```
pip install databento
Collecting databento
  Downloading databento-0.64.0-py3-none-any.whl.metadata (6.9 kB)
Requirement already satisfied: aiohttp<4.0.0,>=3.8.3 in /opt/python/envs/default_3_11/
Collecting databento-dbn<0.43.0,>=0.42.0 (from databento)
  Downloading databento_dbn-0.42.0-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86
Requirement already satisfied: numpy>=1.23.5 in /opt/python/envs/default_3_11/lib/pyth
Requirement already satisfied: pandas>=1.5.3 in /opt/python/envs/default_3_11/lib/pyth
Requirement already satisfied: pyarrow>=13.0.0 in /opt/python/envs/default_3_11/lib/py
Requirement already satisfied: requests>=2.27.0 in /opt/python/envs/default_3_11/lib/p
Collecting zstandard>=0.21.0 (from databento)
  Downloading zstandard-0.25.0-cp311-cp311-manylinux2014_x86_64.manylinux_2_17_x86_64.
Requirement already satisfied: aiohappyeyeballs>=2.5.0 in /opt/python/envs/default_3_1
Requirement already satisfied: aiosignal>=1.4.0 in /opt/python/envs/default_3_11/lib/p
Requirement already satisfied: attrs>=17.3.0 in /opt/python/envs/default_3_11/lib/pyth
Requirement already satisfied: frozenlist>=1.1.1 in /opt/python/envs/default_3_11/lib/
Requirement already satisfied: multidict<7.0,>=4.5 in /opt/python/envs/default_3_11/li
Requirement already satisfied: propcache>=0.2.0 in /opt/python/envs/default_3_11/lib/p
Requirement already satisfied: yarl<2.0,>=1.17.0 in /opt/python/envs/default_3_11/lib/
Requirement already satisfied: python-dateutil>=2.8.2 in /opt/python/envs/default_3_11
Requirement already satisfied: pytz>=2020.1 in /opt/python/envs/default_3_11/lib/pytho
```

```
from __future__ import annotations

import re
from datetime import datetime, timedelta, time as dtime
from zoneinfo import ZoneInfo

import numpy as np
import pandas as pd
import databento as db


# ===== CONFIG =====
API_KEY = "db-iKVuPA7sBdWpefhWQyHPkrSjJpLgH"
client = db.Historical(API_KEY)

# Parent symbol -> expands to all ES contracts
PARENT = "NQ.FUT"

# Month code mapping & symbol regex (e.g., ESZ4, ESH25)
MONTH_MAP = {'F':1,'G':2,'H':3,'J':4,'K':5,'M':6,'N':7,'Q':8,'U':9,'V':10,'X':11,'Z':
12}
SYM_RE = re.compile(r"^([A-Z]{1,3})([FGHJKMNQUVXZ])(\d{1,2})$")  # root + month + 1-
2 digit year
```

```
def _resolve_year(two_digit: int, ref_year: int) -> int:
    """
    Map a 1-2 digit year code to a full year close to the reference year (track-
ing day).
    """
    candidates = [2000 + two_digit, 2010 + two_digit, 2020 + two_digit, 2030 + two_di
git]
    return min(candidates, key=lambda Y: (abs(Y - ref_year), Y))


def _parse_expiry(symbol: str, ref_year: int) -> tuple[int, int]:
    """
    Return (YYYY, MM) for a futures symbol like ESZ4 / ESZ24.
    """
    m = SYM_RE.match(symbol)
    if not m:
        return (9999, 12)  # push unknowns to the back
    _, mon_code, yy = m.groups()
    mm = MONTH_MAP[mon_code]
    y = int(yy)
    yyyy = _resolve_year(y, ref_year)
    return (yyyy, mm)


def _build_bar_ends(start_dt, end_dt, freq="15min"):
    """
    Build bar-end timestamps from RTH open to RTH close inclusive, in America/Chica-
go time.
    Bars are shifted by +3 minutes relative to the regular grid.
    First bar ends at start_dt + 3 minutes (e.g. 08:33 CT), then every `freq` there-
after.
    Last bar is clipped so it does not exceed end_dt (15:00 CT).
    """
    start_dt = pd.Timestamp(start_dt)
    end_dt   = pd.Timestamp(end_dt)

    # Build grid: start at RTH open + 3min, then every `freq`
    first = start_dt + pd.Timedelta("3min")
    grid = pd.date_range(first, end_dt + pd.Timedelta(freq), freq=freq)

    # Clip any value that overshoots the session close (e.g., 15:03 -> 15:00)
    grid = grid.where(grid <= end_dt, end_dt)

    # Drop duplicates if last two collapse at 15:00
    grid = grid.drop_duplicates()
```

```
    return grid



def download_us_rth_day(tracking_day: str,
                        freq: str = "15min",
                        output_csv: str | None = None) -> pd.DataFrame:
    """
    Download TBBO + trades for ES during **US Regular Trading Hours** (09:30-
16:00 ET),
    compute:
      - Bid/Ask at each bar end (last known TBBO)
      - VWAP since RTH open (cumulative from 09:30 ET) at each bar end
    Keep only ONE contract at each timestamp: the front month (nearest expiry).
    Save to CSV if output_csv is provided. Return the final DataFrame.
    """
    chi = ZoneInfo("America/Chicago")
    nyc = ZoneInfo("America/New_York")

    # Tracking day in CT (we anchor on the local date)
    day_ct = datetime.fromisoformat(tracking_day).replace(tzinfo=chi)

    # US RTH window: 09:30-16:00 ET == 08:30-15:00 CT for the same civil day
    rth_open_ct  = day_ct.replace(hour=8, minute=30, second=0, microsecond=0)
    rth_close_ct = day_ct.replace(hour=15, minute=0, second=0, microsecond=0)

    # Bar ends grid in CT, within RTH only
    bar_ends = _build_bar_ends(rth_open_ct, rth_close_ct, freq=freq)

    # === Download raw data (expand parent to all active ES contracts) with-
in RTH window ===
    quotes = client.timeseries.get_range(
        dataset="GLBX.MDP3",
        schema="tbbo",
        symbols=[PARENT],
        stype_in="parent",
        start=rth_open_ct.isoformat(),
        end=rth_close_ct.isoformat(),
    ).to_df()

    trades = client.timeseries.get_range(
        dataset="GLBX.MDP3",
        schema="trades",
        symbols=[PARENT],
        stype_in="parent",
        start=rth_open_ct.isoformat(),
        end=rth_close_ct.isoformat(),
```

```
    ).to_df()

    if quotes.empty and trades.empty:
        raise RuntimeError("No data returned for the requested RTH window.")

    # === Keep only outright futures, drop inter-month spreads ===
    mask_q = quotes["symbol"].astype(str).str.match(SYM_RE)
    quotes = quotes[mask_q].copy()
    if not trades.empty:
        mask_t = trades["symbol"].astype(str).str.match(SYM_RE)
        trades = trades[mask_t].copy()

    # === Normalize timestamps to America/Chicago ===
    quotes["Time"] = pd.to_datetime(quotes["ts_event"], utc=True).dt.tz_convert(chi)
    if not trades.empty:
        trades["Time"] = pd.to_datetime(trades["ts_event"], utc=True).dt.tz_con-
vert(chi)

    # === Prepare TBBO at bar ends: last known before or at each bar end (per sym-
bol) ===
    def resample_tbbo_per_symbol(df_sym: pd.DataFrame) -> pd.DataFrame:
        q = (
            df_sym.loc[:, ["Time", "bid_px_00", "ask_px_00"]]
            .sort_values("Time")
            .drop_duplicates("Time", keep="last")
        )
        target = pd.DataFrame({"Time": bar_ends})
        out = pd.merge_asof(
            target, q, on="Time", direction="backward", allow_exact_matches=True
        )
        out["symbol"] = df_sym["symbol"].iloc[0]
        return out

    tbbo = (
        quotes.sort_values(["symbol", "Time"])
              .groupby("symbol", group_keys=False)
              .apply(resample_tbbo_per_symbol)
    )

    # === VWAP since RTH open: cumulative only within the RTH window (per symbol) ===
    if trades.empty:
        vwap_bars = tbbo.loc[:, ["Time", "symbol"]].copy()
        vwap_bars["vwap_cum"] = np.nan
    else:
        tr = trades.loc[:, ["symbol", "Time", "price", "size"]].copy()
        tr = tr.sort_values(["symbol", "Time"])
        tr["notional"] = tr["price"] * tr["size"]
```

```
        # start cumulation at RTH open by filtering to the RTH window only
        tr = tr[(tr["Time"] >= rth_open_ct) & (tr["Time"] <= rth_close_ct)]
        tr["cum_notional"] = tr.groupby("symbol")["notional"].cumsum()
        tr["cum_volume"]   = tr.groupby("symbol")["size"].cumsum()
        tr = tr.loc[:, ["symbol", "Time", "cum_notional", "cum_volume"]]

        def vwap_per_symbol(df_sym: pd.DataFrame) -> pd.DataFrame:
            s = df_sym.loc[:, ["Time", "cum_notional", "cum_volume"]].drop_duplicates("Time")
            s = s.sort_values("Time")
            target = pd.DataFrame({"Time": bar_ends})
            out = pd.merge_asof(
                target, s, on="Time", direction="backward", allow_exact_matches=True
            )
            out["symbol"] = df_sym["symbol"].iloc[0]
            out["vwap_cum"] = out["cum_notional"] / out["cum_volume"]
            return out.loc[:, ["Time", "symbol", "vwap_cum"]]

        vwap_bars = tr.groupby("symbol", group_keys=False).apply(vwap_per_symbol)

    # === Merge TBBO and VWAP on (Time, symbol) ===
    bars = pd.merge(tbbo, vwap_bars, on=["Time", "symbol"], how="left")

    # === Determine front month (nearest expiry) per timestamp and keep only that ===
    ref_year = day_ct.year
    exp_ym = bars["symbol"].astype(str).map(lambda s: _parse_expiry(s, ref_year))
    bars["exp_year"]  = [ym[0] for ym in exp_ym]
    bars["exp_month"] = [ym[1] for ym in exp_ym]
    bars["expiry_key"] = bars["exp_year"] * 12 + bars["exp_month"]



    # IMPORTANT: columns indexer must be a LIST (not a tuple)
    out = bars.loc[:, ["Time", "symbol", "bid_px_00", "ask_px_00", "vwap_cum", "exp_year", "exp_month", "expiry_key"]].sort_values("Time")



    # Forward-fill VWAP after first valid trade within the day
    # (initial NaNs—before first trade—are left as NaN)
    first_valid = out["vwap_cum"].first_valid_index()
    if first_valid is not None:
        out.loc[first_valid:, "vwap_cum"] = out.loc[first_valid:, "vwap_cum"].ffill()
```

```
    if output_csv:
        out.to_csv(output_csv, index=False)

    return out
```

```
# ===== Example usage =====
df = download_us_rth_day("2010-06-07", freq="15min", output_csv="/data/workspace_-
files/2010-06-07_nasdaq.csv")
display(df.head())
```

	Time	symbol	bid_px_00	ask_px_00	vwap_cum	exp_year	exp_month	expiry_key
0	2010-06-07 08:33:00-05:00	NQM0	1843.75	1844.00	1841.514008	2010	6	24126
27	2010-06-07 08:33:00-05:00	NQU0	1840.25	1841.00	1839.253086	2010	9	24129
1	2010-06-07 08:48:00-05:00	NQM0	1830.50	1830.75	1835.539750	2010	6	24126
28	2010-06-07 08:48:00-05:00	NQU0	1828.00	1828.25	1833.995536	2010	9	24129
2	2010-06-07 09:03:00-05:00	NQM0	1839.25	1839.50	1836.565266	2010	6	24126

```
/tmp/ipykernel_207/291650387.py:145: DeprecationWarning: DataFrameGroupBy.apply operat
  .apply(resample_tbbo_per_symbol)
/tmp/ipykernel_207/291650387.py:173: DeprecationWarning: DataFrameGroupBy.apply operat
  vwap_bars = tr.groupby("symbol", group_keys=False).apply(vwap_per_symbol)
```

```
from time import sleep
import pandas as pd
from pathlib import Path

def run_range(download_fn, start="2010-06-06", end="2010-06-10",
              business_days=True, pause_sec=0.0):
    # Choose trading days: Mon-Fri (business) or every calendar day
    rng = (pd.bdate_range if business_days else pd.date_range)(
        start=start, end=end, freq="D"
    )
    failures = []
    total = len(rng)

    for i, d in enumerate(rng, 1):
        td = d.date().isoformat()
        try:
            print(f"[{i}/{total}] {td} ... ", end="", flush=True)
            download_fn(td)
            print("done")
        except Exception as e:
            print("FAILED")
            failures.append((td, str(e)))
        if pause_sec:
```

```
            sleep(pause_sec)

    print(f"\nFinished: {total - len(failures)} ok, {len(failures)} failed.")
    if failures:
        print("Sample failures (up to 10):")
        for td, msg in failures[:10]:
            print(f"  {td}: {msg}")

# === Example run ===
outdir = Path("/data/workspace_files/nasdaq")
outdir.mkdir(parents=True, exist_ok=True)

# If your function is named download_us_rth_day (per the latest code), use that:
run_range(
    lambda d: download_us_rth_day(d, freq="15min",
                                  output_csv=str(outdir / f"{d}.csv")),
    start="2014-01-01",
    end="2025-01-01",
    business_days=True,
    pause_sec=0.0,
)

# You should now see files like:
# /data/workspace_files/2010-06-07_RTH.csv
# /data/workspace_files/2010-06-08_RTH.csv
```

```
[1/4019] 2014-01-01 ... FAILED
[2/4019] 2014-01-02 ... done
[3/4019] 2014-01-03 ... done
[4/4019] 2014-01-04 ... FAILED
[5/4019] 2014-01-05 ... FAILED
[6/4019] 2014-01-06 ... done
[7/4019] 2014-01-07 ... done
[8/4019] 2014-01-08 ... done
[9/4019] 2014-01-09 ... done
[10/4019] 2014-01-10 ... done
[11/4019] 2014-01-11 ... FAILED
[12/4019] 2014-01-12 ... FAILED
[13/4019] 2014-01-13 ... done
[14/4019] 2014-01-14 ... done
[15/4019] 2014-01-15 ... done
[16/4019] 2014-01-16 ... done
[17/4019] 2014-01-17 ... done
[18/4019] 2014-01-18 ... FAILED
[19/4019] 2014-01-19 ... FAILED
[20/4019] 2014-01-20 ... FAILED
  vwap_bars = tr.groupby("symbol", group_keys=False).apply(vwap_per_symbol)
/tmp/ipykernel_207/291650387.py:145: DeprecationWarning: DataFrameGroupBy.apply operat
  .apply(resample_tbbo_per_symbol)
/tmp/ipykernel_207/291650387.py:173: DeprecationWarning: DataFrameGroupBy.apply operat
  vwap_bars = tr.groupby("symbol", group_keys=False).apply(vwap_per_symbol)
/tmp/ipykernel_207/291650387.py:95: BentoWarning: No data found for the request you su
  quotes = client.timeseries.get_range(
/tmp/ipykernel_207/291650387.py:104: BentoWarning: No data found for the request you s
  trades = client.timeseries.get_range(
/tmp/ipykernel_207/291650387.py:95: BentoWarning: No data found for the request you su
  quotes = client.timeseries.get_range(
/tmp/ipykernel_207/291650387.py:104: BentoWarning: No data found for the request you s
  trades = client.timeseries.get_range(
/tmp/ipykernel_207/291650387.py:145: DeprecationWarning: DataFrameGroupBy.apply operat
  .apply(resample_tbbo_per_symbol)
/tmp/ipykernel_207/291650387.py:173: DeprecationWarning: DataFrameGroupBy.apply operat
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/tmp/ipykernel_207/291650387.py:145: DeprecationWarning: DataFrameGroupBy.apply operat
  .apply(resample_tbbo_per_symbol)
/tmp/ipykernel_207/291650387.py:173: DeprecationWarning: DataFrameGroupBy.apply operat
```

```python
from pathlib import Path
import re
import numpy as np
import pandas as pd

DATE_RE = re.compile(r"(\d{4}-\d{2}-\d{2})\.csv$")
```

```
def load_daily_folder(
    folder: str | Path,
    start: str | None = None,
    end: str | None = None
) -> pd.DataFrame:

    folder = Path(folder)
    files = []
    for fp in sorted(folder.glob("*.csv")):
        m = DATE_RE.search(fp.name)
        if not m:
            continue
        day = m.group(1)
        if start and day < start:
            continue
        if end and day >= end:
            continue
        files.append((day, fp))

    if not files:
        raise FileNotFoundError("No matching CSV files found in the given range.")

    dfs = []
    for day, fp in files:
        print(fp)
        df = pd.read_csv(fp, low_memory=False)

        # --- TEMP datetime in ET for calculations; DO NOT modify df['Time'] ---
        t = pd.to_datetime(df["Time"], errors="coerce")
        # if naive, treat as Chicago local; if tz-aware, keep and convert
        if getattr(t.dtype, "tz", None) is None:
            t = t.dt.tz_localize("America/Chicago")
        time_et_tmp = t.dt.tz_convert("America/New_York")

        ### we compute the time until expiry to rank the contracts
        mmap = {'F':1,'G':2,'H':3,'J':4,'K':5,'M':6,'N':7,'Q':8,'U':9,'V':10,'X':11,'
Z':12}
        m = df['symbol'].astype(str).str.extract(r'([FGHJKMNQUVXZ])(\d{1,2})$')
        df['exp_month'] = m[0].map(mmap).astype('Int64')
        y = pd.to_numeric(m[1], errors='coerce')
        df['exp_year']  = np.where(y < 10, 2020 + y, 2000 + y)   # 4→2024, 24→2024

        base = pd.to_datetime(
            df['exp_year'].astype(int, errors='ignore').astype(str) +
            df['exp_month'].astype(int, errors='ignore').astype(str).str.zfill(2),
            format='%Y%m',
```

```
            errors='coerce'
        )
        third_fri = base + pd.offsets.WeekOfMonth(week=2, weekday=4)  # 3rd Friday
        df['expiry_dt_et'] = third_fri.dt.tz_localize('America/New_York') + pd.-
Timedelta(hours=9, minutes=30)

        # Use temporary ET time for subtraction; df['Time'] is unchanged
        df['time_to_expiry'] = df['expiry_dt_et'] - time_et_tmp
        df['days_to_expiry'] = df['time_to_expiry'].dt.total_seconds() / 86400

        df['expiry_order'] = df.groupby('Time')['ex-
piry_dt_et'].rank(method='dense').astype(int)

        dfs.append(df)

    out = pd.concat(dfs, ignore_index=False, sort=False)
    return out


final_df = load_daily_folder(
    folder="/data/workspace_files/nasdaq",
    start="2010-06-06",
    end="2025-01-01",
)

print(final_df.shape)

# keeping only the useful columns
final_df = final_df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwa
p_cum']]

final_df.to_csv('/data/workspace_files/nasdaq.csv', index=False)
final_df
```

```
/data/workspace_files/nasdaq/2010-06-07.csv
/data/workspace_files/nasdaq/2010-06-08.csv
/data/workspace_files/nasdaq/2010-06-09.csv
/data/workspace_files/nasdaq/2010-06-10.csv
/data/workspace_files/nasdaq/2010-06-11.csv
/data/workspace_files/nasdaq/2010-06-14.csv
/data/workspace_files/nasdaq/2010-06-15.csv
/data/workspace_files/nasdaq/2010-06-16.csv
/data/workspace_files/nasdaq/2010-06-17.csv
/data/workspace_files/nasdaq/2010-06-18.csv
/data/workspace_files/nasdaq/2010-06-21.csv
/data/workspace_files/nasdaq/2010-06-22.csv
/data/workspace_files/nasdaq/2010-06-23.csv
/data/workspace_files/nasdaq/2010-06-24.csv
/data/workspace_files/nasdaq/2010-06-25.csv
/data/workspace_files/nasdaq/2010-06-28.csv
/data/workspace_files/nasdaq/2010-06-29.csv
/data/workspace_files/nasdaq/2010-06-30.csv
/data/workspace_files/nasdaq/2010-07-01.csv
/data/workspace_files/nasdaq/2010-07-02.csv
```

	Time	symbol	expiry_order	bid_px_00	ask_px_00	vwap_cum
0	2010-06-07 08:33:00-05:00	NQM0	1	1843.75	1844.00	1841.514008
1	2010-06-07 08:33:00-05:00	NQU0	2	1840.25	1841.00	1839.253086
2	2010-06-07 08:48:00-05:00	NQM0	1	1830.50	1830.75	1835.539750
3	2010-06-07 08:48:00-05:00	NQU0	2	1828.00	1828.25	1833.995536
4	2010-06-07 09:03:00-05:00	NQM0	1	1839.25	1839.50	1836.565266
...	...	...	...	...	...	...
49	2024-12-31 14:33:00-06:00	NQH5	1	21289.75	21290.50	21321.197330
50	2024-12-31 14:48:00-06:00	NQM5	2	21501.25	21504.25	21563.041818
51	2024-12-31 14:48:00-06:00	NQH5	1	21271.50	21272.00	21319.046374
52	2024-12-31 15:00:00-06:00	NQH5	1	21220.25	21220.75	21311.938467
53	2024-12-31 15:00:00-06:00	NQM5	2	21456.50	21462.00	21549.936120

267705 rows × 6 columns

```
/tmp/ipykernel_3391/2762008146.py:55: PerformanceWarning: Non-vectorized DateOffset be
  third_fri = base + pd.offsets.WeekOfMonth(week=2, weekday=4)  # 3rd Friday
/tmp/ipykernel_3391/2762008146.py:55: PerformanceWarning: Non-vectorized DateOffset be
  third_fri = base + pd.offsets.WeekOfMonth(week=2, weekday=4)  # 3rd Friday
/tmp/ipykernel_3391/2762008146.py:55: PerformanceWarning: Non-vectorized DateOffset be
  third_fri = base + pd.offsets.WeekOfMonth(week=2, weekday=4)  # 3rd Friday
/tmp/ipykernel_3391/2762008146.py:55: PerformanceWarning: Non-vectorized DateOffset be
  third_fri = base + pd.offsets.WeekOfMonth(week=2, weekday=4)  # 3rd Friday
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  third_fri = base + pd.offsets.WeekOfMonth(week=2, weekday=4)  # 3rd Friday
/tmp/ipykernel_3391/2762008146.py:55: PerformanceWarning: Non-vectorized DateOffset be
  third_fri = base + pd.offsets.WeekOfMonth(week=2, weekday=4)  # 3rd Friday
```

```
### Stop opposite band + fix bet size
```

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math
import datetime as dt
import matplotlib.dates as mdates
```

```python
### import the data
df = pd.read_csv('/data/workspace_files/nasdaq.csv')
df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

# (Optional) drop bad rows
df = df.dropna(subset=['Time'])

# Convert UTC -> New York (handles DST)
df['Time'] = df['Time'].dt.tz_convert('America/New_York')

# Build hour/day AFTER conversion
df['hour'] = df['Time'].dt.strftime('%H:%M')
df['day']  = df['Time'].dt.date
```

```
# Keep only US RTH 09:30-16:00 ET
market_open, market_close = dt.time(9, 30), dt.time(16, 0)
df = df[df['Time'].dt.time.between(market_open, market_close)]

# Sanity check (optional)
tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()


### order by symbol, hour and day
df = df.sort_values(['symbol', 'hour', 'day'])

### price
df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


### opening price
df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


# settling price and previous day settling price
daily_settle = (
    df.sort_values(['symbol','day','Time'])
      .groupby(['symbol','day'])['price']
      .last()
      .rename('settl_price')
)

prev_settle = (
    daily_settle.groupby(level='symbol')
                .shift(1)
                .rename('settl_price_previous')
)

df = df.join(daily_settle, on=['symbol','day'])
df = df.join(prev_settle,   on=['symbol','day'])


### move since opening
df["move"] = np.where(
    df["open_price"] > 0,
    np.abs(df["price"] / df["open_price"] - 1),
    np.nan
)


### compute mean of move over the last period
```

```
n = 14
df['sigma_observed'] = (
    df.groupby(['symbol', 'hour'])['move']
      .transform(lambda x: x.rolling(n, min_periods=n).mean())
)


