EPFL

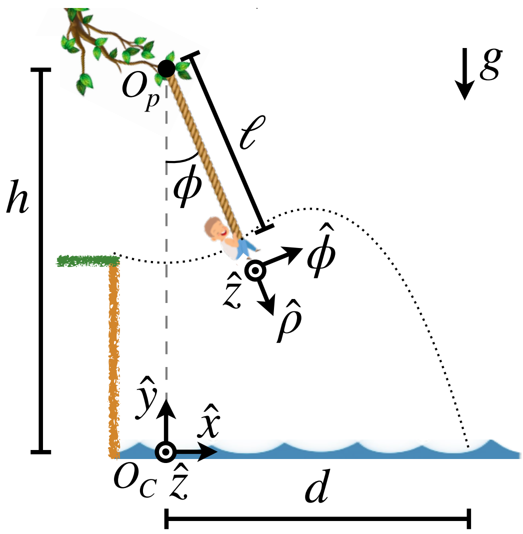
Mock exam 1
PHYS-101(en)
31 October 2023

Problem booklet

Problems

Problem 1 – 10 points – page 2

1. Rope swing (10 points)



You are standing at the edge of a cliff holding the end of a long uniform inextensible rope. You have mass M and can be considered a point mass, while the rope has a total mass m and length ℓ. The other end of the rope is attached to a tree branch above you, a height h above the surface of a lake such that the rope makes an initial angle ϕ_0 with vertical. At time $t = 0$, you hug the rope, pick your feet off the ground, and start to swing over the surface of the lake. At a time $t = t_F$ (shown in figure above), you release the rope and begin to fall. A short time later you land in the water, having had a thrilling experience! We will model different aspects of this situation, taking various simplifications.

You may assume that

- the shape of the rope always remains a straight line,
- you have no initial speed at $t = 0$,
- ϕ_0 (the initial angle of the rope at $t = 0$) satisfies $-\pi/4 < \phi_0 < 0$,
- $h > \ell$,
- $t_F < \pi\sqrt{\ell/g}$,
- air drag is negligible,
- there is no friction at the pivot point O_p, and
- the magnitude of the acceleration due to gravity is g.

Two coordinates systems are shown (which you may use if you wish): one is cylindrical with its origin at the pivot point of the rope O_p and the other is Cartesian with its origin O_C at the surface of the water a distance h directly below the pivot point. You can also use the fact that differential equations of the form

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0$$

have solutions of the form

$$x(t) = A\cos(\omega_0 t + \varphi),$$

where ω_0 is a constant and A and φ are integration constants.

All answers below should be expressed in terms of M, m, ℓ, h, ϕ_0, t_F, g, $\hat{x}$, $\hat{y}$, $\hat{z}$, $\hat{\rho}$, $\hat{\phi}$, and/or any quantities specified in the individual question.

First, we will consider the tension in the rope. To simplify, we will consider it **without** the person.

a. Calculate the tension in the rope $T(\rho)$ as a function of ρ (the radial distance from the pivot point O_p) as well as ϕ (the angle of the rope) and ω (the instantaneous angular speed of the rope). Note that in this part you may assume the **tension is zero at the end of the rope** and include the variables ρ, ϕ, and ω in your answer.

b. At what value of ρ is the rope most likely to break? Note that you should not need to perform any additional calculations.

c. At what value of ϕ is the rope most likely to break? Note that you should not need to perform any additional calculations.

Next, we will consider the trajectory of the person at the end of the rope. To simplify, you may assume that the rope is effectively massless compared to the person (i.e. $m \ll M$) for the remainder of the problem.

d. Draw the free body diagram for the person, labeling all the forces involved.

e. Find the differential equation for $\phi(t)$ (the angular position of the person as a function of time). Note that you may include the variable t in your answer.

f. Assume that the initial angular position is small (i.e. $|\phi_0| \ll 1$), so that $\sin\phi \approx \phi$. Then, solve the differential equation to find the *form* of $\phi(t)$ as well as $\vec{\omega}(t)$ (the angular velocity of the person as a function of time). Note that you may include the variable t as well as the integration constants A and φ in your answer.

g. Use the initial conditions to determine the integration constants and find $\phi(t)$ and $\vec{\omega}(t)$. Note that you may include the variable t in your answer (but not A nor φ).

h. Find x_0 and y_0 (the horizontal and vertical locations where you release the rope respectively) as well as v_{x0}, and v_{y0} (the horizontal and vertical speeds at which you release the rope respectively) in the Cartesian coordinate system shown in the figure.

i. Assume that you are given x_0, y_0, v_{x0}, and v_{y0}. Find d, the horizontal distance from O_p at which you land in the water (shown in the figure above). Note that you may include x_0, y_0, v_{x0}, and v_{y0} in your answer.

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