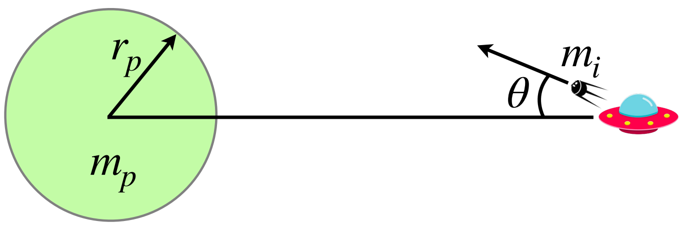
Problem Set 12

Angular momentum

PHYS-101(en)

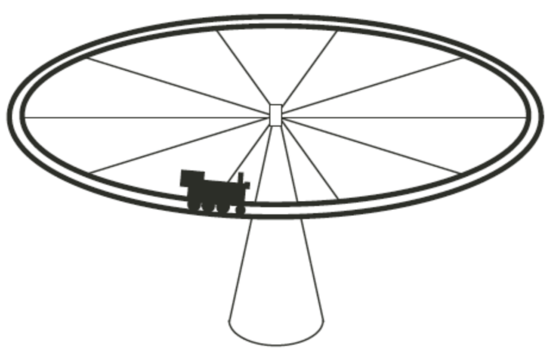
1. Planetary survey

A spaceship is sent to investigate a planet of mass m_p and radius r_p. The ship launches an instrument with mass m_i when it is a distance $5r_p$ from the center of the planet. The instrument has an initial speed v_0 (with respect to the planet), while it is initially traveling at an angle θ with respect to a radial line between the center of the planet and the launch position of the instrument. For what angle θ will the instrument just graze the surface of the planet?



2. Toy Locomotive

A toy locomotive of mass m_L runs on a uniform horizontal circular track of radius R_T and total mass m_T. The track forms the rim of an otherwise massless wheel, which is free to rotate without friction about a vertical axis. The locomotive starts from rest and accelerates without slipping to a final speed of v *relative to the track*. What is the locomotive's final speed v_f *relative to the ground*?

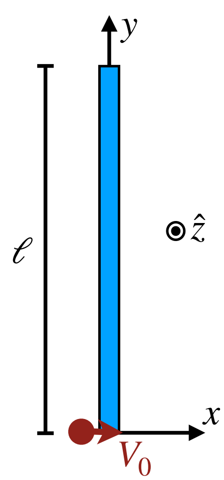


3. Particle-rod collision revisited

We will revisit problem 2 of problem set 6, and you are encouraged to use those results. Note that we use ℓ instead of L to distinguish it from the magnitude of the angular momentum.

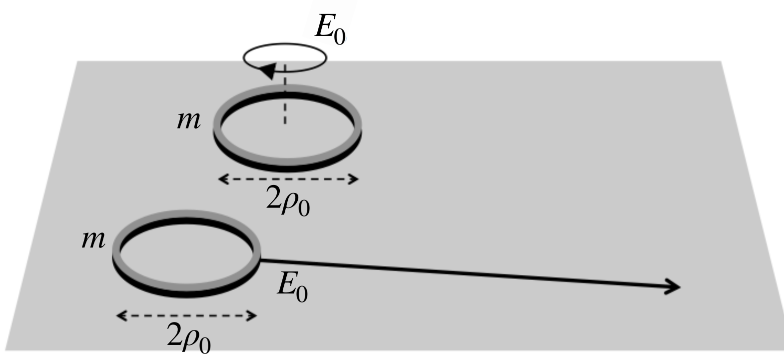
A slender uniform rod of length ℓ and mass M rests along the y-axis on a frictionless, *horizontal* table. A particle of equal mass M is moving along the x-axis at a speed V_0. At $t = 0$, the particle strikes the end of the rod and sticks to it. Note that gravity would be acting in the $\hat{z}$ direction, but does not need to be considered.

1. Calculate the angular velocity $\vec{\omega}$ of the rod-particle system about its *center of mass* after the collision. Express your answer in terms of V_0, ℓ, $\hat{x}$, $\hat{y}$, and $\hat{z}$.
2. Using this result, calculate the position of the particle (stuck at one end of the rod) $\vec{r}_p(t)$ for $t \geq 0$. Express your answer in terms of $\vec{R}_{CM}(t)$, ℓ, $\omega = |\vec{\omega}|$, t, $\hat{x}$, $\hat{y}$, and $\hat{z}$.



4. Former exam question: The ringmaster

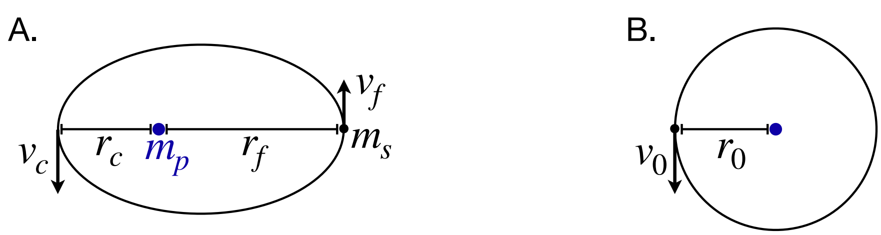
Two identical rings slide on a surface with a coefficient of kinetic friction μ. Each ring has a height h, radius ρ_0, negligible radial thickness, and mass m. At time $t = 0$, each ring is given the same kinetic energy E_0.



1. The first ring has purely translational horizontal motion (and does not rotate). Calculate the time t_1 for the ring to stop due to friction.
2. The second ring has purely rotational motion about its vertical symmetry axis (and there is no translation of its center of mass). Calculate the time t_2 for the ring to stop due to friction. What is the value of the ratio t_2/t_1?
3. What are the answers to parts 1 and 2 if, instead of rings, the objects are uniform disks with the same masses m, heights h, outer radii ρ_0, and energies E_0 as the rings?

5. Optional: Elliptic Orbit

A satellite of mass m_s is in an elliptical orbit around a planet of mass m_p, which is located at one focus of the ellipse. It is not burning any fuel nor expelling any propellant. The satellite has a speed v_f at the distance r_f when it is furthest from the planet.



1. What is the speed v_c of the satellite when it is at its closest distance to the planet? What is this distance r_c? Express your answer in terms of m_s, m_p, G, v_f, and r_f as needed.
2. If the satellite were in a circular orbit of radius $r_0 = r_c$, would its speed v_0 be greater than, equal to, or less than the speed v_c in the original elliptic orbit?