### compute upper bound
df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1 + d
f['sigma_observed'])


### compute lower bound
df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1 -
 df['sigma_observed'])


### get return
df['return'] = df['settl_price']/df['open_price'] - 1


### get variance onf returns among the last 14 days
daily = (df.sort_values(['symbol','day'])
           .groupby(['symbol','day'], as_index=False)['return'].last())

daily['ret_std_14d'] = (
    daily.groupby('symbol')['return']
         .transform(lambda s: s.rolling(14, min_periods=14).std())
)

df = df.merge(daily[['symbol','day','ret_std_14d']], on=['symbol','day'], how='left')


df = df.sort_values(['symbol', 'day', 'hour'])
df[df['expiry_order']==1]
```

	Time	symbol	expiry_order	bid_px_00	ask_px_00	vwap_cum	hour	day	price	open_price	settl_pri
0	2019-03-15 09:33:00-04:00	NQH0	1	NaN	NaN	NaN	09:33	2019-03-15	NaN	NaN	NaN
188	2019-03-15 09:48:00-04:00	NQH0	1	NaN	NaN	7303.251423	09:48	2019-03-15	NaN	NaN	NaN
376	2019-03-15 10:03:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:03	2019-03-15	NaN	NaN	NaN
564	2019-03-15 10:18:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:18	2019-03-15	NaN	NaN	NaN
752	2019-03-15 10:33:00-04:00	NQH0	1	NaN	NaN	7314.065510	10:33	2019-03-15	NaN	NaN	NaN
...	...	...	...	...	...	...	...	...	...	...	...
261856	2018-12-20 15:03:00-05:00	NQZ8	1	6248.00	6250.50	6284.578275	15:03	2018-12-20	6249.250	6336.125	6247.0
262049	2018-12-20 15:18:00-05:00	NQZ8	1	6273.25	6274.50	6283.674965	15:18	2018-12-20	6273.875	6336.125	6247.0
262242	2018-12-20 15:33:00-05:00	NQZ8	1	6270.50	6273.00	6283.447989	15:33	2018-12-20	6271.750	6336.125	6247.0
262435	2018-12-20 15:48:00-05:00	NQZ8	1	6242.25	6242.75	6282.792038	15:48	2018-12-20	6242.500	6336.125	6247.0
262628	2018-12-20 16:00:00-05:00	NQZ8	1	6246.25	6247.75	6279.934103	16:00	2018-12-20	6247.000	6336.125	6247.0

101061 rows × 18 columns

Running strat one day

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
            .sort_values('Time')
            .copy())
    backtest_df = backtest_df.sort_values('hour')

    bid_list = backtest_df['bid_px_00'].values
    ask_list = backtest_df['ask_px_00'].values
    upper_bound = backtest_df['upper_bound'].values
    lower_bound = backtest_df['lower_bound'].values
    vwap_list = backtest_df['vwap_cum'].values
    hour = backtest_df['hour'].values

    if not (len(bid_list) == len(ask_list) == len(vwap_list) == len(hour) == len(up-
per_bound) == len(lower_bound)):
        return initial_cash
    elif any(pd.isna(x) for x in bid_list + ask_list + vwap_list + upper_bound + low-
er_bound):
        return initial_cash



    position = 0
    sigma_target = 0.02
    total_shares_to_hold = math.floor(initial_cash / backtest_df['open_price'].val-
ues[0])




    for i in range(1, len(bid_list) - 1):
        if ask_list[i] > max(upper_bound[i], vwap_list[i]):
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * ask_list[i]
                print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str} a
t {hour[i]}")
```

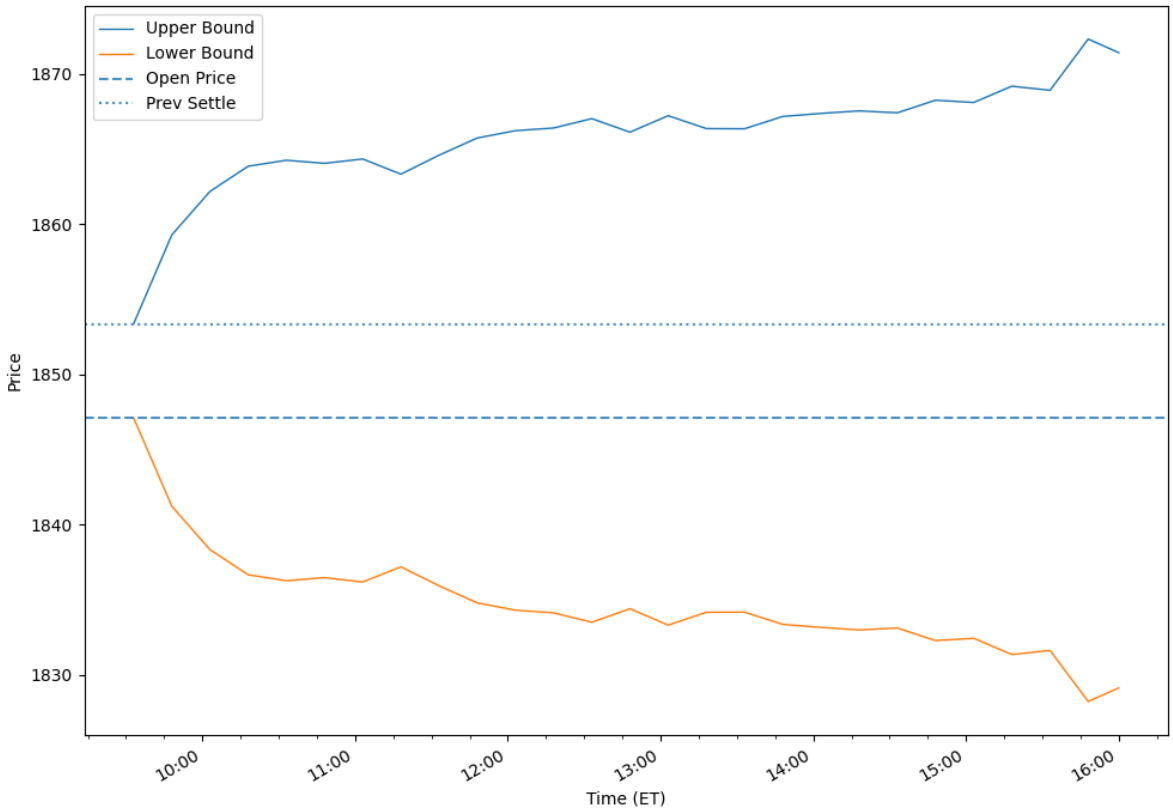
```
        elif bid_list[i] < min(vwap_list[i], lower_bound[i]):
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * bid_list[i]
                print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str} at
 {hour[i]}")

        # elif bid_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
        #     # we must close our position
        #     if position > 0: # we are long so we need to sell
        #         initial_cash += position * bid_list[i]
        #         # print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {
hour[i]}")
        #         position = 0
        #     elif position < 0: # we are short so we need to buy
        #         initial_cash -= abs(position) * ask_list[i]
        #         # print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_st
r} at {hour[i]}")
        #         position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
        initial_cash += position * bid_list[-1]
        print(f"Sold {position} shares at {ask_list[-1]} on {day_str} at {hour[-1]}")
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * ask_list[-1]
        print(f"Bought {abs(position)} shares at {bid_list[-1]} on {day_str} at {hour
[-1]}")
        position = 0

    return initial_cash

backtest_one_day(df, "2010-10-13", 1, 100000)
```

```
Bought 48 shares at 2057.75 on 2010-10-13 at 11:03
Sold 48 shares at 2055.75 on 2010-10-13 at 16:00
99892.0
```

```
# Ensure Time is tz-aware ET
df["Time"] = pd.to_datetime(df["Time"], utc=True).dt.tz_convert("America/New_York")

# Define day as plain date (no tz needed)
```

```
day_str = "2010-07-16"
day = pd.to_datetime(day_str).date()

# Recompute day column in ET if not already done
df["day"] = df["Time"].dt.date

# Filter front-month contract for that day
one = df[(df['expiry_order'] == 1) & (df['day'] == day)].copy()
one = one.sort_values('Time')

# Plot
fig, ax = plt.subplots(figsize=(10, 7))
x = one['Time']

# ax.plot(x, one['price'], label='Mid Price')
if 'upper_bound' in one and one['upper_bound'].notna().any():
    ax.plot(x, one['upper_bound'], label='Upper Bound', linewidth=1)
if 'lower_bound' in one and one['lower_bound'].notna().any():
    ax.plot(x, one['lower_bound'], label='Lower Bound', linewidth=1)

if 'open_price' in one and one['open_price'].notna().any():
    ax.axhline(one['open_price'].iloc[0], linestyle='--', alpha=0.8, la-
bel='Open Price')
if 'settl_price_previous' in one and one['settl_price_previous'].notna().any():
    ax.axhline(one['settl_price_previous'].iloc[0], linestyle=':', alpha=0.8, label='
Prev Settle')

# if 'vwap_cum' in one.columns and one['vwap_cum'].notna().any():
#     ax.plot(x, one['vwap_cum'], label='VWAP (cum)', linewidth=1)

# Tidy x-axis
import matplotlib.dates as mdates
from zoneinfo import ZoneInfo


tz = ZoneInfo("America/New_York")

ax.xaxis.set_major_locator(mdates.HourLocator(tz=tz))
ax.xaxis.set_major_formatter(mdates.DateFormatter('%H:%M', tz=tz))
ax.xaxis.set_minor_locator(mdates.MinuteLocator(byminute=[0,15,30,45], tz=tz))

fig.autofmt_xdate()

ax.set_xlabel("Time (ET)")
```

```
ax.set_ylabel("Price")
ax.legend()
plt.tight_layout()
plt.show()
```

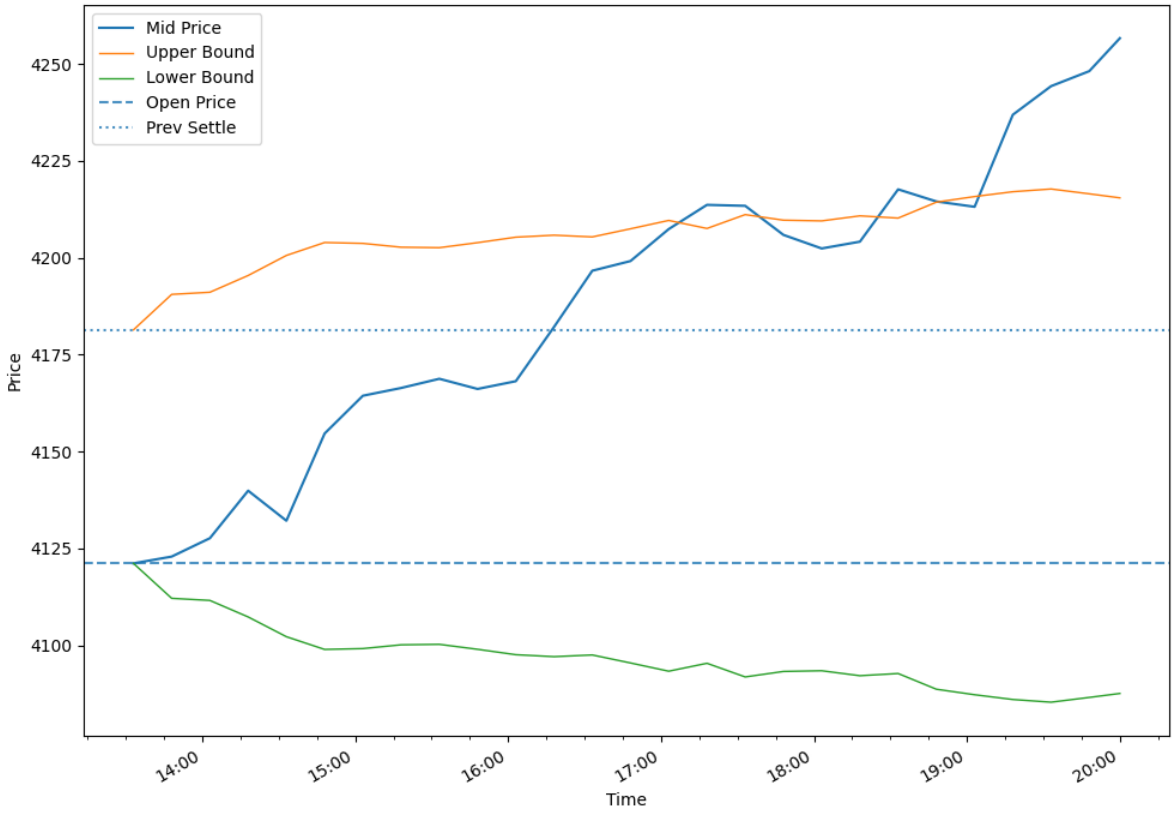



```
backtest_one_day(df, "2010-07-16", 1, 100000)
```

```
Sold 54 shares at 1836.0 on 2010-07-16 at 09:48
Bought 54 shares at 1801.75 on 2010-07-16 at 16:00
101822.5
```

```
day_str = "2015-10-02"
day = pd.to_datetime(day_str).date()

print(backtest_one_day(df, day_str, 1, 100000))
one = df[(df['expiry_order'] == 1) & (df['day'] == day)].copy()
one = one.sort_values('Time')

fig, ax = plt.subplots(figsize=(10, 7))

x = one['Time']

ax.plot(x, one['price'], label='Mid Price')
if 'upper_bound' in one and one['upper_bound'].notna().any():
    ax.plot(x, one['upper_bound'], label='Upper Bound', linewidth=1)
if 'lower_bound' in one and one['lower_bound'].notna().any():
    ax.plot(x, one['lower_bound'], label='Lower Bound', linewidth=1)

# Horizontal lines for open and previous settlement
if 'open_price' in one and one['open_price'].notna().any():
    ax.axhline(one['open_price'].iloc[0], linestyle='--', alpha=0.8, la-
bel='Open Price')
if 'settl_price_previous' in one and one['settl_price_previous'].notna().any():
    ax.axhline(one['settl_price_previous'].iloc[0], linestyle=':', alpha=0.8, label='
Prev Settle')

# Optional: VWAP
# if 'vwap_cum' in one.columns and one['vwap_cum'].notna().any():
#     ax.plot(x, one['vwap_cum'], label='VWAP (cum)', linewidth=1)

# Tidy x-axis: show one tick per hour, minor ticks at 15-min
ax.xaxis.set_major_locator(mdates.HourLocator())
ax.xaxis.set_major_formatter(mdates.DateFormatter('%H:%M'))
ax.xaxis.set_minor_locator(mdates.MinuteLocator(byminute=[0,15,30,45]))
fig.autofmt_xdate()
ax.set_xlabel("Time")
ax.set_ylabel("Price")
ax.legend()
plt.tight_layout()
plt.show()
```

```
Bought 24 shares at 4213.75 on 2015-10-02 at 13:18
Sold 24 shares at 4256.75 on 2015-10-02 at 16:00
101026.0
```



Bactests on all day since 2011

```python
def backtest(backtest_df, rank_to_expiry, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        print(day_ts)
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))
```

```
results = backtest(df, 1)
results
```

```
2010-06-07 00:00:00-04:00
2010-06-08 00:00:00-04:00
2010-06-09 00:00:00-04:00
2010-06-10 00:00:00-04:00
2010-06-11 00:00:00-04:00
2010-06-14 00:00:00-04:00
2010-06-15 00:00:00-04:00
2010-06-16 00:00:00-04:00
2010-06-17 00:00:00-04:00
2010-06-18 00:00:00-04:00
2010-06-21 00:00:00-04:00
2010-06-22 00:00:00-04:00
2010-06-23 00:00:00-04:00
2010-06-24 00:00:00-04:00
Sold 53 shares at 1855.0 on 2010-06-24 00:00:00-04:00 at 10:03
Bought 53 shares at 1843.5 on 2010-06-24 00:00:00-04:00 at 16:00
2010-06-25 00:00:00-04:00
Sold 54 shares at 1834.5 on 2010-06-25 00:00:00-04:00 at 09:48
Bought 54 shares at 1840.25 on 2010-06-25 00:00:00-04:00 at 16:00
2010-06-28 00:00:00-04:00
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	163614.75	164806.50	1191.75	0.007284	True
3739	2024-12-26 00:00:00-05:00	164806.50	164806.50	0.00	0.000000	False
3740	2024-12-27 00:00:00-05:00	164806.50	165006.00	199.50	0.001211	True
3741	2024-12-30 00:00:00-05:00	165006.00	164349.75	-656.25	-0.003977	True
3742	2024-12-31 00:00:00-05:00	164349.75	164831.00	481.25	0.002928	True

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
```

```
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

```
{'total_return': 0.6483099999999999,
 'irr_annual': 0.034218470922113164,
 'vol_annual': 0.10732984746283207,
 'sharpe': 0.36737348868774683,
 'hit_ratio': 0.5287784679089027,
 'mdd': -0.293018433457509,
 'n_days': 3743,
 'n_traded_days': 2415}
```

```
results.sort_values('return')
```

	date	start_cash	end_cash	pnl	return	traded
344	2011-10-04 00:00:00-04:00	94374.00	90515.25	-3858.75	-0.040888	True
2506	2020-03-18 00:00:00-04:00	151446.50	146228.00	-5218.50	-0.034458	True
2499	2020-03-09 00:00:00-04:00	162862.25	157647.25	-5215.00	-0.032021	True
2507	2020-03-19 00:00:00-04:00	146228.00	141753.00	-4475.00	-0.030603	True
2984	2022-01-24 00:00:00-05:00	156325.00	151804.00	-4521.00	-0.028921	True
...	...	...	...	...	...	...
2137	2018-10-10 00:00:00-04:00	137493.75	141593.25	4099.50	0.029816	True
2190	2018-12-26 00:00:00-05:00	156958.25	161742.25	4784.00	0.030479	True
3205	2022-11-30 00:00:00-05:00	173744.00	179500.25	5756.25	0.033131	True
1964	2018-02-08 00:00:00-05:00	124831.50	129030.00	4198.50	0.033633	True
2147	2018-10-24 00:00:00-04:00	138440.50	143632.25	5191.75	0.037502	True

3743 rows × 6 columns

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Clean the series
s = pd.Series(results['return'], dtype='float64')
s = s.replace([np.inf, -np.inf], np.nan).dropna()

fig, ax = plt.subplots(figsize=(8, 5))

# Density histogram
ax.hist(s, bins=100, density=True, alpha=0.3, label='Histogram (density)')

# KDE overlay (if scipy available)
try:
    from scipy.stats import gaussian_kde
    xs = np.linspace(s.quantile(0.001), s.quantile(0.999), 512)
    kde = gaussian_kde(s)
    ax.plot(xs, kde(xs), linewidth=2, label='KDE')
except Exception:
    # If scipy isn't installed, just skip the KDE
    pass

# Nice viewing window (doesn't affect density calc)
qlo, qhi = s.quantile([0.001, 0.999])
ax.set_xlim(qlo, qhi)

ax.set_xlabel('Return')
ax.set_ylabel('Density')
ax.set_title('Return Density')
ax.legend()
plt.tight_layout()
plt.show()
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math
import matplotlib.dates as mdates
import datetime as dt

### import the data
df = pd.read_csv('/data/workspace_files/nasdaq.csv')
df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]
```

```
df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

# (Optional) drop bad rows
df = df.dropna(subset=['Time'])

# Convert UTC -> New York (handles DST)
df['Time'] = df['Time'].dt.tz_convert('America/New_York')

# Build hour/day AFTER conversion
df['hour'] = df['Time'].dt.strftime('%H:%M')
df['day']  = df['Time'].dt.date

# Keep only US RTH 09:30-16:00 ET
market_open, market_close = dt.time(9, 30), dt.time(16, 0)
df = df[df['Time'].dt.time.between(market_open, market_close)]

# Sanity check (optional)
tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()


### order by symbol, hour and day
df = df.sort_values(['symbol', 'hour', 'day'])

### price
df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


### opening price
df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


# settling price and previous day settling price
daily_settle = (
    df.sort_values(['symbol','day','Time'])
      .groupby(['symbol','day'])['price']
      .last()
      .rename('settl_price')
)

prev_settle = (
    daily_settle.groupby(level='symbol')
                .shift(1)
                .rename('settl_price_previous')
)

df = df.join(daily_settle, on=['symbol','day'])
df = df.join(prev_settle,   on=['symbol','day'])
```

```
### move since opening
df["move"] = np.where(
    df["open_price"] > 0,
    np.abs(df["price"] / df["open_price"] - 1),
    np.nan
)


### compute mean of move over the last period
n = 14
df['sigma_observed'] = (
    df.groupby(['symbol', 'hour'])['move']
      .transform(lambda x: x.rolling(n, min_periods=n).mean())
)


### compute upper bound
df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1 + d
f['sigma_observed'])


### compute lower bound
df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1 -
 df['sigma_observed'])


### get return
df['return'] = df['settl_price']/df['open_price'] - 1


### get variance onf returns among the last 14 days
daily = (df.sort_values(['symbol','day'])
           .groupby(['symbol','day'], as_index=False)['return'].last())

daily['ret_std_14d'] = (
    daily.groupby('symbol')['return']
         .transform(lambda s: s.rolling(14, min_periods=14).std())
)

df = df.merge(daily[['symbol','day','ret_std_14d']], on=['symbol','day'], how='left')


df = df.sort_values(['symbol', 'day', 'hour'])
# df[df['expiry_order']==1]
df
```

	Time	symbol	expiry_order	bid_px_00	ask_px_00	vwap_cum	hour	day	price	open_price	settl_pri
0	2019-03-15 09:33:00-04:00	NQH0	1	NaN	NaN	NaN	09:33	2019-03-15	NaN	NaN	NaN
188	2019-03-15 09:48:00-04:00	NQH0	1	NaN	NaN	7303.251423	09:48	2019-03-15	NaN	NaN	NaN
376	2019-03-15 10:03:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:03	2019-03-15	NaN	NaN	NaN
564	2019-03-15 10:18:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:18	2019-03-15	NaN	NaN	NaN
752	2019-03-15 10:33:00-04:00	NQH0	1	NaN	NaN	7314.065510	10:33	2019-03-15	NaN	NaN	NaN
...	...	...	...	...	...	...	...	...	...	...	...
266952	2019-12-19 15:03:00-05:00	NQZ9	3	8632.25	8632.50	8613.137849	15:03	2019-12-19	8632.375	8604.125	8640.62
267140	2019-12-19 15:18:00-05:00	NQZ9	3	8628.75	8629.25	8613.525157	15:18	2019-12-19	8629.000	8604.125	8640.62
267328	2019-12-19 15:33:00-05:00	NQZ9	3	8628.75	8629.00	8613.953049	15:33	2019-12-19	8628.875	8604.125	8640.62
267516	2019-12-19 15:48:00-05:00	NQZ9	3	8631.75	8632.00	8614.566532	15:48	2019-12-19	8631.875	8604.125	8640.62
267704	2019-12-19 16:00:00-05:00	NQZ9	3	8640.50	8640.75	8617.831876	16:00	2019-12-19	8640.625	8604.125	8640.62

267705 rows × 18 columns

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_target):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]

    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
ower_bound) == n) or n == 0:
        return initial_cash

    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
        np.isnan(lower_bound).any()):
```

```
        return initial_cash

    position = 0
    cash = float(initial_cash)

    for i in range(1, n - 1):
        # Long region: price breaking above both local band and trend
        if ask_list[i] > max(upper_bound[i], vwap_list[i]):
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                cash -= shares_to_buy * ask_list[i]

        # Short region: price breaking below both local band and trend
        elif bid_list[i] < min(vwap_list[i], lower_bound[i]):
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold  # positive number
                position -= shares_to_sell
                cash += shares_to_sell * bid_list[i]

        # # Flat/inside band: close any open position
        # elif (bid_list[i] >= min(vwap_list[i], lower_bound[i])) and \
        #      (bid_list[i] <= max(upper_bound[i], vwap_list[i])):
        #     if position > 0:  # close long at bid
        #         cash += position * bid_list[i]
        #         position = 0
        #     elif position < 0:  # close short at ask
        #         cash -= abs(position) * ask_list[i]
        #         position = 0

    # Close at the last tick (use correct side of book)
    if position > 0:      # close long at bid
        cash += position * bid_list[-1]
        position = 0
    elif position < 0:    # close short at ask
        cash -= abs(position) * ask_list[-1]
        position = 0

    return cash

backtest_one_day(df, "2011-10-10", 1, 100000, 0.02)
```

```
100990.0
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_tar-
get)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
```

```
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization, sigma_lever-
age_period, sigma_leverage_bound, sigma_target):

    df = pd.read_csv('/data/workspace_files/nasdaq.csv')
    df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

    # (Optional) drop bad rows
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST)
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')

    # Build hour/day AFTER conversion
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date

    # Keep only US RTH 09:30-16:00 ET
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Sanity check (optional)
    tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()

    ### order by symbol, hour and day
    df = df.sort_values(['symbol', 'hour', 'day'])
```

```
    ### price
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


    ### opening price
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


    # settling price and previous day settling price
    daily_settle = (
        df.sort_values(['symbol','day','Time'])
        .groupby(['symbol','day'])['price']
        .last()
        .rename('settl_price')
    )

    prev_settle = (
        daily_settle.groupby(level='symbol')
                    .shift(1)
                    .rename('settl_price_previous')
    )

    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])


    ### move since opening
    df["move"] = np.where(
        df["open_price"] > 0,
        np.abs(df["price"] / df["open_price"] - 1),
        np.nan
    )


