Problem Set 11

Rigid body rotation and static equilibrium

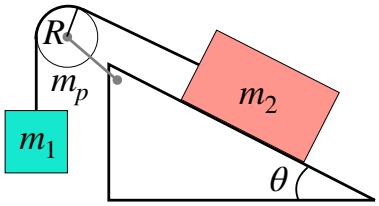
PHYS-101(en)

1. The leaning ladder

A uniform ladder of mass m and length L is leaning against a vertical wall. The angle between the wall and the ladder is α and the coefficient of static friction between the foot of the ladder and the ground is μ. There is no friction between the ladder and the wall. What is the maximum possible angle α_m before the foot of the ladder begins to slip and the ladder falls down? How does it depend on m?

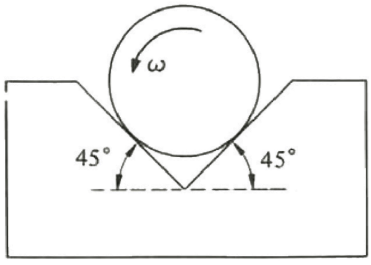
2. Frictionless funicular

A pulley, composed of a uniform wheel of radius R and mass m_p, is mounted on a frictionless horizontal axis (running into the page in the diagram below). The wheel has moment of inertia about its center of mass of $I_p = m_p R^2/2$. A massless inextensible cord is wrapped around the wheel and is used to create a funicular. One end of the cord is attached to a car of mass m_2 that can slide up or down a frictionless inclined plane. The other end of the cord is attached to a counterweight of mass m_1 that hangs off the edge of the inclined plane. The plane is inclined from the horizontal by an angle θ. Once the objects are released from rest, the cord moves without slipping around the wheel. Calculate the speed of the car $v(d)$ as a function of the distance d that it moves *down* the inclined plane using energy conservation. Assume there is no friction nor drag in the system.



3. Rotating cylinder

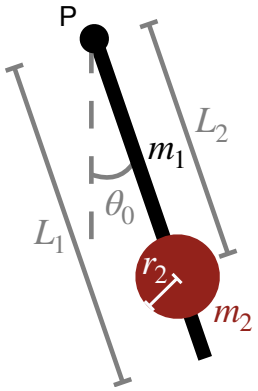
A uniform cylinder of mass m and radius R is rotating about its axis in a V-groove with constant angular velocity ω. The coefficient of kinetic friction between the cylinder and the surface is μ. An external torque $\vec{\tau}_a$ is applied about the axis of the cylinder. What $\vec{\tau}_a$ is required to keep the cylinder rolling at a constant angular velocity?



4. Pendulum and disk

A physical pendulum consists of a uniform rod of mass m_1 and length L_1 pivoting about the axis of rotation P. A uniform disk of mass m_2 and radius r_2 is rigidly attached a distance L_2 from P. The pendulum is initially displaced by an angle θ_0 from vertical and then released from rest. Friction and air drag experienced by the pendulum can be neglected.

1. What is the total moment of inertia of the entire pendulum (i.e. rod and disk) about the axis of rotation P?
2. How far from P is the center of mass of the system?
3. What is the angular speed of the pendulum when it is at the bottom of its swing?



5. Homework: The beam

Consider a thin uniform beam of mass M and length L embedded a short distance into a wall and held up by a rope that makes an angle α with the beam as shown below. The system is in static equilibrium. Determine the vector expressions for the forces experienced by the beam and mark their point of application on the diagram below.