Problem Set 8

Work and energy

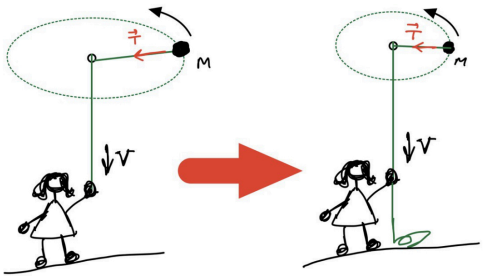
PHYS-101(en)

1. Throwing a ball in the wind

A ball is thrown straight upwards in a strong but steady wind, blowing towards the east. It rises a height H, during which time it moves eastward by a distance D. Assume that the wind exerts a steady force on the ball of magnitude F. How much work does the wind do on the ball, from the time it is thrown to when it reaches its maximum height?

2. Work-kinetic energy theorem and Newton's 2nd law: Tetherball

A ball of mass m is whirling around on a string that passes through a fixed ring located at the center of the circular motion. Initially, the ball is at a radial distance ρ_0 and has an angular speed ω_0. Then, a person grabs the string and pulls it downwards, with a constant speed V, until the ball is a distance ρ_f from the fixed ring. You may neglect the effect of gravity. Show that the work done in pulling the string equals the increase in kinetic energy of the ball.



3. Fragmenting projectile (former exam problem)

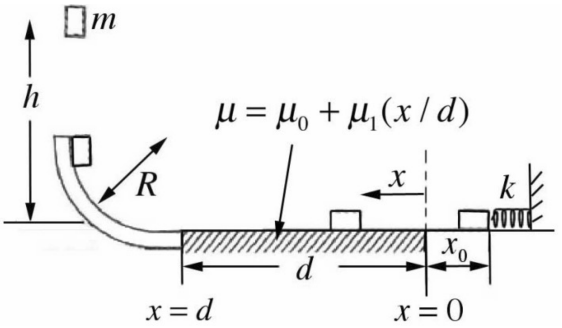
At time $t = 0$, a cannon in the middle of a flat desert fires a projectile of mass M with a speed v_0 at an angle α with respect to the horizontal. At the peak of its trajectory, the projectile explodes into a thousand pieces, which increases the kinetic energy of the system by W. The masses of these fragments are not necessarily equal. However, they all have an identical (but unknown) speed u measured with respect to the projectile just before it exploded. Lastly, you know that the velocities of all the fragments immediately after the explosion are purely horizontal. Neglect air drag.

1. At what time does the explosion occur? At what time do the pieces hit the ground? Why do they all hit the ground at the same time?
2. The pieces land on the ground in a circle. Why? What is the distance between the center of the circle and the cannon? Calculate the speed u and then the radius of the circle.
3. Consider the case where one of the pieces hits the cannon. Calculate W in terms of the initial kinetic energy of the projectile.

4. Travel on surface/loop

An object of mass m is released from rest and is pushed by a spring (with a spring constant k) that has been compressed by a distance x_0. After losing contact with the spring at $x = 0$ (which is the equilibrium position of the spring), the object travels a distance d along a horizontal track with a coefficient of friction that varies with position according to $\mu(x) = \mu_0 + \mu_1(x/d)$.

Following the horizontal track, the object enters a quarter turn of a frictionless loop with radius R. Finally, after exiting the quarter turn of the loop, the object travels vertically upwards to a maximum height h (as measured from the horizontal surface). Find the maximum height h that the object attains in terms of the constants: m, k, x_0, μ_0, μ_1, d, R, and the acceleration due to gravity g (not all may be needed).



5. Homework: Slide

A child's playground slide is $d = 5$ m in length and makes an angle of $\theta = 20$ degrees with respect to the horizontal ground. A child with mass $m = 20$ kg starts from rest at the top of the slide. The coefficient of kinetic friction between the child and the slide is $\mu_k = 0.2$.

1. What is the total work done by the friction force on the child?
2. What is the speed of the child at the bottom of the slide?
3. How long does the child take to slide down the slide?
4. To get to the top of the slide, one must climb a ladder of height h_0. Two children who have the exact same mass climb the ladder. The first child climbs up in t seconds and the other climbs up 5 times faster. Who does more work? Justify your answer mathematically.