    ### compute mean of move over the last period
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
        .transform(lambda x: x.rolling(sigma_leverage_bound, min_periods=sigma_lever-
age_bound).mean())
    )


    ### compute upper bound
    df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1
 + coef_sigma * df['sigma_observed']) + percentage_penalization * df['open_price']
```

```
    ### compute lower bound
    df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1
 - coef_sigma * df['sigma_observed']) - percentage_penalization * df['open_price']


    ### get return
    df['return'] = df['settl_price']/df['open_price'] - 1


    ### get variance onf returns among the last 14 days
    daily = (df.sort_values(['symbol','day'])
            .groupby(['symbol','day'], as_index=False)['return'].last())

    daily[f'ret_std_14d'] = (
        daily.groupby('symbol')['return']
            .transform(lambda s: s.rolling(sigma_leverage_period, min_periods=sig-
ma_leverage_period).std())
    )

    df = df.merge(daily[['symbol','day', f'ret_std_14d']], on=['sym-
bol','day'], how='left')


    df = df.sort_values(['symbol', 'day', 'hour'])

    results = backtest(df, 1, sigma_target)

    return evaluate_strategy(results, risk_free_rate=0.00)
```

```
results = backtest_with_hyperparameters(0.142857, 0.008143, 30, 30, 0.02)
results
```

```
{'total_return': 0.7512125000000001,
 'irr_annual': 0.03844369096551703,
 'vol_annual': 0.06546663729533318,
 'sharpe': 0.6089773317872827,
 'hit_ratio': 0.5588235294117647,
 'mdd': -0.17309699134294576,
 'n_days': 3743,
 'n_traded_days': 918}
```

```
import itertools
import math
import numpy as np
import pandas as pd
```

```
def run_grid_search(
    backtest_fn,
    p1_values,
    p2_values,
    p3_values,
    p4_values,
    p5_values,
    verbose=True,
):
    """
    backtest_fn: callable like backtest_with_hyperparameters(p1, p2) -> dict
    p1_values, p2_values: iterables of parameter candidates
    """
    rows = []
    for p1, p2, p3, p4, p5 in itertools.product(p1_values, p2_values, p3_values, p4_-
values, p5_values):
        try:
            res = backtest_fn(p1, p2, p3, p4, p5)  # expected to re-
turn a dict with keys shown below
            # Ensure required keys exist (coerce missing to NaN)
            record = {
                "param1": p1,
                "param2": p2,
                "param3": p3,
                "param4": p4,
                "param5": p5,
                "total_return": res.get("total_return", np.nan),
                "irr_annual": res.get("irr_annual", np.nan),
                "vol_annual": res.get("vol_annual", np.nan),
                "sharpe": res.get("sharpe", np.nan),
                "hit_ratio": res.get("hit_ratio", np.nan),
                "mdd": res.get("mdd", np.nan),
                "n_days": res.get("n_days", np.nan),
                "n_traded_days": res.get("n_traded_days", np.nan),
                "error": None,
            }
        except Exception as e:
            record = {
                "param1": p1,
                "param2": p2,
                "param3": p3,
                "param4": p4,
                "param5": p5,
                "total_return": np.nan,
                "irr_annual": np.nan,
                "vol_annual": np.nan,
                "sharpe": np.nan,
```

```
                "hit_ratio": np.nan,
                "mdd": np.nan,
                "n_days": np.nan,
                "n_traded_days": np.nan,
                "error": str(e),
            }
        rows.append(record)
        print(record)

    df = pd.DataFrame(rows)

    # Keep only successful runs with finite metrics
    ok = df[df["error"].isna()].copy()
    for c in ["total_return", "sharpe"]:
        ok = ok[np.isfinite(ok[c])]

    if ok.empty:
        if verbose:
            print("No successful runs with finite metrics.")
        # Still return the full df for debugging
        return {
            "all_results": df.sort_values(["param1", "param2", "param3", "param4", "p
aram5"]).reset_index(drop=True),
            "best_total_return": None,
            "best_sharpe": None,
        }

    # Best by total return
    idx_ret = ok["total_return"].idxmax()
    best_ret = ok.loc[idx_ret].to_dict()

    # Best by Sharpe
    idx_sharpe = ok["sharpe"].idxmax()
    best_sharpe = ok.loc[idx_sharpe].to_dict()

    if verbose:
        print("=== Best by total_return ===")
        print(
            f"param1={best_ret['param1']}, param2={best_ret['param2']}, param3=
{best_ret['param3']}, param4={best_ret['param4']}, param5={best_ret['param5']}"
            f"total_return={best_ret['total_return']:.6f}, "
            f"sharpe={best_ret['sharpe']:.6f}, vol_annual={best_ret['vol_annu-
al']:.6f}, "
            f"hit_ratio={best_ret['hit_ratio']:.4f}, mdd={best_ret['mdd']:.4f}, "
            f"n_days={int(best_ret['n_days']) if not math.isnan(best_ret['n_-
days']) else 'NA'}, "
            f"n_traded_days={int(best_ret['n_traded_days']) if not math.is-
```

```
nan(best_ret['n_traded_days']) else 'NA'}"
        )
        print("\n=== Best by sharpe ===")
        print(
            f"param1={best_sharpe['param1']}, param2={best_sharpe['param2']}, param3=
{best_sharpe['param3']}, param4={best_sharpe['param4']}, param5=
{best_sharpe['param5']}"
            f"sharpe={best_sharpe['sharpe']:.6f}, "
            f"total_return={best_sharpe['total_return']:.6f}, vol_annual=
{best_sharpe['vol_annual']:.6f}, "
            f"hit_ratio={best_sharpe['hit_ratio']:.4f}, mdd=
{best_sharpe['mdd']:.4f}, "
            f"n_days={int(best_sharpe['n_days']) if not math.isnan(best_sharpe['n_-
days']) else 'NA'}, "
            f"n_traded_days={int(best_sharpe['n_traded_days']) if not math.is-
nan(best_sharpe['n_traded_days']) else 'NA'}"
        )

    return {
        "all_results": df.sort_values(["param1", "param2", 'param3', 'param4', 'param
5']).reset_index(drop=True),
        "best_total_return": best_ret,
        "best_sharpe": best_sharpe,
    }
```

```
p1_grid = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4, 1.5]
p2_grid = [0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009, 0.01, 0.011
, 0.012, 0.013, 0.014, 0.015]
p3_grid = [14]
p4_grid = [14]
p5_grid = [0.02]

results = run_grid_search(backtest_with_hyperparameters, p1_grid, p2_grid, p3_-
grid, p4_grid, p5_grid,  verbose=True)

# Access the DataFrame of all runs:
all_runs_df = results["all_results"]
# Top 10 by total_return:
top10_return = all_runs_df.sort_values("total_return", ascending=False).head(10)
# Top 10 by Sharpe:
top10_sharpe = all_runs_df.sort_values("sharpe", ascending=False).head(10)

all_runs_df
```

```
{'param1': 0.1, 'param2': 0.001, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.002, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.003, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.004, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.005, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.006, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.007, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.008, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.009, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.01, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_ret
{'param1': 0.1, 'param2': 0.011, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.012, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.013, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.014, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.015, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.001, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.002, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.003, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.004, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.005, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
```

	param1	param2	param3	param4	param5	total_return	irr_annual	vol_annual	sharpe	hit_ratio	mdd	n_day
0	0.1	0.001	14	14	0.02	0.020072	0.001339	0.128107	0.074643	0.510297	-0.292869	3743
1	0.1	0.002	14	14	0.02	0.214655	0.013178	0.122256	0.168389	0.519179	-0.235324	3743
2	0.1	0.003	14	14	0.02	0.431105	0.024426	0.114290	0.268528	0.528280	-0.256866	3743
3	0.1	0.004	14	14	0.02	0.767467	0.039090	0.106772	0.412752	0.540844	-0.201978	3743
4	0.1	0.005	14	14	0.02	0.976152	0.046927	0.099077	0.512593	0.553619	-0.187821	3743
...	...	...	...	...	...	...	...	...	...	...	...	...
220	1.5	0.011	14	14	0.02	-0.046810	-0.003222	0.027559	-0.103183	0.528090	-0.149638	3743
221	1.5	0.012	14	14	0.02	0.036177	0.002396	0.022536	0.117592	0.563380	-0.110425	3743
222	1.5	0.013	14	14	0.02	0.033065	0.002193	0.019991	0.119700	0.547170	-0.092806	3743
223	1.5	0.014	14	14	0.02	-0.007387	-0.000499	0.018778	-0.017012	0.461538	-0.077169	3743
224	1.5	0.015	14	14	0.02	-0.009328	-0.000631	0.018062	-0.025708	0.483871	-0.074620	3743

225 rows × 14 columns

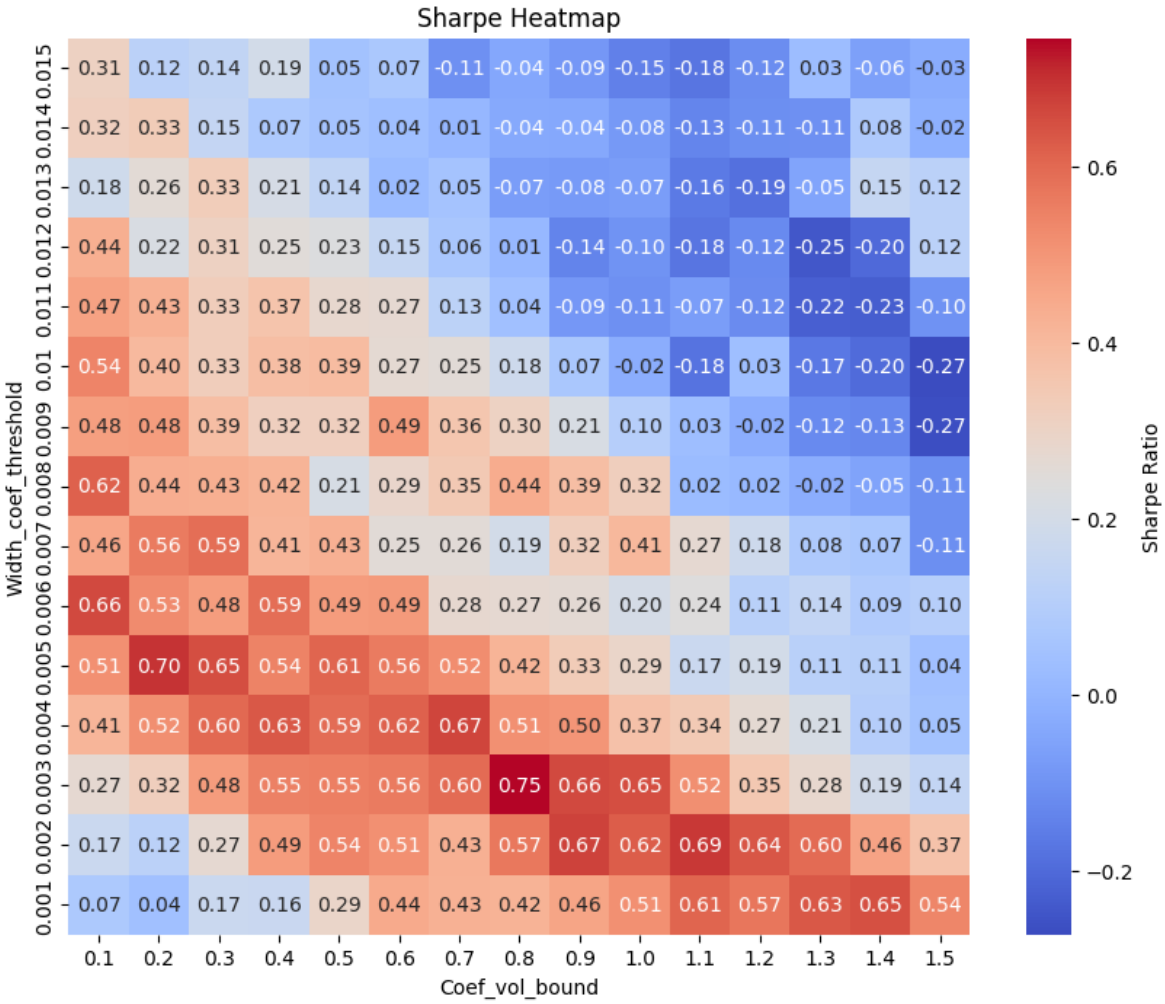
top10_return

	param1	param2	param3	param4	param5	total_return	irr_annual	vol_annual	sharpe	hit_ratio	mdd	n_days
19	0.2	0.005	14	14	0.02	1.478110	0.063003	0.094332	0.695071	0.568182	-0.166300	3743
107	0.8	0.003	14	14	0.02	1.422332	0.061375	0.084758	0.745294	0.555631	-0.209073	3743
121	0.9	0.002	14	14	0.02	1.304520	0.057818	0.089897	0.670342	0.554282	-0.218632	3743
5	0.1	0.006	14	14	0.02	1.300140	0.057683	0.091598	0.658234	0.560672	-0.182268	3743
48	0.4	0.004	14	14	0.02	1.279812	0.057051	0.095087	0.631339	0.555107	-0.202619	3743
34	0.3	0.005	14	14	0.02	1.272318	0.056816	0.091320	0.651040	0.562578	-0.197030	3743
33	0.3	0.004	14	14	0.02	1.235110	0.055642	0.098596	0.598737	0.558698	-0.191641	3743
151	1.1	0.002	14	14	0.02	1.176963	0.053771	0.080879	0.688073	0.558072	-0.175883	3743
61	0.5	0.002	14	14	0.02	1.170285	0.053553	0.107641	0.538725	0.543029	-0.218375	3743
150	1.1	0.001	14	14	0.02	1.149090	0.052857	0.091841	0.606879	0.548258	-0.228528	3743

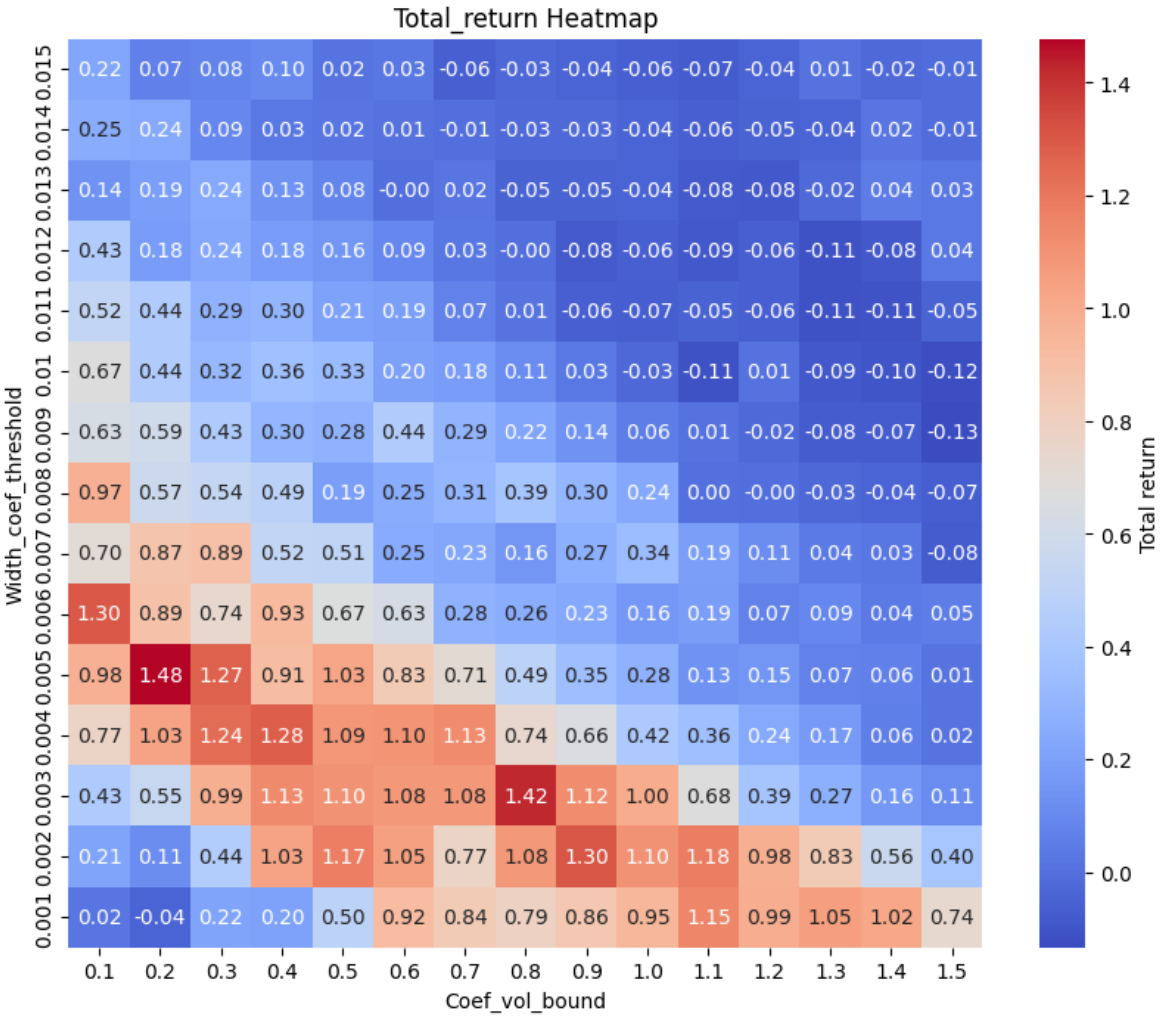
top10_sharpe

	param1	param2	param3	param4	param5	total_return	irr_annual	vol_annual	sharpe	hit_ratio	mdd	n_days
107	0.8	0.003	14	14	0.02	1.422332	0.061375	0.084758	0.745294	0.555631	-0.209073	3743
19	0.2	0.005	14	14	0.02	1.478110	0.063003	0.094332	0.695071	0.568182	-0.166300	3743
151	1.1	0.002	14	14	0.02	1.176963	0.053771	0.080879	0.688073	0.558072	-0.175883	3743
121	0.9	0.002	14	14	0.02	1.304520	0.057818	0.089897	0.670342	0.554282	-0.218632	3743
93	0.7	0.004	14	14	0.02	1.131222	0.052265	0.081389	0.666776	0.564394	-0.242721	3743
122	0.9	0.003	14	14	0.02	1.124300	0.052035	0.082022	0.659618	0.564989	-0.216565	3743
5	0.1	0.006	14	14	0.02	1.300140	0.057683	0.091598	0.658234	0.560672	-0.182268	3743
34	0.3	0.005	14	14	0.02	1.272318	0.056816	0.091320	0.651040	0.562578	-0.197030	3743
137	1.0	0.003	14	14	0.02	0.998955	0.047736	0.076127	0.650663	0.565506	-0.159539	3743
195	1.4	0.001	14	14	0.02	1.017492	0.048387	0.077610	0.647736	0.560976	-0.178268	3743

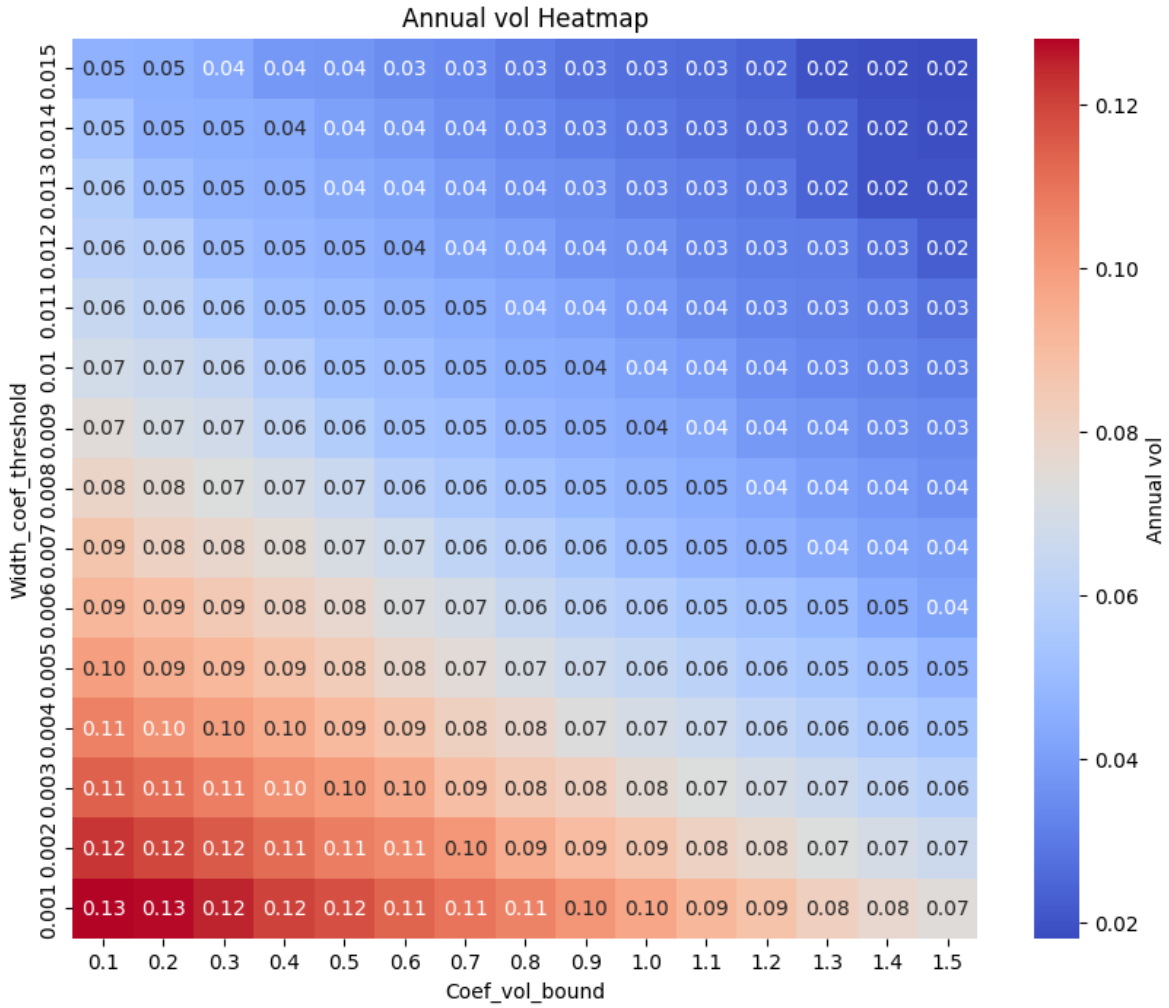
```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="sharpe")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Sharpe Ratio"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Sharpe Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Sharpe Heatmap
Width_coef_threshold
0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.01 0.011 0.012 0.013 0.014 0.015
0.31 0.12 0.14 0.19 0.05 0.07 -0.11 -0.04 -0.09 -0.15 -0.18 -0.12 0.03 -0.06 -0.03
0.32 0.33 0.15 0.07 0.05 0.04 0.01 -0.04 -0.04 -0.08 -0.13 -0.11 -0.11 0.08 -0.02
0.18 0.26 0.33 0.21 0.14 0.02 0.05 -0.07 -0.08 -0.07 -0.16 -0.19 -0.05 0.15 0.12
0.44 0.22 0.31 0.25 0.23 0.15 0.06 0.01 -0.14 -0.10 -0.18 -0.12 -0.25 -0.20 0.12
0.47 0.43 0.33 0.37 0.28 0.27 0.13 0.04 -0.09 -0.11 -0.07 -0.12 -0.22 -0.23 -0.10
0.54 0.40 0.33 0.38 0.39 0.27 0.25 0.18 0.07 -0.02 -0.18 0.03 -0.17 -0.20 -0.27
0.48 0.48 0.39 0.32 0.32 0.49 0.36 0.30 0.21 0.10 0.03 -0.02 -0.12 -0.13 -0.27
0.62 0.44 0.43 0.42 0.21 0.29 0.35 0.44 0.39 0.32 0.02 0.02 -0.02 -0.05 -0.11
0.46 0.56 0.59 0.41 0.43 0.25 0.26 0.19 0.32 0.41 0.27 0.18 0.08 0.07 -0.11
0.66 0.53 0.48 0.59 0.49 0.49 0.28 0.27 0.26 0.20 0.24 0.11 0.14 0.09 0.10
0.51 0.70 0.65 0.54 0.61 0.56 0.52 0.42 0.33 0.29 0.17 0.19 0.11 0.11 0.04
0.41 0.52 0.60 0.63 0.59 0.62 0.67 0.51 0.50 0.37 0.34 0.27 0.21 0.10 0.05
0.27 0.32 0.48 0.55 0.55 0.56 0.60 0.75 0.66 0.65 0.52 0.35 0.28 0.19 0.14
0.17 0.12 0.27 0.49 0.54 0.51 0.43 0.57 0.67 0.62 0.69 0.64 0.60 0.46 0.37
0.07 0.04 0.17 0.16 0.29 0.44 0.43 0.42 0.46 0.51 0.61 0.57 0.63 0.65 0.54
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
0.6
0.4
0.2
0.0
−0.2
Sharpe Ratio

