and an initial vertical speed of zero. The gravitational acceleration is g. Consider the Cartesian coordinate system shown in the figure where $\hat{x}$ is to the right and $\hat{y}$ is downwards.

1. What is the component of the acceleration in the $\hat{x}$ direction for $t > 0$? Express your answer in terms of v_x (the component of the velocity in the x direction), β, g, m, and u as needed.
2. What is the component of the acceleration in the $\hat{y}$ direction for $t > 0$? Express your answer in terms of v_y (the component of the velocity in the y direction), β, g, m, and u as needed.
3. Using your result from part 1, find $v_x(t)$, the horizontal component of the ball's velocity, as a function of time t. Express your answer in terms of β, g, m, u, and t as needed.
4. Using your result from part 2, find $v_y(t)$, the vertical component of the ball's velocity, as a function of time t. Express your answer in terms of β, g, m, u, and t as needed.
5. What is $v_{x\infty}$, the value of the horizontal component of the ball's terminal velocity? Express your answer in terms of β, g, m, and u as needed.
6. What is $v_{y\infty}$, the value of the vertical component of the ball's terminal velocity? Express your answer in terms of β, g, m, and u as needed.