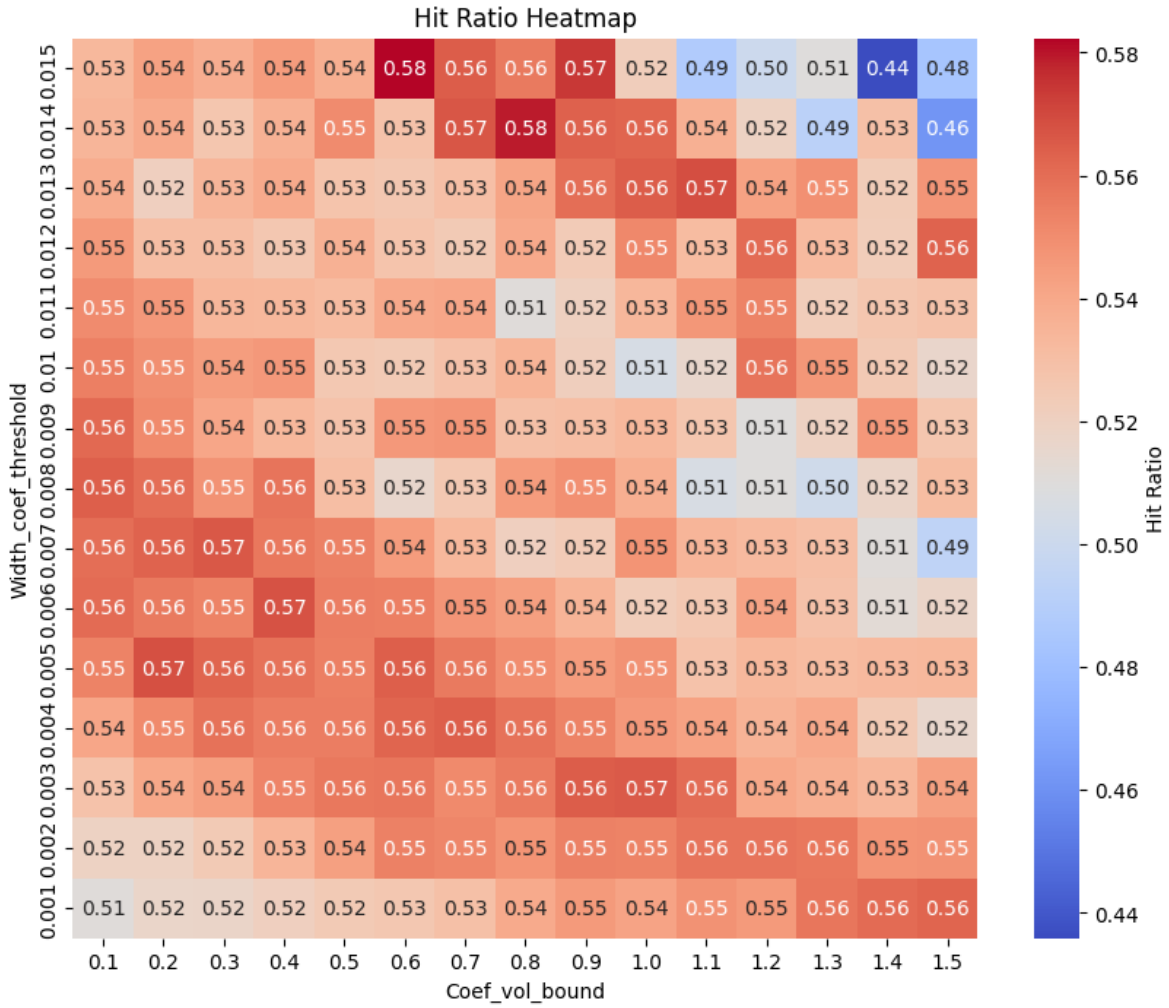
```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="total_re-
turn")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Total return"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Total_return Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Total_return Heatmap
Width_coef_threshold
0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.01 0.011 0.012 0.013 0.014 0.015
0.22 0.07 0.08 0.10 0.02 0.03 -0.06 -0.03 -0.04 -0.06 -0.07 -0.04 0.01 -0.02 -0.01
0.25 0.24 0.09 0.03 0.02 0.01 -0.01 -0.03 -0.03 -0.04 -0.06 -0.05 -0.04 0.02 -0.01
0.14 0.19 0.24 0.13 0.08 -0.00 0.02 -0.05 -0.05 -0.04 -0.08 -0.08 -0.02 0.04 0.03
0.43 0.18 0.24 0.18 0.16 0.09 0.03 -0.00 -0.08 -0.06 -0.09 -0.06 -0.11 -0.08 0.04
0.52 0.44 0.29 0.30 0.21 0.19 0.07 0.01 -0.06 -0.07 -0.05 -0.06 -0.11 -0.11 -0.05
0.67 0.44 0.32 0.36 0.33 0.20 0.18 0.11 0.03 -0.03 -0.11 0.01 -0.09 -0.10 -0.12
0.63 0.59 0.43 0.30 0.28 0.44 0.29 0.22 0.14 0.06 0.01 -0.02 -0.08 -0.07 -0.13
0.97 0.57 0.54 0.49 0.19 0.25 0.31 0.39 0.30 0.24 0.00 -0.00 -0.03 -0.04 -0.07
0.70 0.87 0.89 0.52 0.51 0.25 0.23 0.16 0.27 0.34 0.19 0.11 0.04 0.03 -0.08
1.30 0.89 0.74 0.93 0.67 0.63 0.28 0.26 0.23 0.16 0.19 0.07 0.09 0.04 0.05
0.98 1.48 1.27 0.91 1.03 0.83 0.71 0.49 0.35 0.28 0.13 0.15 0.07 0.06 0.01
0.77 1.03 1.24 1.28 1.09 1.10 1.13 0.74 0.66 0.42 0.36 0.24 0.17 0.06 0.02
0.43 0.55 0.99 1.13 1.10 1.08 1.08 1.42 1.12 1.00 0.68 0.39 0.27 0.16 0.11
0.21 0.11 0.44 1.03 1.17 1.05 0.77 1.08 1.30 1.10 1.18 0.98 0.83 0.56 0.40
0.02 -0.04 0.22 0.20 0.50 0.92 0.84 0.79 0.86 0.95 1.15 0.99 1.05 1.02 0.74
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
Total return
1.4 1.2 1.0 0.8 0.6 0.4 0.2 0.0

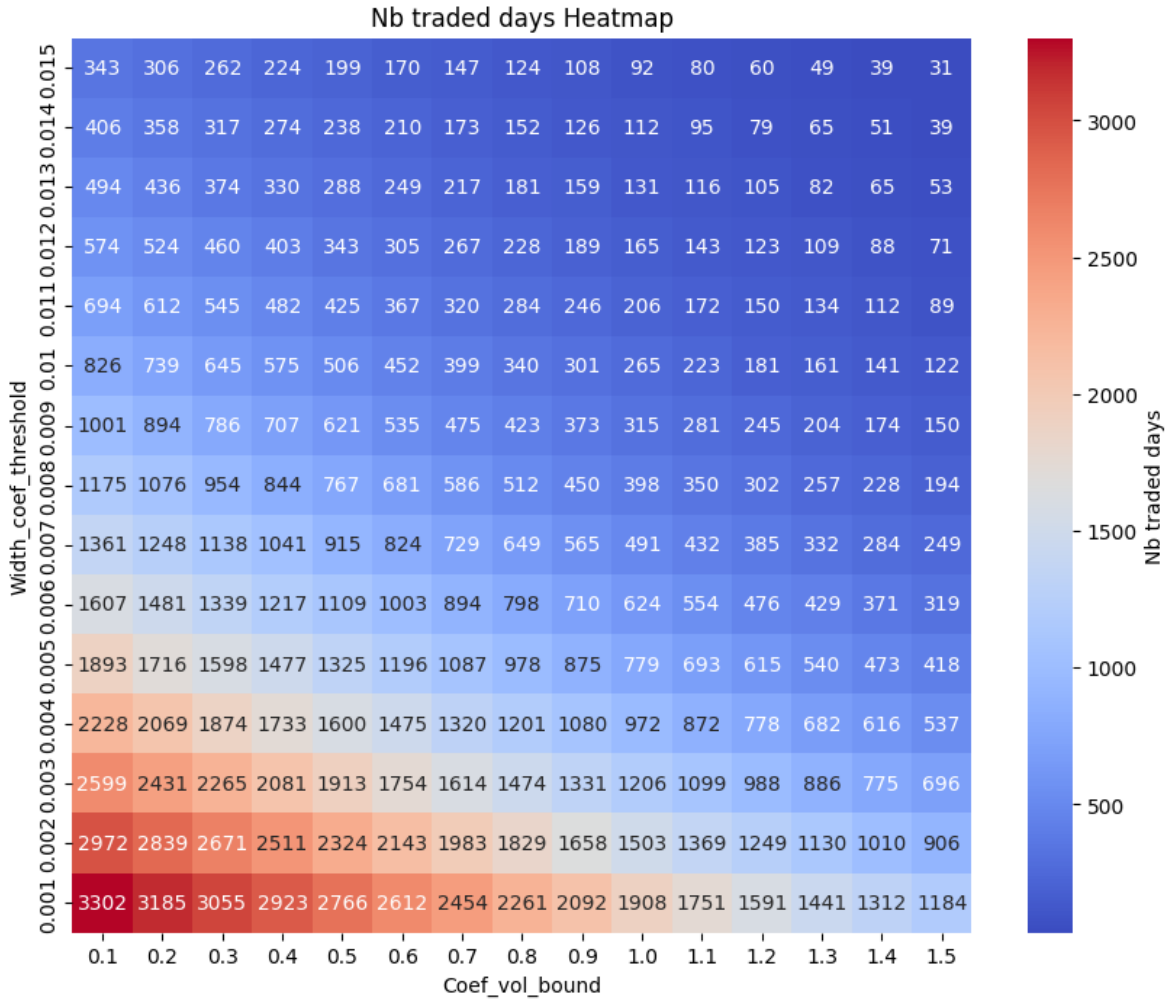
```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="vol_annual"
)

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Annual vol"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Annual vol Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Annual vol Heatmap
Width_coef_threshold
0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.01 0.011 0.012 0.013 0.014 0.015
0.05 0.05 0.04 0.04 0.04 0.03 0.03 0.03 0.03 0.03 0.03 0.02 0.02 0.02 0.02
0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.03 0.03 0.02 0.02 0.02
0.06 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.03 0.02 0.02 0.02
0.06 0.06 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.03 0.02
0.06 0.06 0.06 0.05 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.03
0.07 0.07 0.06 0.06 0.05 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03
0.07 0.07 0.07 0.06 0.06 0.05 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03
0.08 0.08 0.07 0.07 0.07 0.06 0.06 0.05 0.05 0.05 0.05 0.04 0.04 0.04 0.04
0.09 0.08 0.08 0.08 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.05 0.04 0.04 0.04
0.09 0.09 0.09 0.08 0.08 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.05 0.05 0.04
0.10 0.09 0.09 0.09 0.08 0.08 0.07 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.05
0.11 0.10 0.10 0.10 0.09 0.09 0.08 0.08 0.07 0.07 0.07 0.06 0.06 0.06 0.05
0.11 0.11 0.11 0.10 0.10 0.10 0.09 0.08 0.08 0.08 0.07 0.07 0.07 0.06 0.06
0.12 0.12 0.12 0.11 0.11 0.11 0.10 0.09 0.09 0.09 0.08 0.08 0.07 0.07 0.07
0.13 0.13 0.12 0.12 0.12 0.11 0.11 0.11 0.10 0.10 0.09 0.09 0.08 0.08 0.07
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
Annual vol
0.12
0.10
0.08
0.06
0.04
0.02

```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="hit_ratio")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Hit Ratio"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Hit Ratio Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Hit Ratio Heatmap
Width_coef_threshold
0.001 0.51 0.52 0.52 0.52 0.52 0.53 0.53 0.54 0.55 0.54 0.55 0.55 0.56 0.56 0.56
0.002 0.52 0.52 0.52 0.53 0.54 0.55 0.55 0.55 0.55 0.55 0.56 0.56 0.56 0.55 0.55
0.003 0.53 0.54 0.54 0.55 0.56 0.56 0.55 0.56 0.56 0.57 0.56 0.54 0.54 0.53 0.54
0.004 0.54 0.55 0.56 0.56 0.56 0.56 0.56 0.56 0.55 0.55 0.54 0.54 0.54 0.52 0.52
0.005 0.55 0.57 0.56 0.56 0.55 0.56 0.56 0.55 0.55 0.55 0.53 0.53 0.53 0.53 0.53
0.006 0.56 0.56 0.55 0.57 0.56 0.55 0.55 0.54 0.54 0.52 0.53 0.54 0.53 0.51 0.52
0.007 0.56 0.56 0.57 0.56 0.55 0.54 0.53 0.52 0.52 0.55 0.53 0.53 0.53 0.51 0.49
0.008 0.56 0.56 0.55 0.56 0.53 0.52 0.53 0.54 0.55 0.54 0.51 0.51 0.50 0.52 0.53
0.009 0.56 0.55 0.54 0.53 0.53 0.55 0.55 0.53 0.53 0.53 0.53 0.51 0.52 0.55 0.53
0.01 0.55 0.55 0.54 0.55 0.53 0.52 0.53 0.54 0.52 0.51 0.52 0.56 0.55 0.52 0.52
0.011 0.55 0.55 0.53 0.53 0.53 0.54 0.54 0.51 0.52 0.53 0.55 0.55 0.52 0.53 0.53
0.012 0.55 0.53 0.53 0.53 0.54 0.53 0.52 0.54 0.52 0.55 0.53 0.56 0.53 0.52 0.56
0.013 0.54 0.52 0.53 0.54 0.53 0.53 0.53 0.54 0.56 0.56 0.57 0.54 0.55 0.52 0.55
0.014 0.53 0.54 0.53 0.54 0.55 0.53 0.57 0.58 0.56 0.56 0.54 0.52 0.49 0.53 0.46
0.015 0.53 0.54 0.54 0.54 0.54 0.58 0.56 0.56 0.57 0.52 0.49 0.50 0.51 0.44 0.48
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
Hit Ratio
0.58
0.56
0.54
0.52
0.50
0.48
0.46
0.44

```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="n_traded_-
days")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".0f",
    cbar_kws={"label": "Nb traded days"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Nb traded days Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```




```
### stop same band + vwap
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math
import datetime as dt
import matplotlib.dates as mdates
```

```
### import the data
df = pd.read_csv('/data/workspace_files/nasdaq.csv')
df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

# (Optional) drop bad rows
df = df.dropna(subset=['Time'])
```

```
# Convert UTC -> New York (handles DST)
df['Time'] = df['Time'].dt.tz_convert('America/New_York')

# Build hour/day AFTER conversion
df['hour'] = df['Time'].dt.strftime('%H:%M')
df['day']  = df['Time'].dt.date

# Keep only US RTH 09:30-16:00 ET
market_open, market_close = dt.time(9, 30), dt.time(16, 0)
df = df[df['Time'].dt.time.between(market_open, market_close)]

# Sanity check (optional)
tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()
print("Min/Max ET times in df:", tmin, tmax)  # expect between 09:30 and 16:00


### order by symbol, hour and day
df = df.sort_values(['symbol', 'hour', 'day'])

### price
df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


### opening price
df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


# settling price and previous day settling price
daily_settle = (
    df.sort_values(['symbol','day','Time'])
      .groupby(['symbol','day'])['price']
      .last()
      .rename('settl_price')
)

prev_settle = (
    daily_settle.groupby(level='symbol')
                .shift(1)
                .rename('settl_price_previous')
)

df = df.join(daily_settle, on=['symbol','day'])
df = df.join(prev_settle,   on=['symbol','day'])


### move since opening
df["move"] = np.where(
```

```
    df["open_price"] > 0,
    np.abs(df["price"] / df["open_price"] - 1),
    np.nan
)


### compute mean of move over the last period
n = 14
df['sigma_observed'] = (
    df.groupby(['symbol', 'hour'])['move']
      .transform(lambda x: x.rolling(n, min_periods=n).mean())
)


### compute upper bound
df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1 + d
f['sigma_observed'])


### compute lower bound
df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1 -
 df['sigma_observed'])


### get return
df['return'] = df['settl_price']/df['open_price'] - 1


### get variance onf returns among the last 14 days
daily = (df.sort_values(['symbol','day'])
           .groupby(['symbol','day'], as_index=False)['return'].last())

daily['ret_std_14d'] = (
    daily.groupby('symbol')['return']
         .transform(lambda s: s.rolling(14, min_periods=14).std())
)

df = df.merge(daily[['symbol','day','ret_std_14d']], on=['symbol','day'], how='left')


df = df.sort_values(['symbol', 'day', 'hour'])
# df[df['expiry_order']==1]
df
```

```
Min/Max ET times in df: 09:33:00 16:00:00
```

	Time	symbol	expiry_order	bid_px_00	ask_px_00	vwap_cum	hour	day	price	open_price	settl_pri
0	2019-03-15 09:33:00-04:00	NQH0	1	NaN	NaN	NaN	09:33	2019-03-15	NaN	NaN	NaN
188	2019-03-15 09:48:00-04:00	NQH0	1	NaN	NaN	7303.251423	09:48	2019-03-15	NaN	NaN	NaN
376	2019-03-15 10:03:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:03	2019-03-15	NaN	NaN	NaN
564	2019-03-15 10:18:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:18	2019-03-15	NaN	NaN	NaN
752	2019-03-15 10:33:00-04:00	NQH0	1	NaN	NaN	7314.065510	10:33	2019-03-15	NaN	NaN	NaN
...	...	...	...	...	...	...	...	...	...	...	...
266952	2019-12-19 15:03:00-05:00	NQZ9	3	8632.25	8632.50	8613.137849	15:03	2019-12-19	8632.375	8604.125	8640.62
267140	2019-12-19 15:18:00-05:00	NQZ9	3	8628.75	8629.25	8613.525157	15:18	2019-12-19	8629.000	8604.125	8640.62
267328	2019-12-19 15:33:00-05:00	NQZ9	3	8628.75	8629.00	8613.953049	15:33	2019-12-19	8628.875	8604.125	8640.62
267516	2019-12-19 15:48:00-05:00	NQZ9	3	8631.75	8632.00	8614.566532	15:48	2019-12-19	8631.875	8604.125	8640.62
267704	2019-12-19 16:00:00-05:00	NQZ9	3	8640.50	8640.75	8617.831876	16:00	2019-12-19	8640.625	8604.125	8640.62

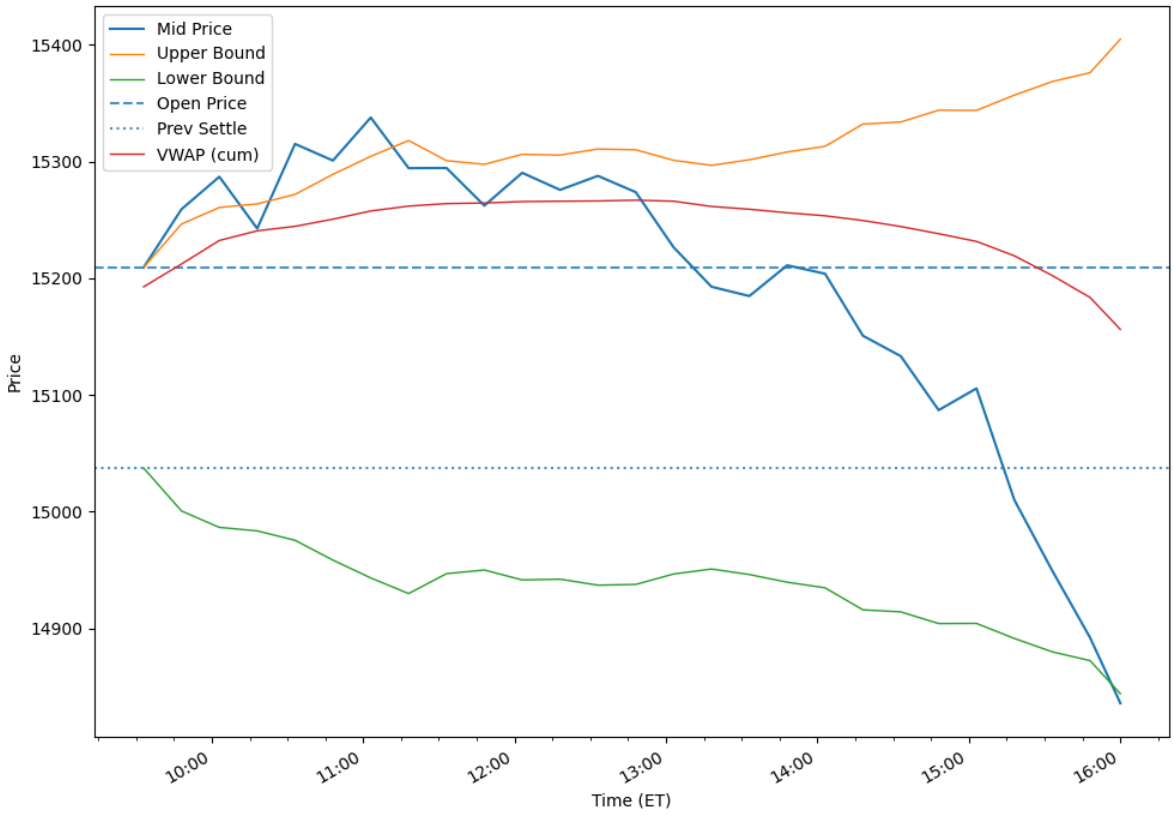
267705 rows × 18 columns

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
            .sort_values('Time')
            .copy())
    backtest_df = backtest_df.sort_values('hour')

    bid_list = backtest_df['bid_px_00'].values
    ask_list = backtest_df['ask_px_00'].values
    upper_bound = backtest_df['upper_bound'].values
    lower_bound = backtest_df['lower_bound'].values
    vwap_list = backtest_df['vwap_cum'].values
    hour = backtest_df['hour'].values

    if not (len(bid_list) == len(ask_list) == len(vwap_list) == len(hour) == len(up-
per_bound) == len(lower_bound)):
        return initial_cash
    elif any(pd.isna(x) for x in bid_list + ask_list + vwap_list + upper_bound + low-
er_bound):
        return initial_cash



    position = 0
    sigma_target = 0.02
    total_shares_to_hold = math.floor(initial_cash / backtest_df['open_price'].val-
ues[0])


    count = 0
    for i in range(1, len(bid_list) - 1):
        if ask_list[i] > max(upper_bound[i], vwap_list[i]) and count==0:
            count == 1
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * ask_list[i]
                print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str} a
t {hour[i]}")

        elif bid_list[i] < min(vwap_list[i], lower_bound[i]) and count == 0:
            count == 1
            # we must be short and sell more than what we already have
```

```
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * bid_list[i]
                print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str} at
 {hour[i]}")

        elif bid_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * bid_list[i]
                print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {hour
[i]}")
                position = 0
            elif position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * ask_list[i]
                print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str} a
t {hour[i]}")
                position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
        initial_cash += position * bid_list[-1]
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * ask_list[-1]
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
# Ensure Time is tz-aware ET
df["Time"] = pd.to_datetime(df["Time"], utc=True).dt.tz_convert("America/New_York")

# Define day as plain date (no tz needed)
day_str = "2022-01-20"
day = pd.to_datetime(day_str).date()

# Recompute day column in ET if not already done
df["day"] = df["Time"].dt.date

# Filter front-month contract for that day
one = df[(df['expiry_order'] == 1) & (df['day'] == day)].copy()
one = one.sort_values('Time')
```

```
# Plot
fig, ax = plt.subplots(figsize=(10, 7))
x = one['Time']

ax.plot(x, one['price'], label='Mid Price')
if 'upper_bound' in one and one['upper_bound'].notna().any():
    ax.plot(x, one['upper_bound'], label='Upper Bound', linewidth=1)
if 'lower_bound' in one and one['lower_bound'].notna().any():
    ax.plot(x, one['lower_bound'], label='Lower Bound', linewidth=1)

if 'open_price' in one and one['open_price'].notna().any():
    ax.axhline(one['open_price'].iloc[0], linestyle='--', alpha=0.8, la-
bel='Open Price')
if 'settl_price_previous' in one and one['settl_price_previous'].notna().any():
    ax.axhline(one['settl_price_previous'].iloc[0], linestyle=':', alpha=0.8, label='
Prev Settle')

if 'vwap_cum' in one.columns and one['vwap_cum'].notna().any():
    ax.plot(x, one['vwap_cum'], label='VWAP (cum)', linewidth=1)

# Tidy x-axis
import matplotlib.dates as mdates
from zoneinfo import ZoneInfo


tz = ZoneInfo("America/New_York")

ax.xaxis.set_major_locator(mdates.HourLocator(tz=tz))
ax.xaxis.set_major_formatter(mdates.DateFormatter('%H:%M', tz=tz))
ax.xaxis.set_minor_locator(mdates.MinuteLocator(byminute=[0,15,30,45], tz=tz))

fig.autofmt_xdate()

ax.set_xlabel("Time (ET)")
ax.set_ylabel("Price")
ax.legend()
plt.tight_layout()
plt.show()
```

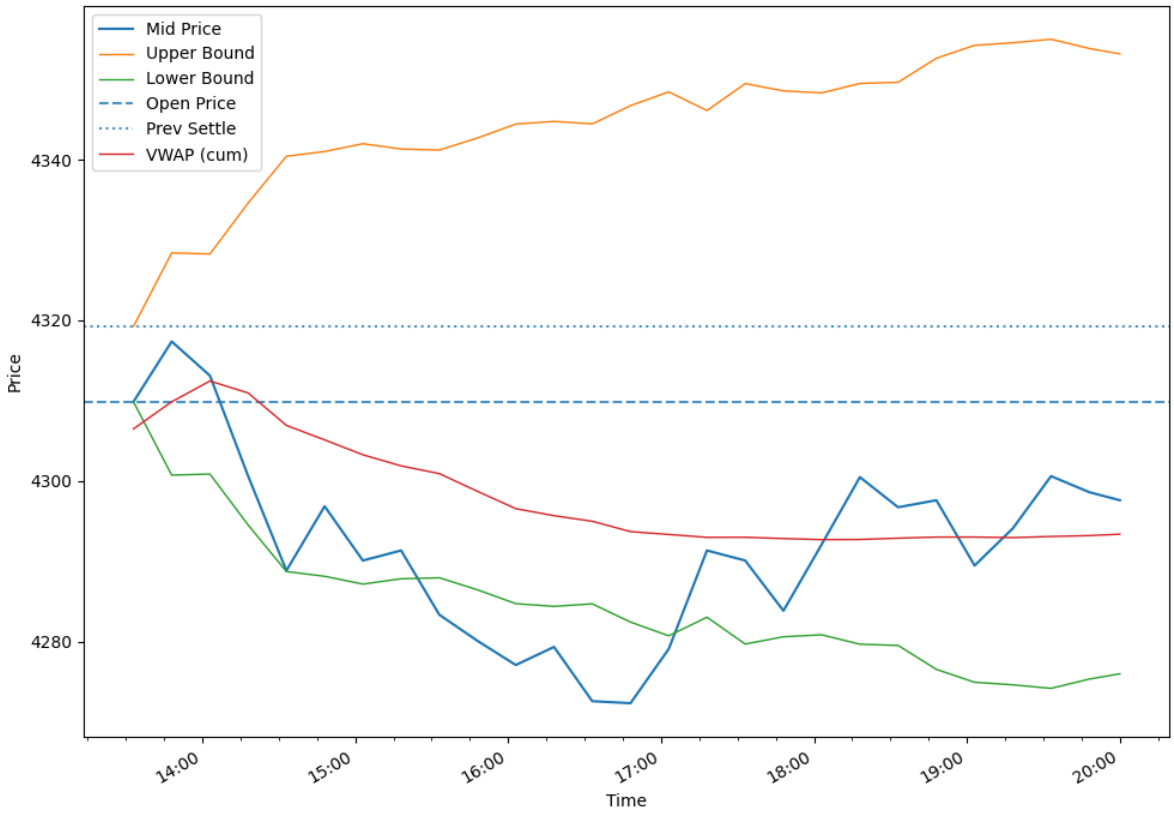



```
backtest_one_day(df, "2022-01-20", 1, 100000)
```

```
Bought 6 shares at 15259.5 on 2022-01-20 at 09:48
Sold 6 shares at 15242.25 on 2022-01-20 at 10:18
Bought 6 shares at 15315.5 on 2022-01-20 at 10:33
Sold 6 shares at 15294.0 on 2022-01-20 at 11:18
99767.5
```

```
day_str = "2015-10-06"
day = pd.to_datetime(day_str).date()

print(backtest_one_day(df, day_str, 1, 100000))
one = df[(df['expiry_order'] == 1) & (df['day'] == day)].copy()
one = one.sort_values('Time')

fig, ax = plt.subplots(figsize=(10, 7))

x = one['Time']

ax.plot(x, one['price'], label='Mid Price')
if 'upper_bound' in one and one['upper_bound'].notna().any():
    ax.plot(x, one['upper_bound'], label='Upper Bound', linewidth=1)
if 'lower_bound' in one and one['lower_bound'].notna().any():
    ax.plot(x, one['lower_bound'], label='Lower Bound', linewidth=1)

# Horizontal lines for open and previous settlement
if 'open_price' in one and one['open_price'].notna().any():
    ax.axhline(one['open_price'].iloc[0], linestyle='--', alpha=0.8, la-
bel='Open Price')
if 'settl_price_previous' in one and one['settl_price_previous'].notna().any():
    ax.axhline(one['settl_price_previous'].iloc[0], linestyle=':', alpha=0.8, label='
Prev Settle')

# Optional: VWAP
if 'vwap_cum' in one.columns and one['vwap_cum'].notna().any():
    ax.plot(x, one['vwap_cum'], label='VWAP (cum)', linewidth=1)

# Tidy x-axis: show one tick per hour, minor ticks at 15-min
ax.xaxis.set_major_locator(mdates.HourLocator())
ax.xaxis.set_major_formatter(mdates.DateFormatter('%H:%M'))
ax.xaxis.set_minor_locator(mdates.MinuteLocator(byminute=[0,15,30,45]))
fig.autofmt_xdate()
ax.set_xlabel("Time")
ax.set_ylabel("Price")
ax.legend()
plt.tight_layout()
plt.show()
```

```
Sold 23 shares at 4288.75 on 2015-10-06 at 10:33
Bought 23 shares at 4297.0 on 2015-10-06 at 10:48
Sold 23 shares at 4283.25 on 2015-10-06 at 11:33
Bought 23 shares at 4291.5 on 2015-10-06 at 13:18
99620.5
```



Backtest on all day since 2011

```python
def backtest(backtest_df, rank_to_expiry, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        print(day_ts)
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))
```

```
results = backtest(df, 1)
results
```

```
2010-06-07 00:00:00-04:00
2010-06-08 00:00:00-04:00
2010-06-09 00:00:00-04:00
2010-06-10 00:00:00-04:00
2010-06-11 00:00:00-04:00
2010-06-14 00:00:00-04:00
2010-06-15 00:00:00-04:00
2010-06-16 00:00:00-04:00
2010-06-17 00:00:00-04:00
2010-06-18 00:00:00-04:00
2010-06-21 00:00:00-04:00
2010-06-22 00:00:00-04:00
2010-06-23 00:00:00-04:00
2010-06-24 00:00:00-04:00
Sold 53 shares at 1855.0 on 2010-06-24 00:00:00-04:00 at 10:03
Bought 53 shares at 1857.5 on 2010-06-24 00:00:00-04:00 at 10:33
Sold 53 shares at 1853.5 on 2010-06-24 00:00:00-04:00 at 10:48
Bought 53 shares at 1856.5 on 2010-06-24 00:00:00-04:00 at 12:03
Sold 53 shares at 1853.75 on 2010-06-24 00:00:00-04:00 at 12:18
Bought 53 shares at 1859.0 on 2010-06-24 00:00:00-04:00 at 13:18
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.00	100000.00	0.00	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	210540.00	212072.25	1532.25	0.007278	True
3739	2024-12-26 00:00:00-05:00	212072.25	212072.25	0.00	0.000000	False
3740	2024-12-27 00:00:00-05:00	212072.25	212601.00	528.75	0.002493	True
3741	2024-12-30 00:00:00-05:00	212601.00	212875.50	274.50	0.001291	True
3742	2024-12-31 00:00:00-05:00	212875.50	211046.25	-1829.25	-0.008593	True

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
```

```
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

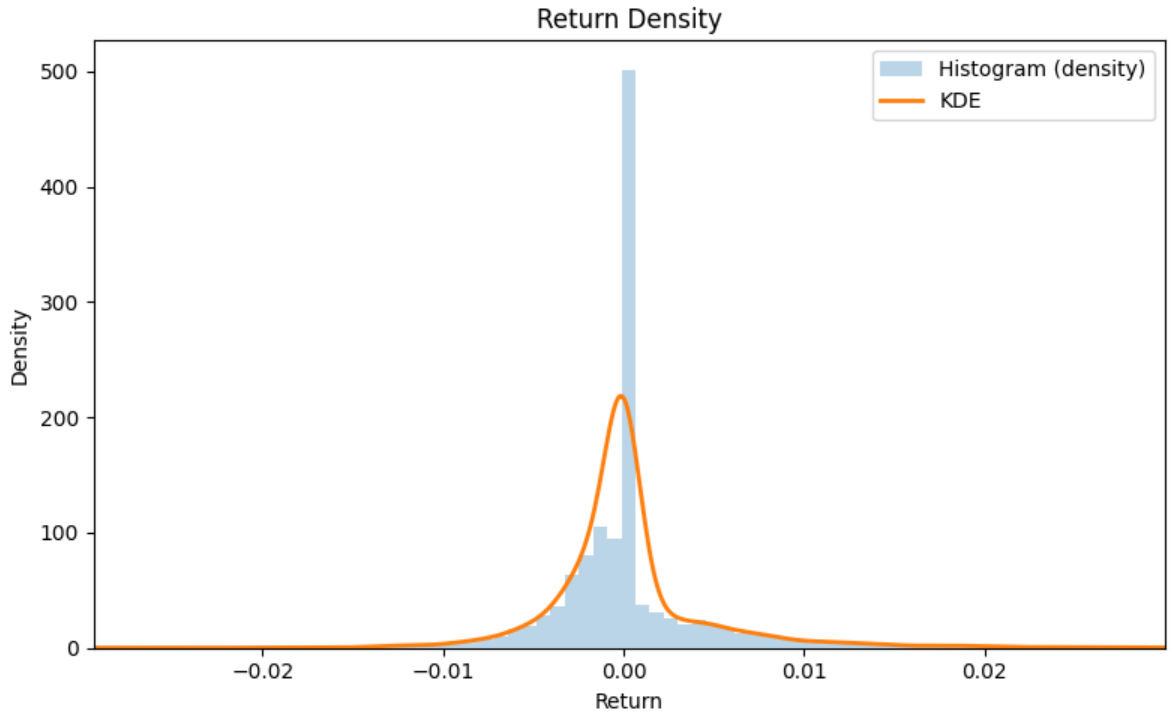
```
{'total_return': 1.1104625000000001,
 'irr_annual': 0.05157182978947472,
 'vol_annual': 0.07804053072698804,
 'sharpe': 0.6832862922827433,
 'hit_ratio': 0.38930348258706465,
 'mdd': -0.1900108014561963,
 'n_days': 3743,
 'n_traded_days': 2412}
```

```
results.sort_values('return')
```

	date	start_cash	end_cash	pnl	return	traded
344	2011-10-04 00:00:00-04:00	94374.00	90515.25	-3858.75	-0.040888	True
2506	2020-03-18 00:00:00-04:00	151446.50	146228.00	-5218.50	-0.034458	True
2499	2020-03-09 00:00:00-04:00	162862.25	157647.25	-5215.00	-0.032021	True
2507	2020-03-19 00:00:00-04:00	146228.00	141753.00	-4475.00	-0.030603	True
2984	2022-01-24 00:00:00-05:00	156325.00	151804.00	-4521.00	-0.028921	True
...	...	...	...	...	...	...
2137	2018-10-10 00:00:00-04:00	137493.75	141593.25	4099.50	0.029816	True
2190	2018-12-26 00:00:00-05:00	156958.25	161742.25	4784.00	0.030479	True
3205	2022-11-30 00:00:00-05:00	173744.00	179500.25	5756.25	0.033131	True
1964	2018-02-08 00:00:00-05:00	124831.50	129030.00	4198.50	0.033633	True
2147	2018-10-24 00:00:00-04:00	138440.50	143632.25	5191.75	0.037502	True

3743 rows × 6 columns

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Clean the series
s = pd.Series(results['return'], dtype='float64')
s = s.replace([np.inf, -np.inf], np.nan).dropna()

fig, ax = plt.subplots(figsize=(8, 5))

# Density histogram
ax.hist(s, bins=100, density=True, alpha=0.3, label='Histogram (density)')

# KDE overlay (if scipy available)
try:
    from scipy.stats import gaussian_kde
    xs = np.linspace(s.quantile(0.001), s.quantile(0.999), 512)
    kde = gaussian_kde(s)
    ax.plot(xs, kde(xs), linewidth=2, label='KDE')
except Exception:
    # If scipy isn't installed, just skip the KDE
    pass

# Nice viewing window (doesn't affect density calc)
qlo, qhi = s.quantile([0.001, 0.999])
ax.set_xlim(qlo, qhi)

ax.set_xlabel('Return')
ax.set_ylabel('Density')
ax.set_title('Return Density')
ax.legend()
plt.tight_layout()
plt.show()
```




```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math

import datetime as dt

import matplotlib.dates as mdates
```

```
### import the data
df = pd.read_csv('/data/workspace_files/nasdaq.csv')
df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

# (Optional) drop bad rows
df = df.dropna(subset=['Time'])

# Convert UTC -> New York (handles DST)
df['Time'] = df['Time'].dt.tz_convert('America/New_York')

# Build hour/day AFTER conversion
df['hour'] = df['Time'].dt.strftime('%H:%M')
df['day']  = df['Time'].dt.date
```

```
# Keep only US RTH 09:30-16:00 ET
market_open, market_close = dt.time(9, 30), dt.time(16, 0)
df = df[df['Time'].dt.time.between(market_open, market_close)]

# Sanity check (optional)
tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()


### order by symbol, hour and day
df = df.sort_values(['symbol', 'hour', 'day'])

### price
df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


### opening price
df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


# settling price and previous day settling price
daily_settle = (
    df.sort_values(['symbol','day','Time'])
      .groupby(['symbol','day'])['price']
      .last()
      .rename('settl_price')
)

prev_settle = (
    daily_settle.groupby(level='symbol')
                .shift(1)
                .rename('settl_price_previous')
)

df = df.join(daily_settle, on=['symbol','day'])
df = df.join(prev_settle,   on=['symbol','day'])


### move since opening
df["move"] = np.where(
    df["open_price"] > 0,
    np.abs(df["price"] / df["open_price"] - 1),
    np.nan
)


### compute mean of move over the last period
n = 14
```

```
df['sigma_observed'] = (
    df.groupby(['symbol', 'hour'])['move']
      .transform(lambda x: x.rolling(n, min_periods=n).mean())
)


### compute upper bound
df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1 + d
f['sigma_observed'])


### compute lower bound
df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1 -
 df['sigma_observed'])


### get return
df['return'] = df['settl_price']/df['open_price'] - 1


### get variance onf returns among the last 14 days
daily = (df.sort_values(['symbol','day'])
           .groupby(['symbol','day'], as_index=False)['return'].last())

daily['ret_std_14d'] = (
    daily.groupby('symbol')['return']
         .transform(lambda s: s.rolling(14, min_periods=14).std())
)

df = df.merge(daily[['symbol','day','ret_std_14d']], on=['symbol','day'], how='left')


df = df.sort_values(['symbol', 'day', 'hour'])
# df[df['expiry_order']==1]
df
```

	Time	symbol	expiry_order	bid_px_00	ask_px_00	vwap_cum	hour	day	price	open_price	settl_pri
0	2019-03-15 09:33:00-04:00	NQH0	1	NaN	NaN	NaN	09:33	2019-03-15	NaN	NaN	NaN
188	2019-03-15 09:48:00-04:00	NQH0	1	NaN	NaN	7303.251423	09:48	2019-03-15	NaN	NaN	NaN
376	2019-03-15 10:03:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:03	2019-03-15	NaN	NaN	NaN
564	2019-03-15 10:18:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:18	2019-03-15	NaN	NaN	NaN
752	2019-03-15 10:33:00-04:00	NQH0	1	NaN	NaN	7314.065510	10:33	2019-03-15	NaN	NaN	NaN
...	...	...	...	...	...	...	...	...	...	...	...
266952	2019-12-19 15:03:00-05:00	NQZ9	3	8632.25	8632.50	8613.137849	15:03	2019-12-19	8632.375	8604.125	8640.62
267140	2019-12-19 15:18:00-05:00	NQZ9	3	8628.75	8629.25	8613.525157	15:18	2019-12-19	8629.000	8604.125	8640.62
267328	2019-12-19 15:33:00-05:00	NQZ9	3	8628.75	8629.00	8613.953049	15:33	2019-12-19	8628.875	8604.125	8640.62
267516	2019-12-19 15:48:00-05:00	NQZ9	3	8631.75	8632.00	8614.566532	15:48	2019-12-19	8631.875	8604.125	8640.62
267704	2019-12-19 16:00:00-05:00	NQZ9	3	8640.50	8640.75	8617.831876	16:00	2019-12-19	8640.625	8604.125	8640.62

267705 rows × 18 columns

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_target):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]

    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
ower_bound) == n) or n == 0:
        return initial_cash

    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
        np.isnan(lower_bound).any()):
```

```
        return initial_cash

    position = 0
    cash = float(initial_cash)


    for i in range(1, len(bid_list) - 1):
        if ask_list[i] > max(upper_bound[i], vwap_list[i]):
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * ask_list[i]
                # print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")

        elif bid_list[i] < min(vwap_list[i], lower_bound[i]):
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * bid_list[i]
                # print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str}
at {hour[i]}")

        elif bid_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * bid_list[i]
                # print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {ho
ur[i]}")
                position = 0
            elif position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * ask_list[i]
                # print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")
                position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
        initial_cash += position * bid_list[-1]
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * ask_list[-1]
        position = 0
```

```
    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_tar-
get)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
```

```
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization, sigma_lever-
age_period, sigma_leverage_bound, sigma_target):

    df = pd.read_csv('/data/workspace_files/nasdaq.csv')
    df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

    # (Optional) drop bad rows
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST)
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')

    # Build hour/day AFTER conversion
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date

    # Keep only US RTH 09:30-16:00 ET
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Sanity check (optional)
    tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()

    ### order by symbol, hour and day
    df = df.sort_values(['symbol', 'hour', 'day'])
```

```
    ### price
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


    ### opening price
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


    # settling price and previous day settling price
    daily_settle = (
        df.sort_values(['symbol','day','Time'])
        .groupby(['symbol','day'])['price']
        .last()
        .rename('settl_price')
    )

    prev_settle = (
        daily_settle.groupby(level='symbol')
                    .shift(1)
                    .rename('settl_price_previous')
    )

    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])


    ### move since opening
    df["move"] = np.where(
        df["open_price"] > 0,
        np.abs(df["price"] / df["open_price"] - 1),
        np.nan
    )


    ### compute mean of move over the last period
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
        .transform(lambda x: x.rolling(sigma_leverage_bound, min_periods=sigma_lever-
age_bound).mean())
    )


    ### compute upper bound
    df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1
 + coef_sigma * df['sigma_observed']) + percentage_penalization * df['open_price']
```

```
    ### compute lower bound
    df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1
 - coef_sigma * df['sigma_observed']) - percentage_penalization * df['open_price']


    ### get return
    df['return'] = df['settl_price']/df['open_price'] - 1


    ### get variance onf returns among the last 14 days
    daily = (df.sort_values(['symbol','day'])
            .groupby(['symbol','day'], as_index=False)['return'].last())

    daily[f'ret_std_14d'] = (
        daily.groupby('symbol')['return']
            .transform(lambda s: s.rolling(sigma_leverage_period, min_periods=sig-
ma_leverage_period).std())
    )

    df = df.merge(daily[['symbol','day', f'ret_std_14d']], on=['sym-
bol','day'], how='left')


    df = df.sort_values(['symbol', 'day', 'hour'])

    results = backtest(df, 1, sigma_target)

    return evaluate_strategy(results, risk_free_rate=0.00)
```

```
results = backtest_with_hyperparameters(1, 0, 14, 14, 0.02)
results
```

```
{'total_return': 1.1104625000000001,
 'irr_annual': 0.05157182978947472,
 'vol_annual': 0.07804053072698804,
 'sharpe': 0.6832862922827433,
 'hit_ratio': 0.38930348258706465,
 'mdd': -0.1900108014561963,
 'n_days': 3743,
 'n_traded_days': 2412}
```

```
import itertools
import math
import numpy as np
import pandas as pd
```

```
def run_grid_search(
    backtest_fn,
    p1_values,
    p2_values,
    p3_values,
    p4_values,
    p5_values,
    verbose=True,
):
    """
    backtest_fn: callable like backtest_with_hyperparameters(p1, p2) -> dict
    p1_values, p2_values: iterables of parameter candidates
    """
    rows = []
    for p1, p2, p3, p4, p5 in itertools.product(p1_values, p2_values, p3_values, p4_-
values, p5_values):
        try:
            res = backtest_fn(p1, p2, p3, p4, p5)  # expected to re-
turn a dict with keys shown below
            # Ensure required keys exist (coerce missing to NaN)
            record = {
                "param1": p1,
                "param2": p2,
                "param3": p3,
                "param4": p4,
                "param5": p5,
                "total_return": res.get("total_return", np.nan),
                "irr_annual": res.get("irr_annual", np.nan),
                "vol_annual": res.get("vol_annual", np.nan),
                "sharpe": res.get("sharpe", np.nan),
                "hit_ratio": res.get("hit_ratio", np.nan),
                "mdd": res.get("mdd", np.nan),
                "n_days": res.get("n_days", np.nan),
                "n_traded_days": res.get("n_traded_days", np.nan),
                "error": None,
            }
        except Exception as e:
            record = {
                "param1": p1,
                "param2": p2,
                "param3": p3,
                "param4": p4,
                "param5": p5,
                "total_return": np.nan,
                "irr_annual": np.nan,
                "vol_annual": np.nan,
                "sharpe": np.nan,
```

```
                "hit_ratio": np.nan,
                "mdd": np.nan,
                "n_days": np.nan,
                "n_traded_days": np.nan,
                "error": str(e),
            }
        rows.append(record)
        print(record)

    df = pd.DataFrame(rows)

    # Keep only successful runs with finite metrics
    ok = df[df["error"].isna()].copy()
    for c in ["total_return", "sharpe"]:
        ok = ok[np.isfinite(ok[c])]

    if ok.empty:
        if verbose:
            print("No successful runs with finite metrics.")
        # Still return the full df for debugging
        return {
            "all_results": df.sort_values(["param1", "param2", "param3", "param4", "p
aram5"]).reset_index(drop=True),
            "best_total_return": None,
            "best_sharpe": None,
        }

    # Best by total return
    idx_ret = ok["total_return"].idxmax()
    best_ret = ok.loc[idx_ret].to_dict()

    # Best by Sharpe
    idx_sharpe = ok["sharpe"].idxmax()
    best_sharpe = ok.loc[idx_sharpe].to_dict()

    if verbose:
        print("=== Best by total_return ===")
        print(
            f"param1={best_ret['param1']}, param2={best_ret['param2']}, param3=
{best_ret['param3']}, param4={best_ret['param4']}, param5={best_ret['param5']}"
            f"total_return={best_ret['total_return']:.6f}, "
            f"sharpe={best_ret['sharpe']:.6f}, vol_annual={best_ret['vol_annu-
al']:.6f}, "
            f"hit_ratio={best_ret['hit_ratio']:.4f}, mdd={best_ret['mdd']:.4f}, "
            f"n_days={int(best_ret['n_days']) if not math.isnan(best_ret['n_-
days']) else 'NA'}, "
            f"n_traded_days={int(best_ret['n_traded_days']) if not math.is-
```

```
nan(best_ret['n_traded_days']) else 'NA'}"
        )
        print("\n=== Best by sharpe ===")
        print(
            f"param1={best_sharpe['param1']}, param2={best_sharpe['param2']}, param3=
{best_sharpe['param3']}, param4={best_sharpe['param4']}, param5=
{best_sharpe['param5']}"
            f"sharpe={best_sharpe['sharpe']:.6f}, "
            f"total_return={best_sharpe['total_return']:.6f}, vol_annual=
{best_sharpe['vol_annual']:.6f}, "
            f"hit_ratio={best_sharpe['hit_ratio']:.4f}, mdd=
{best_sharpe['mdd']:.4f}, "
            f"n_days={int(best_sharpe['n_days']) if not math.isnan(best_sharpe['n_-
days']) else 'NA'}, "
            f"n_traded_days={int(best_sharpe['n_traded_days']) if not math.is-
nan(best_sharpe['n_traded_days']) else 'NA'}"
        )

    return {
        "all_results": df.sort_values(["param1", "param2", 'param3', 'param4', 'param
5']).reset_index(drop=True),
        "best_total_return": best_ret,
        "best_sharpe": best_sharpe,
    }
```

```
p1_grid = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4, 1.5]
p2_grid = [0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009, 0.01, 0.011
, 0.012]
p3_grid = [14]
p4_grid = [14]
p5_grid = [0.02]

results = run_grid_search(backtest_with_hyperparameters, p1_grid, p2_grid, p3_-
grid, p4_grid, p5_grid,  verbose=True)

# Access the DataFrame of all runs:
all_runs_df = results["all_results"]
# Top 10 by total_return:
top10_return = all_runs_df.sort_values("total_return", ascending=False).head(10)
# Top 10 by Sharpe:
top10_sharpe = all_runs_df.sort_values("sharpe", ascending=False).head(10)

all_runs_df
```

```
{'param1': 0.1, 'param2': 0.001, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.002, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.003, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.004, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.005, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.006, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.007, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.008, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.009, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.01, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_ret
{'param1': 0.1, 'param2': 0.011, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.1, 'param2': 0.012, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.001, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.002, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.003, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.004, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.005, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.006, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.007, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
{'param1': 0.2, 'param2': 0.008, 'param3': 14, 'param4': 14, 'param5': 0.02, 'total_re
```

	param1	param2	param3	param4	param5	total_return	irr_annual	vol_annual	sharpe	hit_ratio	mdd	n_days
0	0.1	0.001	14	14	0.02	0.677820	0.035455	0.106489	0.380409	0.427707	-0.174998	3743
1	0.1	0.002	14	14	0.02	1.092335	0.050961	0.098356	0.554502	0.428523	-0.177869	3743
2	0.1	0.003	14	14	0.02	1.380985	0.060145	0.090787	0.688692	0.430604	-0.157938	3743
3	0.1	0.004	14	14	0.02	1.556550	0.065235	0.084498	0.790137	0.436911	-0.150478	3743
4	0.1	0.005	14	14	0.02	1.720127	0.069693	0.077791	0.904955	0.448549	-0.147763	3743
...	...	...	...	...	...	...	...	...	...	...	...	...
175	1.5	0.008	14	14	0.02	-0.148142	-0.010737	0.022295	-0.473022	0.365979	-0.173375	3743
176	1.5	0.009	14	14	0.02	-0.131012	-0.009410	0.020119	-0.459858	0.353333	-0.157837	3743
177	1.5	0.010	14	14	0.02	-0.150578	-0.010927	0.019565	-0.551771	0.344262	-0.182324	3743
178	1.5	0.011	14	14	0.02	-0.140315	-0.010127	0.018707	-0.534741	0.348315	-0.163018	3743
179	1.5	0.012	14	14	0.02	-0.063477	-0.004406	0.015793	-0.271663	0.352113	-0.100068	3743

180 rows × 14 columns

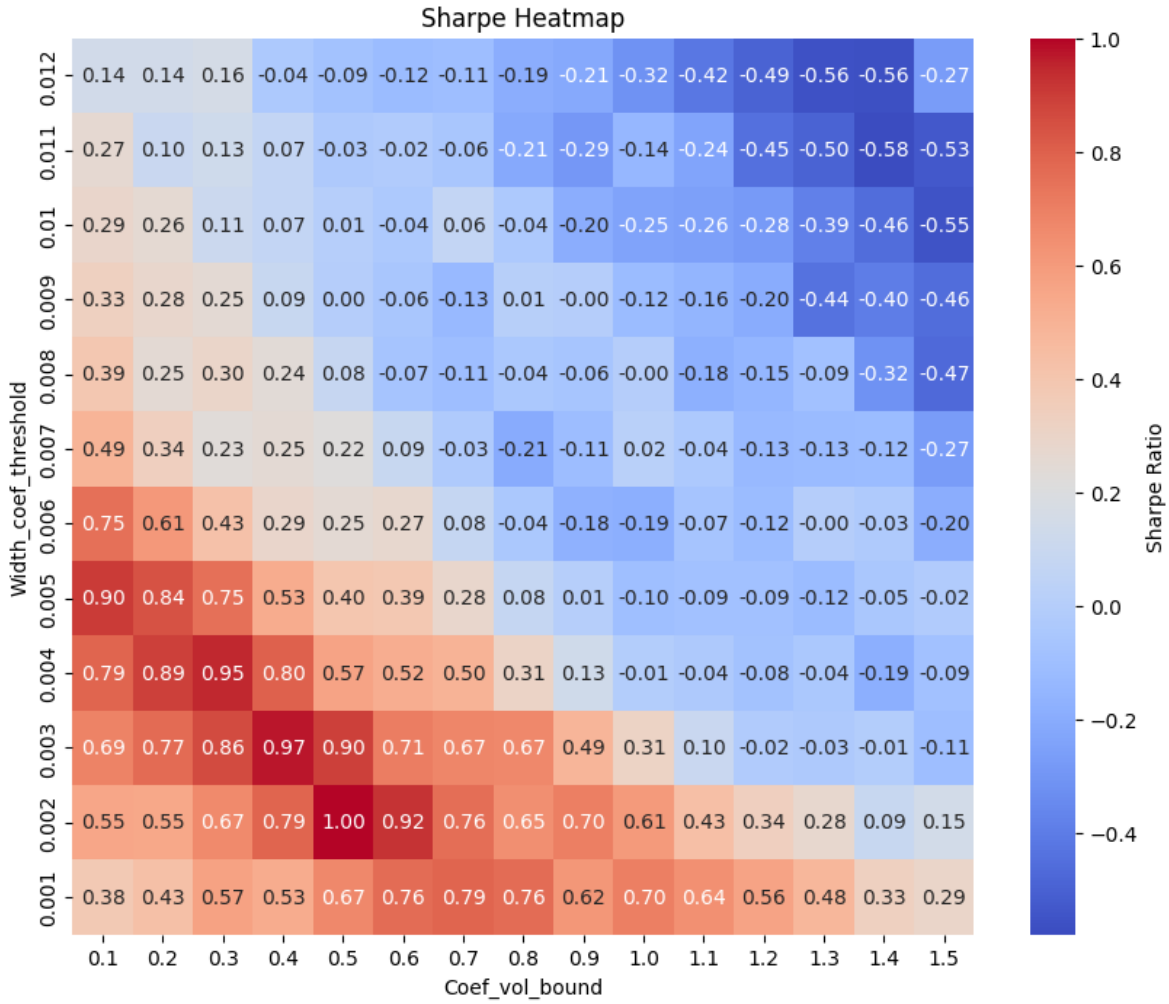
top10_return

	param1	param2	param3	param4	param5	total_return	irr_annual	vol_annual	sharpe	hit_ratio	mdd	n_days	r
49	0.5	0.002	14	14	0.02	2.196662	0.081382	0.081464	1.001114	0.421370	-0.152302	3743	2
38	0.4	0.003	14	14	0.02	1.953700	0.075642	0.078209	0.971418	0.429259	-0.147253	3743	2
61	0.6	0.002	14	14	0.02	1.785347	0.071400	0.078078	0.922330	0.419701	-0.184928	3743	2
27	0.3	0.004	14	14	0.02	1.778217	0.071216	0.075636	0.947313	0.433993	-0.144990	3743	1
26	0.3	0.003	14	14	0.02	1.778083	0.071212	0.083614	0.864511	0.429960	-0.149389	3743	2
15	0.2	0.004	14	14	0.02	1.769293	0.070983	0.080457	0.892572	0.432511	-0.145430	3743	2
4	0.1	0.005	14	14	0.02	1.720127	0.069693	0.077791	0.904955	0.448549	-0.147763	3743	1
37	0.4	0.002	14	14	0.02	1.598135	0.066393	0.086484	0.786490	0.413972	-0.155472	3743	2
50	0.5	0.003	14	14	0.02	1.584977	0.066029	0.074483	0.895667	0.427973	-0.159796	3743	1
3	0.1	0.004	14	14	0.02	1.556550	0.065235	0.084498	0.790137	0.436911	-0.150478	3743	2

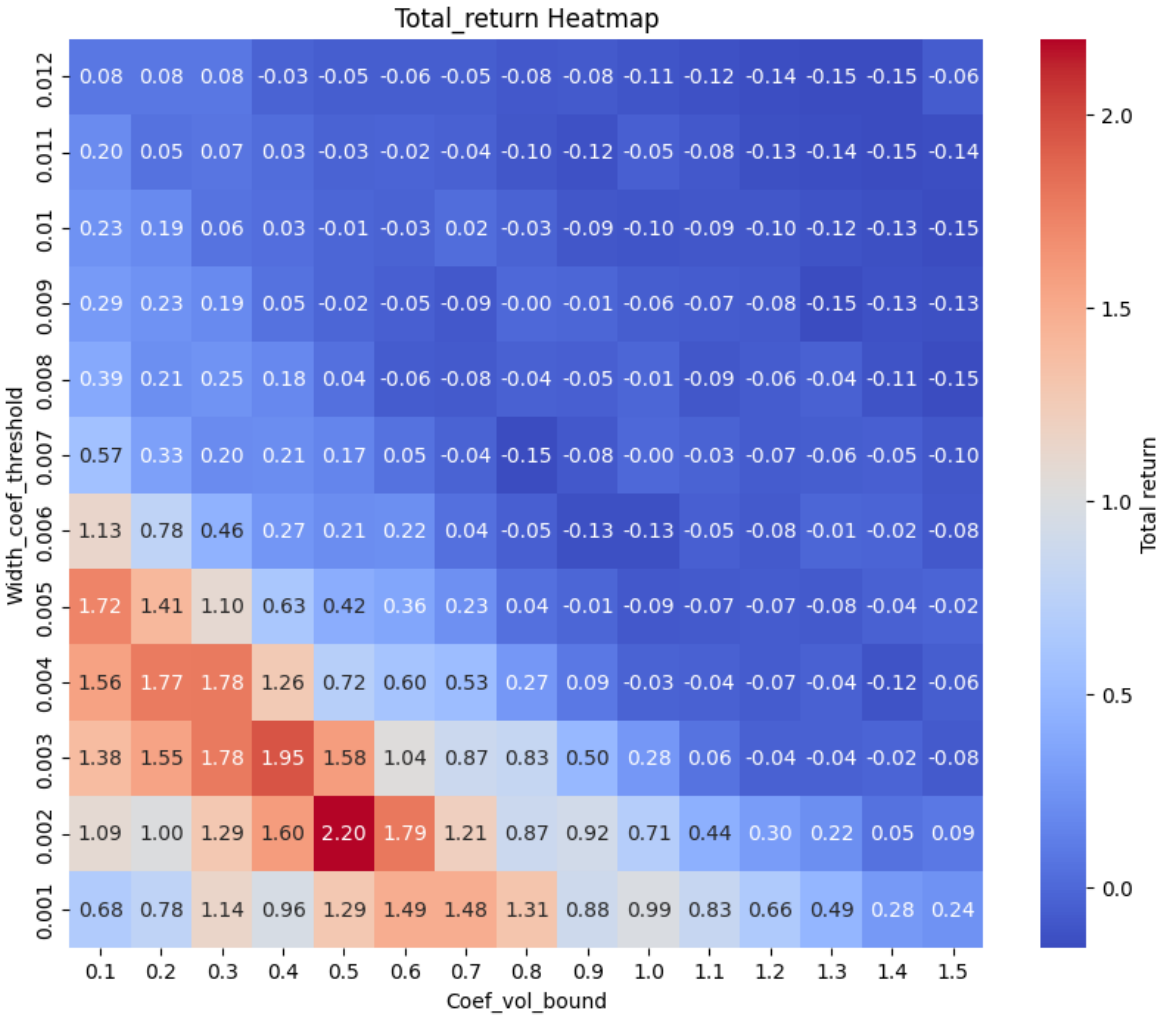
top10_sharpe

	param1	param2	param3	param4	param5	total_return	irr_annual	vol_annual	sharpe	hit_ratio	mdd	n_days	r
49	0.5	0.002	14	14	0.02	2.196662	0.081382	0.081464	1.001114	0.421370	-0.152302	3743	2
38	0.4	0.003	14	14	0.02	1.953700	0.075642	0.078209	0.971418	0.429259	-0.147253	3743	2
27	0.3	0.004	14	14	0.02	1.778217	0.071216	0.075636	0.947313	0.433993	-0.144990	3743	1
61	0.6	0.002	14	14	0.02	1.785347	0.071400	0.078078	0.922330	0.419701	-0.184928	3743	2
4	0.1	0.005	14	14	0.02	1.720127	0.069693	0.077791	0.904955	0.448549	-0.147763	3743	1
50	0.5	0.003	14	14	0.02	1.584977	0.066029	0.074483	0.895667	0.427973	-0.159796	3743	1
15	0.2	0.004	14	14	0.02	1.769293	0.070983	0.080457	0.892572	0.432511	-0.145430	3743	2
26	0.3	0.003	14	14	0.02	1.778083	0.071212	0.083614	0.864511	0.429960	-0.149389	3743	2
16	0.2	0.005	14	14	0.02	1.409218	0.060987	0.073593	0.841174	0.453116	-0.142285	3743	1
39	0.4	0.004	14	14	0.02	1.259818	0.056424	0.071597	0.802423	0.438799	-0.169479	3743	1

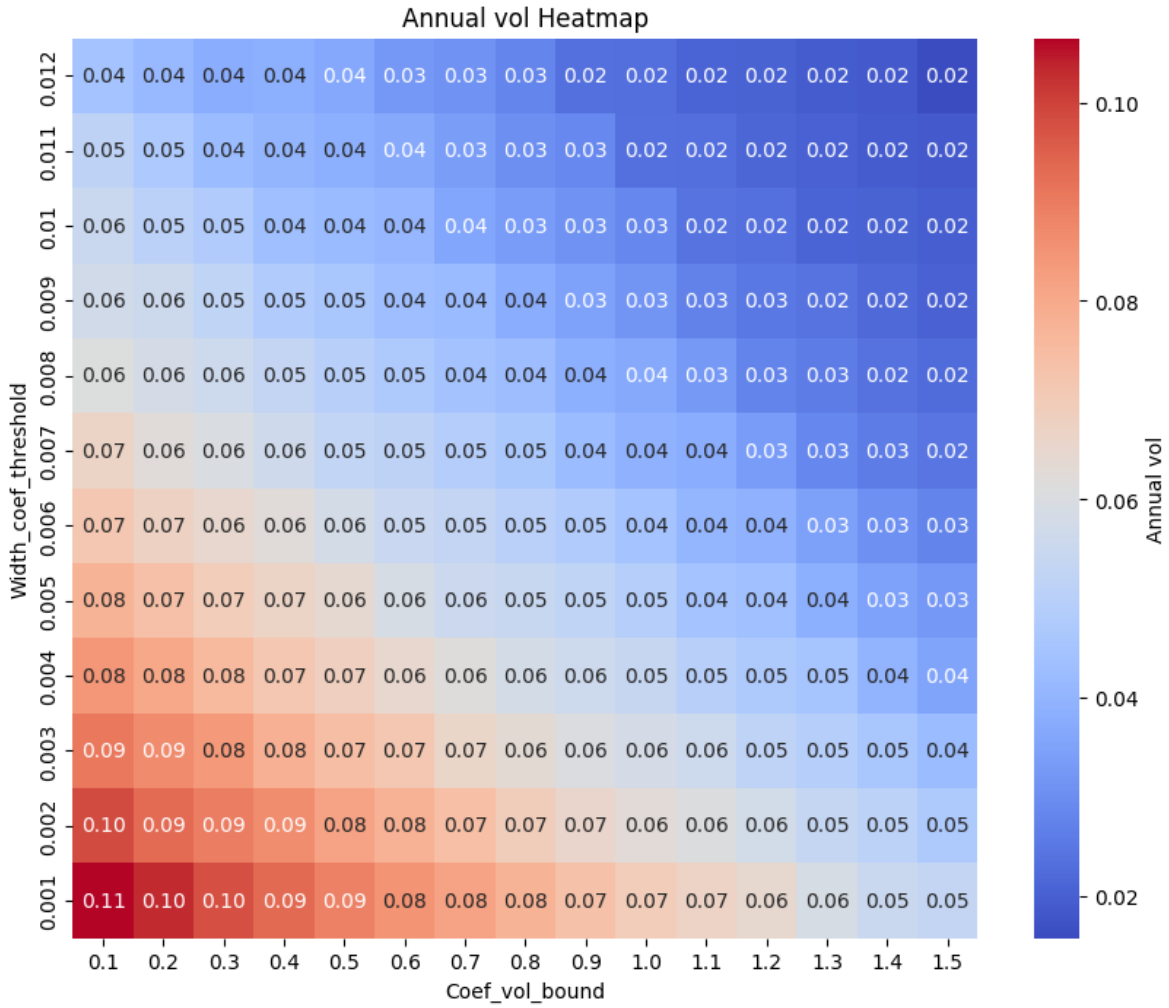
```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="sharpe")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Sharpe Ratio"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Sharpe Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Sharpe Heatmap
Width_coef_threshold
0.012 0.14 0.14 0.16 -0.04 -0.09 -0.12 -0.11 -0.19 -0.21 -0.32 -0.42 -0.49 -0.56 -0.56 -0.27
0.011 0.27 0.10 0.13 0.07 -0.03 -0.02 -0.06 -0.21 -0.29 -0.14 -0.24 -0.45 -0.50 -0.58 -0.53
0.01 0.29 0.26 0.11 0.07 0.01 -0.04 0.06 -0.04 -0.20 -0.25 -0.26 -0.28 -0.39 -0.46 -0.55
0.009 0.33 0.28 0.25 0.09 0.00 -0.06 -0.13 0.01 -0.00 -0.12 -0.16 -0.20 -0.44 -0.40 -0.46
0.008 0.39 0.25 0.30 0.24 0.08 -0.07 -0.11 -0.04 -0.06 -0.00 -0.18 -0.15 -0.09 -0.32 -0.47
0.007 0.49 0.34 0.23 0.25 0.22 0.09 -0.03 -0.21 -0.11 0.02 -0.04 -0.13 -0.13 -0.12 -0.27
0.006 0.75 0.61 0.43 0.29 0.25 0.27 0.08 -0.04 -0.18 -0.19 -0.07 -0.12 -0.00 -0.03 -0.20
0.005 0.90 0.84 0.75 0.53 0.40 0.39 0.28 0.08 0.01 -0.10 -0.09 -0.09 -0.12 -0.05 -0.02
0.004 0.79 0.89 0.95 0.80 0.57 0.52 0.50 0.31 0.13 -0.01 -0.04 -0.08 -0.04 -0.19 -0.09
0.003 0.69 0.77 0.86 0.97 0.90 0.71 0.67 0.67 0.49 0.31 0.10 -0.02 -0.03 -0.01 -0.11
0.002 0.55 0.55 0.67 0.79 1.00 0.92 0.76 0.65 0.70 0.61 0.43 0.34 0.28 0.09 0.15
0.001 0.38 0.43 0.57 0.53 0.67 0.76 0.79 0.76 0.62 0.70 0.64 0.56 0.48 0.33 0.29
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
Sharpe Ratio
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4

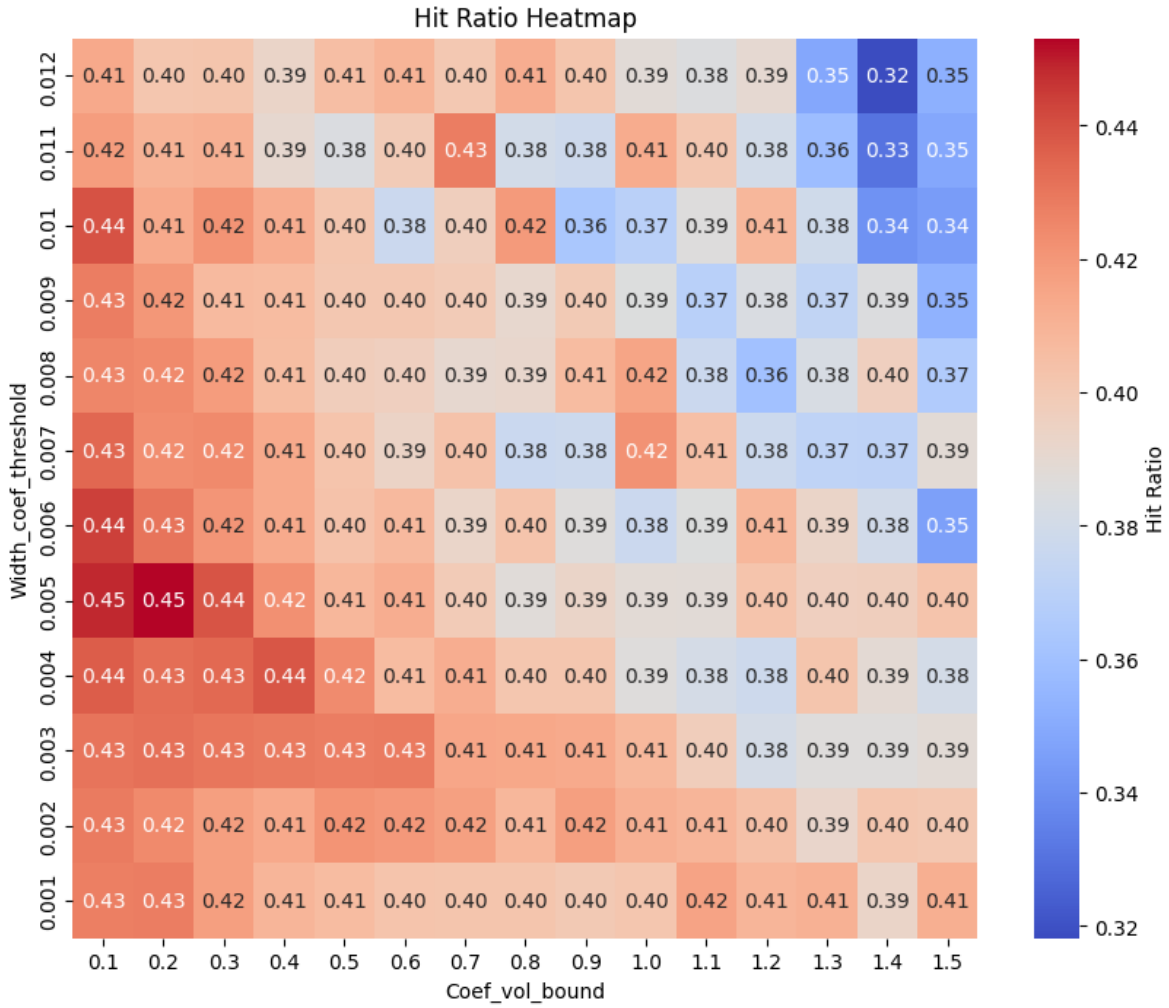
```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="total_re-
turn")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Total return"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Total_return Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Total_return Heatmap
Width_coef_threshold
0.012 0.08 0.08 0.08 -0.03 -0.05 -0.06 -0.05 -0.08 -0.08 -0.11 -0.12 -0.14 -0.15 -0.15 -0.06
0.011 0.20 0.05 0.07 0.03 -0.03 -0.02 -0.04 -0.10 -0.12 -0.05 -0.08 -0.13 -0.14 -0.15 -0.14
0.01 0.23 0.19 0.06 0.03 -0.01 -0.03 0.02 -0.03 -0.09 -0.10 -0.09 -0.10 -0.12 -0.13 -0.15
0.009 0.29 0.23 0.19 0.05 -0.02 -0.05 -0.09 -0.00 -0.01 -0.06 -0.07 -0.08 -0.15 -0.13 -0.13
0.008 0.39 0.21 0.25 0.18 0.04 -0.06 -0.08 -0.04 -0.05 -0.01 -0.09 -0.06 -0.04 -0.11 -0.15
0.007 0.57 0.33 0.20 0.21 0.17 0.05 -0.04 -0.15 -0.08 -0.00 -0.03 -0.07 -0.06 -0.05 -0.10
0.006 1.13 0.78 0.46 0.27 0.21 0.22 0.04 -0.05 -0.13 -0.13 -0.05 -0.08 -0.01 -0.02 -0.08
0.005 1.72 1.41 1.10 0.63 0.42 0.36 0.23 0.04 -0.01 -0.09 -0.07 -0.07 -0.08 -0.04 -0.02
0.004 1.56 1.77 1.78 1.26 0.72 0.60 0.53 0.27 0.09 -0.03 -0.04 -0.07 -0.04 -0.12 -0.06
0.003 1.38 1.55 1.78 1.95 1.58 1.04 0.87 0.83 0.50 0.28 0.06 -0.04 -0.04 -0.02 -0.08
0.002 1.09 1.00 1.29 1.60 2.20 1.79 1.21 0.87 0.92 0.71 0.44 0.30 0.22 0.05 0.09
0.001 0.68 0.78 1.14 0.96 1.29 1.49 1.48 1.31 0.88 0.99 0.83 0.66 0.49 0.28 0.24
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
Total return
2.0
1.5
1.0
0.5
0.0

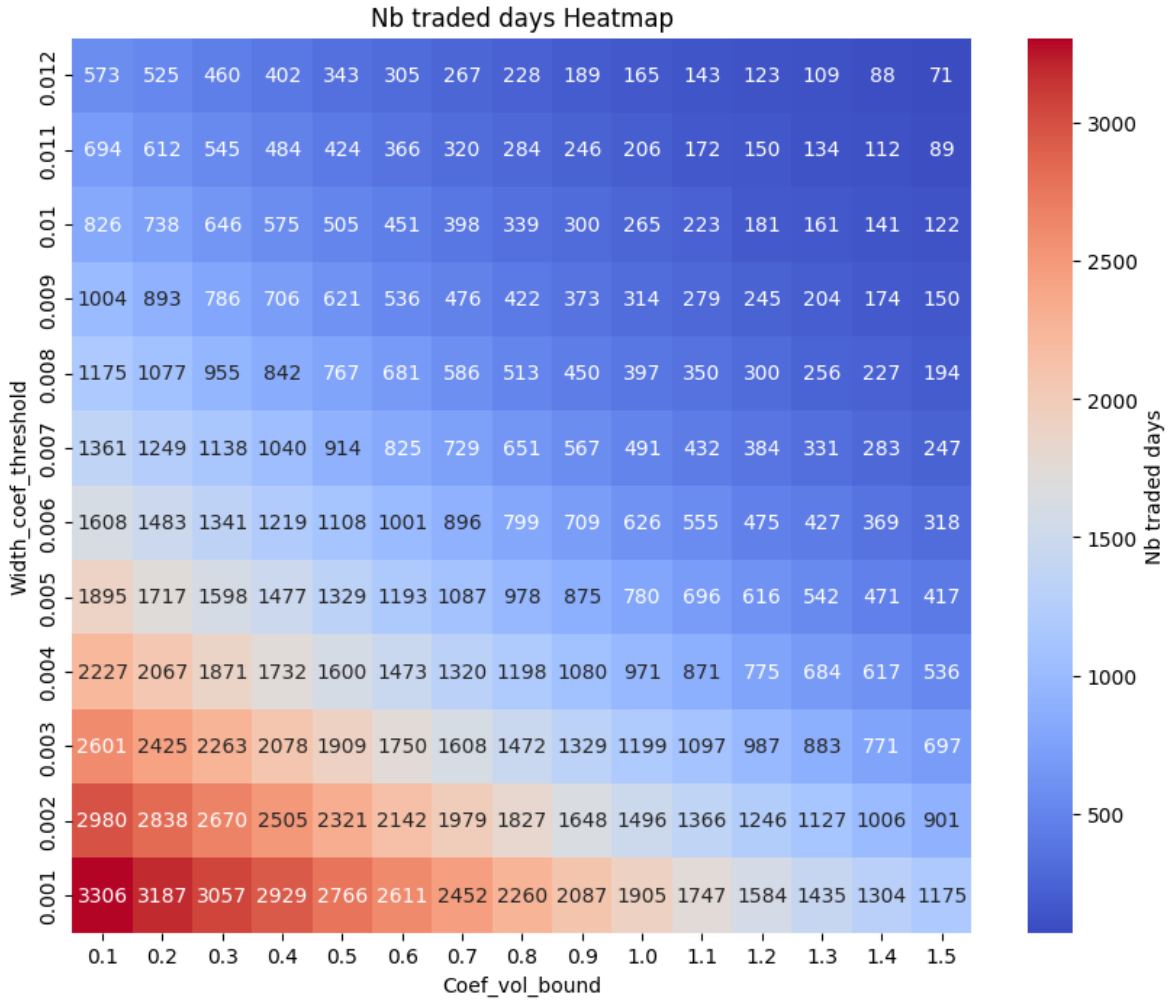
```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="vol_annual"
)

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Annual vol"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Annual vol Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Annual vol Heatmap
Width_coef_threshold
0.012 0.04 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.02 0.02 0.02 0.02 0.02 0.02 0.02
0.011 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.02 0.02 0.02 0.02 0.02 0.02
0.01 0.06 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.02 0.02 0.02 0.02 0.02
0.009 0.06 0.06 0.05 0.05 0.05 0.04 0.04 0.04 0.03 0.03 0.03 0.03 0.02 0.02 0.02
0.008 0.06 0.06 0.06 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.02 0.02
0.007 0.07 0.06 0.06 0.06 0.05 0.05 0.05 0.05 0.04 0.04 0.04 0.03 0.03 0.03 0.02
0.006 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.05 0.05 0.04 0.04 0.04 0.03 0.03 0.03
0.005 0.08 0.07 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.05 0.04 0.04 0.04 0.03 0.03
0.004 0.08 0.08 0.08 0.07 0.07 0.06 0.06 0.06 0.06 0.05 0.05 0.05 0.05 0.04 0.04
0.003 0.09 0.09 0.08 0.08 0.07 0.07 0.07 0.06 0.06 0.06 0.06 0.05 0.05 0.05 0.04
0.002 0.10 0.09 0.09 0.09 0.08 0.08 0.07 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.05
0.001 0.11 0.10 0.10 0.09 0.09 0.08 0.08 0.08 0.07 0.07 0.07 0.06 0.06 0.05 0.05
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
Annual vol
0.10
0.08
0.06
0.04
0.02

```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="hit_ratio")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".2f",
    cbar_kws={"label": "Hit Ratio"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Hit Ratio Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```


Hit Ratio Heatmap
Width_coef_threshold
0.012 0.41 0.40 0.40 0.39 0.41 0.41 0.40 0.41 0.40 0.39 0.38 0.39 0.35 0.32 0.35
0.011 0.42 0.41 0.41 0.39 0.38 0.40 0.43 0.38 0.38 0.41 0.40 0.38 0.36 0.33 0.35
0.01 0.44 0.41 0.42 0.41 0.40 0.38 0.40 0.42 0.36 0.37 0.39 0.41 0.38 0.34 0.34
0.009 0.43 0.42 0.41 0.41 0.40 0.40 0.40 0.39 0.40 0.39 0.37 0.38 0.37 0.39 0.35
0.008 0.43 0.42 0.42 0.41 0.40 0.40 0.39 0.39 0.41 0.42 0.38 0.36 0.38 0.40 0.37
0.007 0.43 0.42 0.42 0.41 0.40 0.39 0.40 0.38 0.38 0.42 0.41 0.38 0.37 0.37 0.39
0.006 0.44 0.43 0.42 0.41 0.40 0.41 0.39 0.40 0.39 0.38 0.39 0.41 0.39 0.38 0.35
0.005 0.45 0.45 0.44 0.42 0.41 0.41 0.40 0.39 0.39 0.39 0.39 0.40 0.40 0.40 0.40
0.004 0.44 0.43 0.43 0.44 0.42 0.41 0.41 0.40 0.40 0.39 0.38 0.38 0.40 0.39 0.38
0.003 0.43 0.43 0.43 0.43 0.43 0.43 0.41 0.41 0.41 0.41 0.40 0.38 0.39 0.39 0.39
0.002 0.43 0.42 0.42 0.41 0.42 0.42 0.42 0.41 0.42 0.41 0.41 0.40 0.39 0.40 0.40
0.001 0.43 0.43 0.42 0.41 0.41 0.40 0.40 0.40 0.40 0.40 0.42 0.41 0.41 0.39 0.41
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
Coef_vol_bound
Hit Ratio
0.44
0.42
0.40
0.38
0.36
0.34
0.32

```
import seaborn as sns
import matplotlib.pyplot as plt

pivot = all_runs_df.pivot_table(index="param2", columns="param1", values="n_traded_-
days")

# Ensure axes are sorted ascending so (0,0) is bottom-left
pivot = pivot.sort_index(ascending=False).sort_index(axis=1, ascending=True)

plt.figure(figsize=(10,8))
ax = sns.heatmap(
    pivot,
    cmap="coolwarm",
    annot=True, fmt=".0f",
    cbar_kws={"label": "Nb traded days"},
    xticklabels=pivot.columns,
    yticklabels=pivot.index
)


ax.set_title("Nb traded days Heatmap")
ax.set_xlabel("Coef_vol_bound")
ax.set_ylabel("Width_coef_threshold")
plt.show()
```




```
results = backtest_with_hyperparameters(1, 0, 14, 14, 0.02)
results
```

```
{'total_return': 1.1104625000000001,
 'irr_annual': 0.05157182978947472,
 'vol_annual': 0.07804053072698804,
 'sharpe': 0.6832862922827433,
 'hit_ratio': 0.38930348258706465,
 'mdd': -0.1900108014561963,
 'n_days': 3743,
 'n_traded_days': 2412}
```

```
### stop same band + vwap + dynamic size bet
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math
import datetime as dt
import matplotlib.dates as mdates
```

```
### import the data
df = pd.read_csv('/data/workspace_files/nasdaq.csv')
df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

# (Optional) drop bad rows
df = df.dropna(subset=['Time'])

# Convert UTC -> New York (handles DST)
df['Time'] = df['Time'].dt.tz_convert('America/New_York')

# Build hour/day AFTER conversion
df['hour'] = df['Time'].dt.strftime('%H:%M')
df['day']  = df['Time'].dt.date

# Keep only US RTH 09:30–16:00 ET
market_open, market_close = dt.time(9, 30), dt.time(16, 0)
df = df[df['Time'].dt.time.between(market_open, market_close)]

# Sanity check (optional)
tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()
print("Min/Max ET times in df:", tmin, tmax)  # expect between 09:30 and 16:00


### order by symbol, hour and day
df = df.sort_values(['symbol', 'hour', 'day'])

### price
df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


### opening price
df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


# settling price and previous day settling price
daily_settle = (
```

```
    df.sort_values(['symbol','day','Time'])
      .groupby(['symbol','day'])['price']
      .last()
      .rename('settl_price')
)

prev_settle = (
    daily_settle.groupby(level='symbol')
                .shift(1)
                .rename('settl_price_previous')
)

df = df.join(daily_settle, on=['symbol','day'])
df = df.join(prev_settle,   on=['symbol','day'])


### move since opening
df["move"] = np.where(
    df["open_price"] > 0,
    np.abs(df["price"] / df["open_price"] - 1),
    np.nan
)


### compute mean of move over the last period
n = 14
df['sigma_observed'] = (
    df.groupby(['symbol', 'hour'])['move']
      .transform(lambda x: x.rolling(n, min_periods=n).mean())
)


### compute upper bound
df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1 + d
f['sigma_observed'])


### compute lower bound
df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1 -
 df['sigma_observed'])


### get return
df['return'] = df['settl_price']/df['open_price'] - 1


### get variance onf returns among the last 14 days
```

```
daily = (df.sort_values(['symbol','day'])
           .groupby(['symbol','day'], as_index=False)['return'].last())

daily['ret_std_14d'] = (
    daily.groupby('symbol')['return']
         .transform(lambda s: s.rolling(14, min_periods=14).std())
)

df = df.merge(daily[['symbol','day','ret_std_14d']], on=['symbol','day'], how='left')


df = df.sort_values(['symbol', 'day', 'hour'])
# df[df['expiry_order']==1]
df
```

```
Min/Max ET times in df: 09:33:00 16:00:00
```

	Time	symbol	expiry_order	bid_px_00	ask_px_00	vwap_cum	hour	day	price	open_price	settl_pri
0	2019-03-15 09:33:00-04:00	NQH0	1	NaN	NaN	NaN	09:33	2019-03-15	NaN	NaN	NaN
188	2019-03-15 09:48:00-04:00	NQH0	1	NaN	NaN	7303.251423	09:48	2019-03-15	NaN	NaN	NaN
376	2019-03-15 10:03:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:03	2019-03-15	NaN	NaN	NaN
564	2019-03-15 10:18:00-04:00	NQH0	1	NaN	NaN	7370.000000	10:18	2019-03-15	NaN	NaN	NaN
752	2019-03-15 10:33:00-04:00	NQH0	1	NaN	NaN	7314.065510	10:33	2019-03-15	NaN	NaN	NaN
...	...	...	...	...	...	...	...	...	...	...	...
266952	2019-12-19 15:03:00-05:00	NQZ9	3	8632.25	8632.50	8613.137849	15:03	2019-12-19	8632.375	8604.125	8640.62
267140	2019-12-19 15:18:00-05:00	NQZ9	3	8628.75	8629.25	8613.525157	15:18	2019-12-19	8629.000	8604.125	8640.62
267328	2019-12-19 15:33:00-05:00	NQZ9	3	8628.75	8629.00	8613.953049	15:33	2019-12-19	8628.875	8604.125	8640.62
267516	2019-12-19 15:48:00-05:00	NQZ9	3	8631.75	8632.00	8614.566532	15:48	2019-12-19	8631.875	8604.125	8640.62
267704	2019-12-19 16:00:00-05:00	NQZ9	3	8640.50	8640.75	8617.831876	16:00	2019-12-19	8640.625	8604.125	8640.62

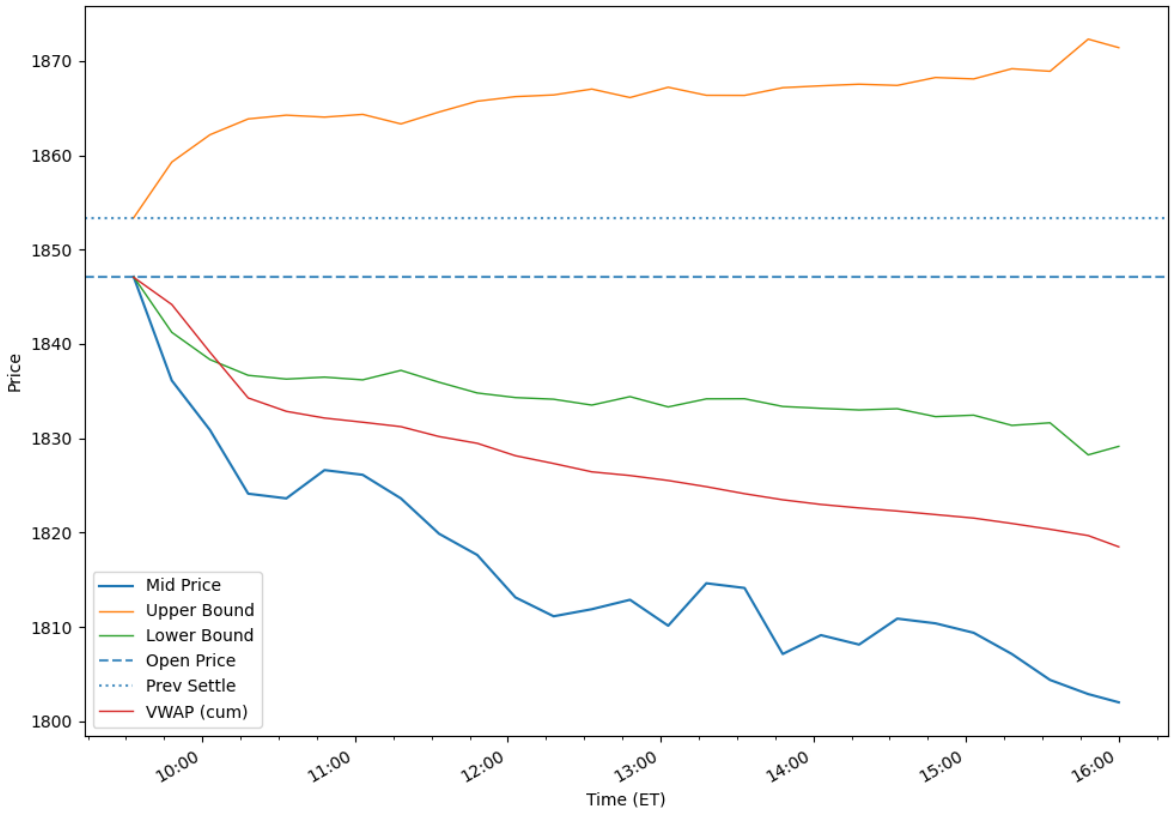
267705 rows × 18 columns

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash):

    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
            .sort_values('Time')
            .copy())
    backtest_df = backtest_df.sort_values('hour')

    bid_list = backtest_df['bid_px_00'].values
    ask_list = backtest_df['ask_px_00'].values
    upper_bound = backtest_df['upper_bound'].values
    lower_bound = backtest_df['lower_bound'].values
    vwap_list = backtest_df['vwap_cum'].values
    hour = backtest_df['hour'].values

    if not (len(bid_list) == len(ask_list) == len(vwap_list) == len(hour) == len(up-
per_bound) == len(lower_bound)):
        return initial_cash
    elif any(pd.isna(x) for x in bid_list + ask_list + vwap_list + upper_bound + low-
er_bound):
        return initial_cash



    position = 0
    sigma_target = 0.02

    total_shares_to_hold = math.floor(initial_cash * min(4, sigma_target/backtest_df[
'ret_std_14d'].values[0]) / backtest_df['open_price'].values[0])


    count = 0
    for i in range(1, len(bid_list) - 1):
        if ask_list[i] > max(upper_bound[i], vwap_list[i]) and count==0:
            count == 1
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * ask_list[i]
                print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str} a
t {hour[i]}")

        elif bid_list[i] < min(vwap_list[i], lower_bound[i]) and count == 0:
```

```
            count == 1
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * bid_list[i]
                print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str} at
 {hour[i]}")

        elif bid_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * bid_list[i]
                print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {hour
[i]}")
                position = 0
            elif position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * ask_list[i]
                print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str} a
t {hour[i]}")
                position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
        initial_cash += position * bid_list[-1]
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * ask_list[-1]
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
# Ensure Time is tz-aware ET
df["Time"] = pd.to_datetime(df["Time"], utc=True).dt.tz_convert("America/New_York")

# Define day as plain date (no tz needed)
day_str = "2010-07-16"
day = pd.to_datetime(day_str).date()

# Recompute day column in ET if not already done
df["day"] = df["Time"].dt.date

# Filter front-month contract for that day
```

```
one = df[(df['expiry_order'] == 1) & (df['day'] == day)].copy()
one = one.sort_values('Time')

# Plot
fig, ax = plt.subplots(figsize=(10, 7))
x = one['Time']

ax.plot(x, one['price'], label='Mid Price')
if 'upper_bound' in one and one['upper_bound'].notna().any():
    ax.plot(x, one['upper_bound'], label='Upper Bound', linewidth=1)
if 'lower_bound' in one and one['lower_bound'].notna().any():
    ax.plot(x, one['lower_bound'], label='Lower Bound', linewidth=1)

if 'open_price' in one and one['open_price'].notna().any():
    ax.axhline(one['open_price'].iloc[0], linestyle='--', alpha=0.8, la-
bel='Open Price')
if 'settl_price_previous' in one and one['settl_price_previous'].notna().any():
    ax.axhline(one['settl_price_previous'].iloc[0], linestyle=':', alpha=0.8, label='
Prev Settle')

if 'vwap_cum' in one.columns and one['vwap_cum'].notna().any():
    ax.plot(x, one['vwap_cum'], label='VWAP (cum)', linewidth=1)

# Tidy x-axis
import matplotlib.dates as mdates
from zoneinfo import ZoneInfo


tz = ZoneInfo("America/New_York")

ax.xaxis.set_major_locator(mdates.HourLocator(tz=tz))
ax.xaxis.set_major_formatter(mdates.DateFormatter('%H:%M', tz=tz))
ax.xaxis.set_minor_locator(mdates.MinuteLocator(byminute=[0,15,30,45], tz=tz))

fig.autofmt_xdate()

ax.set_xlabel("Time (ET)")
ax.set_ylabel("Price")
ax.legend()
plt.tight_layout()
plt.show()
```

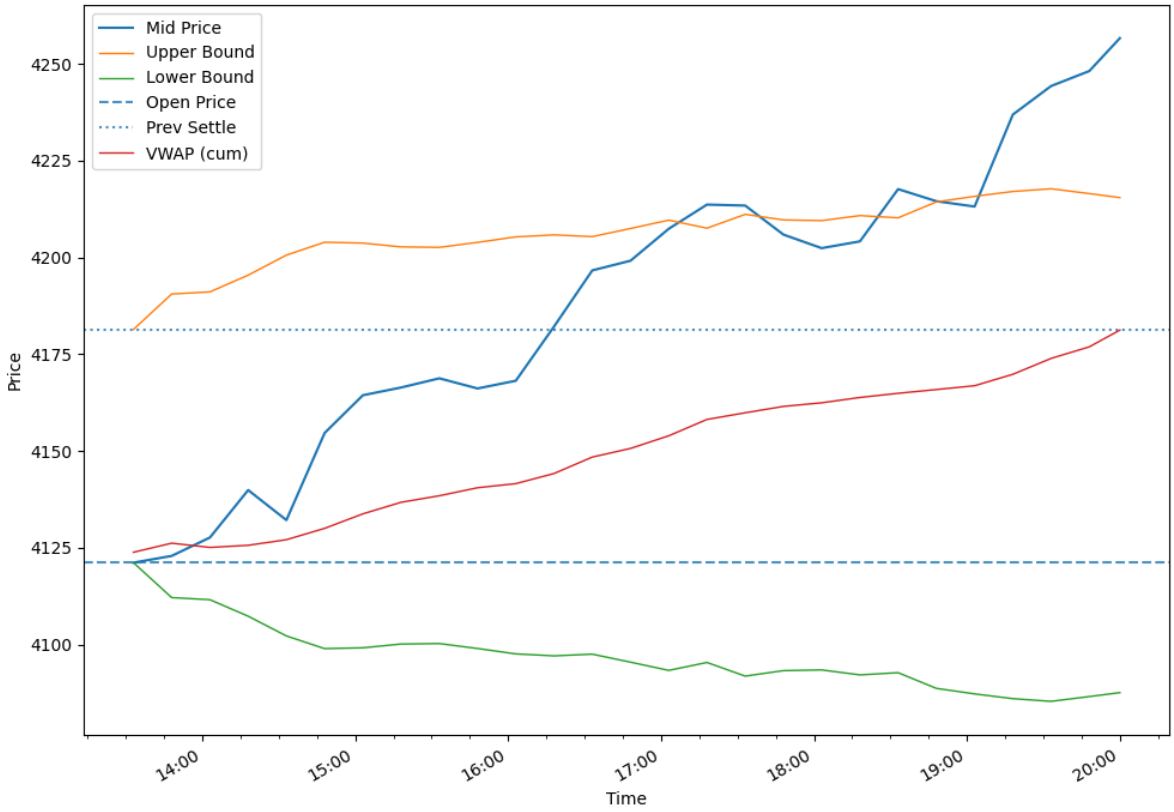



```
backtest_one_day(df, "2010-07-16", 1, 100000)
```

```
Sold 82 shares at 1836.0 on 2010-07-16 at 09:48
102767.5
```

```
day_str = "2015-10-02"
day = pd.to_datetime(day_str).date()

print(backtest_one_day(df, day_str, 1, 100000))
one = df[(df['expiry_order'] == 1) & (df['day'] == day)].copy()
one = one.sort_values('Time')

fig, ax = plt.subplots(figsize=(10, 7))

x = one['Time']

ax.plot(x, one['price'], label='Mid Price')
if 'upper_bound' in one and one['upper_bound'].notna().any():
    ax.plot(x, one['upper_bound'], label='Upper Bound', linewidth=1)
if 'lower_bound' in one and one['lower_bound'].notna().any():
    ax.plot(x, one['lower_bound'], label='Lower Bound', linewidth=1)

# Horizontal lines for open and previous settlement
if 'open_price' in one and one['open_price'].notna().any():
    ax.axhline(one['open_price'].iloc[0], linestyle='--', alpha=0.8, la-
bel='Open Price')
if 'settl_price_previous' in one and one['settl_price_previous'].notna().any():
    ax.axhline(one['settl_price_previous'].iloc[0], linestyle=':', alpha=0.8, label='
Prev Settle')

# Optional: VWAP
if 'vwap_cum' in one.columns and one['vwap_cum'].notna().any():
    ax.plot(x, one['vwap_cum'], label='VWAP (cum)', linewidth=1)

# Tidy x-axis: show one tick per hour, minor ticks at 15-min
ax.xaxis.set_major_locator(mdates.HourLocator())
ax.xaxis.set_major_formatter(mdates.DateFormatter('%H:%M'))
ax.xaxis.set_minor_locator(mdates.MinuteLocator(byminute=[0,15,30,45]))
fig.autofmt_xdate()
ax.set_xlabel("Time")
ax.set_ylabel("Price")
ax.legend()
plt.tight_layout()
plt.show()
```

```
Bought 38 shares at 4213.75 on 2015-10-02 at 13:18
Sold 38 shares at 4205.75 on 2015-10-02 at 13:48
Bought 38 shares at 4217.75 on 2015-10-02 at 14:33
Sold 38 shares at 4213.0 on 2015-10-02 at 15:03
Bought 38 shares at 4237.0 on 2015-10-02 at 15:18
100256.5
```



```python
def backtest(backtest_df, rank_to_expiry, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        print(day_ts)
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))
```

```
results = backtest(df, 1)
results
```

```
2010-06-07 00:00:00-04:00
2010-06-08 00:00:00-04:00
2010-06-09 00:00:00-04:00
2010-06-10 00:00:00-04:00
2010-06-11 00:00:00-04:00
2010-06-14 00:00:00-04:00
2010-06-15 00:00:00-04:00
2010-06-16 00:00:00-04:00
2010-06-17 00:00:00-04:00
2010-06-18 00:00:00-04:00
2010-06-21 00:00:00-04:00
2010-06-22 00:00:00-04:00
2010-06-23 00:00:00-04:00
2010-06-24 00:00:00-04:00
Sold 79 shares at 1855.0 on 2010-06-24 00:00:00-04:00 at 10:03
Bought 79 shares at 1857.5 on 2010-06-24 00:00:00-04:00 at 10:33
Sold 79 shares at 1853.5 on 2010-06-24 00:00:00-04:00 at 10:48
Bought 79 shares at 1856.5 on 2010-06-24 00:00:00-04:00 at 12:03
Sold 79 shares at 1853.75 on 2010-06-24 00:00:00-04:00 at 12:18
Bought 79 shares at 1859.0 on 2010-06-24 00:00:00-04:00 at 13:18
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.00	100000.00	0.0	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.00	100000.00	0.0	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.00	100000.00	0.0	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.00	100000.00	0.0	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.00	100000.00	0.0	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	219293.25	222017.25	2724.0	0.012422	True
3739	2024-12-26 00:00:00-05:00	222017.25	222017.25	0.0	0.000000	False
3740	2024-12-27 00:00:00-05:00	222017.25	222957.25	940.0	0.004234	True
3741	2024-12-30 00:00:00-05:00	222957.25	223475.75	518.5	0.002326	True
3742	2024-12-31 00:00:00-05:00	223475.75	220223.75	-3252.0	-0.014552	True

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
```

```
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

```
{'total_return': 1.2022374999999998,
 'irr_annual': 0.0545897852302224,
 'vol_annual': 0.15833871376768047,
 'sharpe': 0.41429261020156816,
 'hit_ratio': 0.38930348258706465,
 'mdd': -0.39369256680225745,
 'n_days': 3743,
 'n_traded_days': 2412}
```

```
results.sort_values('return')
```

	date	start_cash	end_cash	pnl	return	traded
344	2011-10-04 00:00:00-04:00	74483.50	70882.00	-3601.50	-0.048353	True
2972	2022-01-06 00:00:00-05:00	140500.00	134569.00	-5931.00	-0.042214	True
187	2011-02-24 00:00:00-05:00	83842.25	80391.25	-3451.00	-0.041161	True
1590	2016-08-26 00:00:00-04:00	106571.75	102210.75	-4361.00	-0.040921	True
2890	2021-09-13 00:00:00-04:00	143066.25	137642.25	-5424.00	-0.037913	True
...	...	...	...	...	...	...
1775	2017-05-17 00:00:00-04:00	95465.25	100285.75	4820.50	0.050495	True
2137	2018-10-10 00:00:00-04:00	127501.00	134105.75	6604.75	0.051802	True
1600	2016-09-09 00:00:00-04:00	99722.25	105173.25	5451.00	0.054662	True
877	2013-11-07 00:00:00-05:00	81504.75	85988.50	4483.75	0.055012	True
1792	2017-06-09 00:00:00-04:00	101777.00	107560.50	5783.50	0.056825	True

3743 rows × 6 columns

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Clean the series
s = pd.Series(results['return'], dtype='float64')
s = s.replace([np.inf, -np.inf], np.nan).dropna()

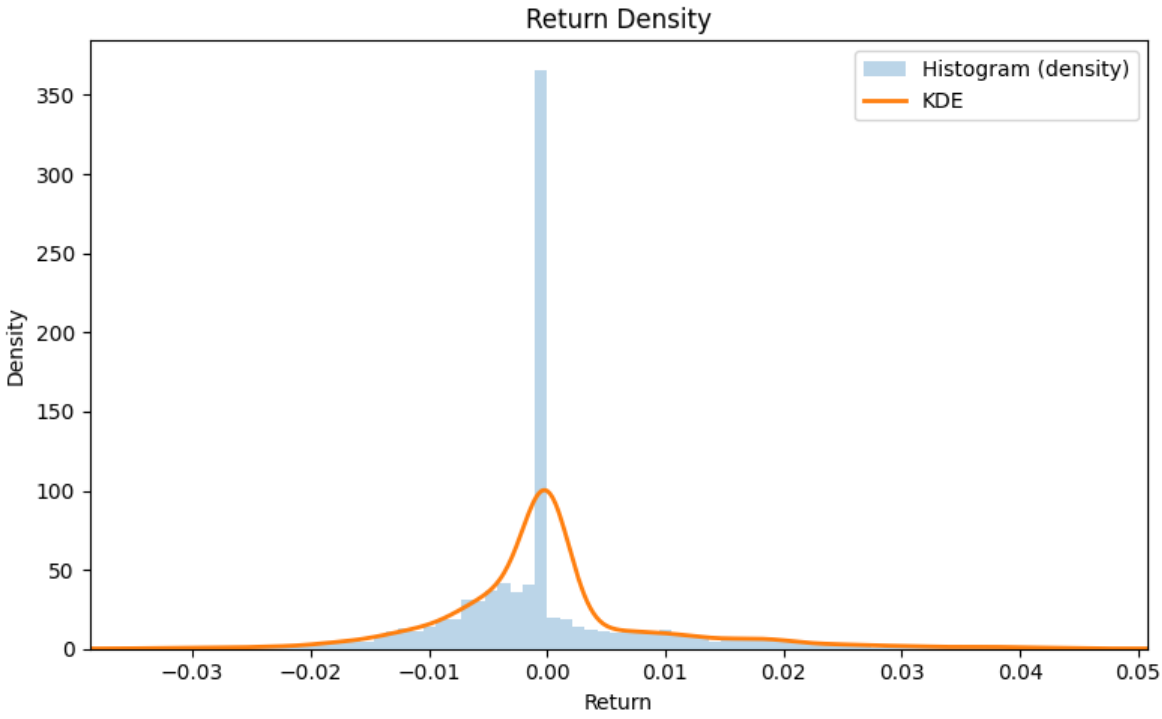
fig, ax = plt.subplots(figsize=(8, 5))

# Density histogram
ax.hist(s, bins=100, density=True, alpha=0.3, label='Histogram (density)')

# KDE overlay (if scipy available)
try:
    from scipy.stats import gaussian_kde
    xs = np.linspace(s.quantile(0.001), s.quantile(0.999), 512)
    kde = gaussian_kde(s)
    ax.plot(xs, kde(xs), linewidth=2, label='KDE')
except Exception:
    # If scipy isn't installed, just skip the KDE
    pass

# Nice viewing window (doesn't affect density calc)
qlo, qhi = s.quantile([0.001, 0.999])
ax.set_xlim(qlo, qhi)

ax.set_xlabel('Return')
ax.set_ylabel('Density')
ax.set_title('Return Density')
ax.legend()
plt.tight_layout()
plt.show()
```




```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math

import datetime as dt

import matplotlib.dates as mdates
```

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_target):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]
```

```
    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()
    price_list   = backtest_df['price'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
ower_bound) == len(price_list) == n) or n == 0:
        return initial_cash

    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
        np.isnan(lower_bound).any() or np.isnan(price_list).any()):
        return initial_cash

    position = 0
    cash = float(initial_cash)

    for i in range(1, len(price_list) - 1):
        if price_list[i] > max(upper_bound[i], vwap_list[i]):
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * price_list[i] * 1.00001
                # print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")

        elif price_list[i] < min(vwap_list[i], lower_bound[i]):
```

```
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * price_list[i] * 0.99999
                # print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str}
at {hour[i]}")

        elif price_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * price_list[i] * 0.99999
                # print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {ho
ur[i]}")
                position = 0
            elif position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * price_list[i] * 1.00001
                # print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")
                position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
        initial_cash += position * price_list[-1] * 0.99999
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * price_list[-1] * 1.00001
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_tar-
get)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization, sigma_lever-
age_period, sigma_leverage_bound, sigma_target):

    df = pd.read_csv('/data/workspace_files/nasdaq.csv')
    df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

    # (Optional) drop bad rows
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST)
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')

    # Build hour/day AFTER conversion
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date

    # Keep only US RTH 09:30-16:00 ET
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Sanity check (optional)
    tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()

    ### order by symbol, hour and day
    df = df.sort_values(['symbol', 'hour', 'day'])

    ### price
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


    ### opening price
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


    # settling price and previous day settling price
    daily_settle = (
        df.sort_values(['symbol','day','Time'])
        .groupby(['symbol','day'])['price']
        .last()
        .rename('settl_price')
    )

    prev_settle = (
        daily_settle.groupby(level='symbol')
```

```
                    .shift(1)
                    .rename('settl_price_previous')
    )

    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])


    ### move since opening
    df["move"] = np.where(
        df["open_price"] > 0,
        np.abs(df["price"] / df["open_price"] - 1),
        np.nan
    )


    ### compute mean of move over the last period
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
        .transform(lambda x: x.rolling(sigma_leverage_bound, min_periods=sigma_lever-
age_bound).mean())
    )


    ### compute upper bound
    df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1
 + coef_sigma * df['sigma_observed']) + percentage_penalization * df['open_price']


    ### compute lower bound
    df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1
 - coef_sigma * df['sigma_observed']) - percentage_penalization * df['open_price']


    ### get return
    df['return'] = df['settl_price']/df['open_price'] - 1


    ### get variance onf returns among the last 14 days
    daily = (df.sort_values(['symbol','day'])
            .groupby(['symbol','day'], as_index=False)['return'].last())

    daily[f'ret_std_14d'] = (
        daily.groupby('symbol')['return']
            .transform(lambda s: s.rolling(sigma_leverage_period, min_periods=sig-
ma_leverage_period).std())
    )
```

```
    df = df.merge(daily[['symbol','day', f'ret_std_14d']], on=['sym-
bol','day'], how='left')


    df = df.sort_values(['symbol', 'day', 'hour'])

    results = backtest(df, 1, sigma_target)

    return results
```

```
results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2)
results
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	375872.516755	378089.918236	2217.401481	0.005899	True
3739	2024-12-26 00:00:00-05:00	378089.918236	378089.918236	0.000000	0.000000	False
3740	2024-12-27 00:00:00-05:00	378089.918236	379093.073425	1003.155189	0.002653	True
3741	2024-12-30 00:00:00-05:00	379093.073425	378720.321267	-372.752158	-0.000983	True
3742	2024-12-31 00:00:00-05:00	378720.321267	377592.755574	-1127.565694	-0.002977	True

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
```

```
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
```

```
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

```
{'total_return': 2.77592755573733,
 'irr_annual': 0.09357483221743235,
 'vol_annual': 0.08155040941442078,
 'sharpe': 1.1376730719827486,
 'hit_ratio': 0.4264705882352941,
 'mdd': -0.1516056300345067,
 'n_days': 3743,
 'n_traded_days': 2312}
```

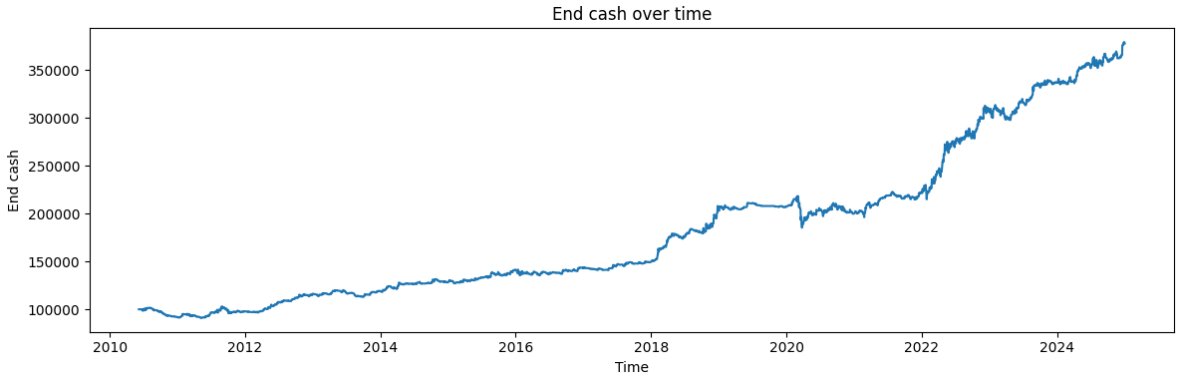
```
s = results.copy()

# sp_df = pd.read_csv("/data/workspace_files/SP500.csv")
# sp_df['SP500'] = 189095.59434874536 * sp_df['SP500'] / 1938.76
# sp_df

# If you have a Time column, make it the datetime index
if "Time" in s.columns:
    s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
    s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

# if "observation_date" in sp_df.columns:
#     sp_df["observation_date"] = pd.to_datetime(sp_df["observation_date"], er-
rors="coerce")
#     sp_df = sp_df.dropna(subset=["observation_date"]).sort_values("observation_-
date").set_index("observation_date")

# Plot end_cash over time
plt.figure(figsize=(12,4))
plt.plot(s.date, s["end_cash"].astype(float))
# plt.plot(sp_df['SP500'].astype(float))
plt.title("End cash over time")
plt.xlabel("Time")
plt.ylabel("End cash")
plt.tight_layout()
plt.show()
```



only positive momentum

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math

import datetime as dt

import matplotlib.dates as mdates
```

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization, sigma_lever-
age_period, sigma_leverage_bound, sigma_target):

    df = pd.read_csv('/data/workspace_files/nasdaq.csv')
    df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

    # (Optional) drop bad rows
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST)
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')

    # Build hour/day AFTER conversion
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date

    # Keep only US RTH 09:30-16:00 ET
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Sanity check (optional)
    tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()

    ### order by symbol, hour and day
    df = df.sort_values(['symbol', 'hour', 'day'])

    ### price
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


    ### opening price
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')
```

```
    # settling price and previous day settling price
    daily_settle = (
        df.sort_values(['symbol','day','Time'])
        .groupby(['symbol','day'])['price']
        .last()
        .rename('settl_price')
    )

    prev_settle = (
        daily_settle.groupby(level='symbol')
                    .shift(1)
                    .rename('settl_price_previous')
    )

    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])


    ### move since opening
    df["move"] = np.where(
        df["open_price"] > 0,
        np.abs(df["price"] / df["open_price"] - 1),
        np.nan
    )


    ### compute mean of move over the last period
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
        .transform(lambda x: x.rolling(sigma_leverage_bound, min_periods=sigma_lever-
age_bound).mean())
    )


    ### compute upper bound
    df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1
 + coef_sigma * df['sigma_observed']) + percentage_penalization * df['open_price']


    ### compute lower bound
    df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1
 - coef_sigma * df['sigma_observed']) - percentage_penalization * df['open_price']


    ### get return
    df['return'] = df['settl_price']/df['open_price'] - 1
```

```
    ### get variance onf returns among the last 14 days
    daily = (df.sort_values(['symbol','day'])
            .groupby(['symbol','day'], as_index=False)['return'].last())

    daily[f'ret_std_14d'] = (
        daily.groupby('symbol')['return']
            .transform(lambda s: s.rolling(sigma_leverage_period, min_periods=sig-
ma_leverage_period).std())
    )

    df = df.merge(daily[['symbol','day', f'ret_std_14d']], on=['sym-
bol','day'], how='left')


    df = df.sort_values(['symbol', 'day', 'hour'])

    results = backtest(df, 1, sigma_target)

    return results
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_tar-
get)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_target):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]

    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()
    price_list   = backtest_df['price'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
ower_bound) == len(price_list) == n) or n == 0:
        return initial_cash

    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
```

```
        np.isnan(lower_bound).any() or np.isnan(price_list).any()):
        return initial_cash

    position = 0
    cash = float(initial_cash)

    for i in range(1, len(price_list) - 1):
        if price_list[i] > max(upper_bound[i], vwap_list[i]):
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * price_list[i] * 1.00001
                # print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")

        # elif price_list[i] < min(vwap_list[i], lower_bound[i]):
        #     # we must be short and sell more than what we already have
        #     if -total_shares_to_hold < position:
        #         shares_to_sell = position + total_shares_to_hold
        #         position -= shares_to_sell
        #         initial_cash += shares_to_sell * price_list[i] * 0.99999
        #         # print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str
} at {hour[i]}")

        else:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * price_list[i] * 0.99999
                # print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {ho
ur[i]}")
                position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
        initial_cash += position * price_list[-1] * 0.99999
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization, sigma_lever-
age_period, sigma_leverage_bound, sigma_target):

    df = pd.read_csv('/data/workspace_files/nasdaq.csv')
    df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]
```

```
df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

# (Optional) drop bad rows
df = df.dropna(subset=['Time'])

# Convert UTC -> New York (handles DST)
df['Time'] = df['Time'].dt.tz_convert('America/New_York')

# Build hour/day AFTER conversion
df['hour'] = df['Time'].dt.strftime('%H:%M')
df['day']  = df['Time'].dt.date

# Keep only US RTH 09:30-16:00 ET
market_open, market_close = dt.time(9, 30), dt.time(16, 0)
df = df[df['Time'].dt.time.between(market_open, market_close)]

# Sanity check (optional)
tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()

### order by symbol, hour and day
df = df.sort_values(['symbol', 'hour', 'day'])

### price
df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


### opening price
df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


# settling price and previous day settling price
daily_settle = (
    df.sort_values(['symbol','day','Time'])
    .groupby(['symbol','day'])['price']
    .last()
    .rename('settl_price')
)

prev_settle = (
    daily_settle.groupby(level='symbol')
                .shift(1)
                .rename('settl_price_previous')
)

df = df.join(daily_settle, on=['symbol','day'])
df = df.join(prev_settle,   on=['symbol','day'])
```

```
    ### move since opening
    df["move"] = np.where(
        df["open_price"] > 0,
        np.abs(df["price"] / df["open_price"] - 1),
        np.nan
    )


    ### compute mean of move over the last period
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
        .transform(lambda x: x.rolling(sigma_leverage_bound, min_periods=sigma_lever-
age_bound).mean())
    )


    ### compute upper bound
    df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1
 + coef_sigma * df['sigma_observed']) + percentage_penalization * df['open_price']


    ### compute lower bound
    df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1
 - coef_sigma * df['sigma_observed']) - percentage_penalization * df['open_price']


    ### get return
    df['return'] = df['settl_price']/df['open_price'] - 1


    ### get variance onf returns among the last 14 days
    daily = (df.sort_values(['symbol','day'])
            .groupby(['symbol','day'], as_index=False)['return'].last())

    daily[f'ret_std_14d'] = (
        daily.groupby('symbol')['return']
            .transform(lambda s: s.rolling(sigma_leverage_period, min_periods=sig-
ma_leverage_period).std())
    )

    df = df.merge(daily[['symbol','day', f'ret_std_14d']], on=['sym-
bol','day'], how='left')


    df = df.sort_values(['symbol', 'day', 'hour'])
```

```
    results = backtest(df, 1, sigma_target)

    return results
```

```
results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2)
results
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	246798.848844	248233.638037	1434.789194	0.005814	True
3739	2024-12-26 00:00:00-05:00	248233.638037	248233.638037	0.000000	0.000000	False
3740	2024-12-27 00:00:00-05:00	248233.638037	248233.638037	0.000000	0.000000	False
3741	2024-12-30 00:00:00-05:00	248233.638037	248233.638037	0.000000	0.000000	False
3742	2024-12-31 00:00:00-05:00	248233.638037	248233.638037	0.000000	0.000000	False

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)
```

```
    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
```

```
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

```
{'total_return': 1.482336380374937,
 'irr_annual': 0.06312482246920781,
 'vol_annual': 0.051355188095698494,
 'sharpe': 1.2176129210070048,
 'hit_ratio': 0.4423380726698262,
 'mdd': -0.06261665510758299,
 'n_days': 3743,
 'n_traded_days': 1266}
```

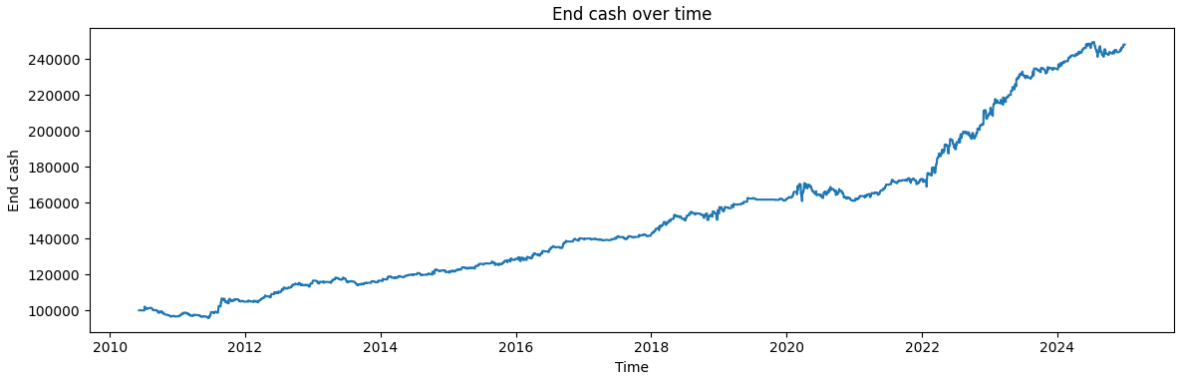
```
s = results.copy()

sp_df = pd.read_csv("/data/workspace_files/SP500.csv")
sp_df['SP500'] = 0
sp_df

# If you have a Time column, make it the datetime index
if "Time" in s.columns:
    s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
    s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

if "observation_date" in sp_df.columns:
    sp_df["observation_date"] = pd.to_datetime(sp_df["observation_date"], errors="co-
erce")
    sp_df = sp_df.dropna(subset=["observation_date"]).sort_values("observation_-
date").set_index("observation_date")

# Plot end_cash over time
plt.figure(figsize=(12,4))
plt.plot(s.date, s["end_cash"].astype(float))
plt.title("End cash over time")
plt.xlabel("Time")
plt.ylabel("End cash")
plt.tight_layout()
plt.show()
```




```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math

import datetime as dt

import matplotlib.dates as mdates
```

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization, sigma_lever-
age_period, sigma_leverage_bound, sigma_target):

    df = pd.read_csv('/data/workspace_files/nasdaq.csv')
    df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

    # (Optional) drop bad rows
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST)
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')

    # Build hour/day AFTER conversion
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date

    # Keep only US RTH 09:30-16:00 ET
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Sanity check (optional)
    tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()

    ### order by symbol, hour and day
    df = df.sort_values(['symbol', 'hour', 'day'])

    ### price
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


    ### opening price
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


    # settling price and previous day settling price
    daily_settle = (
        df.sort_values(['symbol','day','Time'])
        .groupby(['symbol','day'])['price']
        .last()
        .rename('settl_price')
    )

    prev_settle = (
        daily_settle.groupby(level='symbol')
```

```
                    .shift(1)
                    .rename('settl_price_previous')
    )

    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])


    ### move since opening
    df["move"] = np.where(
        df["open_price"] > 0,
        np.abs(df["price"] / df["open_price"] - 1),
        np.nan
    )


    ### compute mean of move over the last period
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
        .transform(lambda x: x.rolling(sigma_leverage_bound, min_periods=sigma_lever-
age_bound).mean())
    )


    ### compute upper bound
    df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1
 + coef_sigma * df['sigma_observed']) + percentage_penalization * df['open_price']


    ### compute lower bound
    df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1
 - coef_sigma * df['sigma_observed']) - percentage_penalization * df['open_price']


    ### get return
    df['return'] = df['settl_price']/df['open_price'] - 1


    ### get variance onf returns among the last 14 days
    daily = (df.sort_values(['symbol','day'])
            .groupby(['symbol','day'], as_index=False)['return'].last())

    daily[f'ret_std_14d'] = (
        daily.groupby('symbol')['return']
            .transform(lambda s: s.rolling(sigma_leverage_period, min_periods=sig-
ma_leverage_period).std())
    )
```

```
    df = df.merge(daily[['symbol','day', f'ret_std_14d']], on=['sym-
bol','day'], how='left')


    df = df.sort_values(['symbol', 'day', 'hour'])

    results = backtest(df, 1, sigma_target)

    return results
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_tar-
get)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_target):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]

    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()
    price_list   = backtest_df['price'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
ower_bound) == len(price_list) == n) or n == 0:
        return initial_cash

    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
```

```
        np.isnan(lower_bound).any() or np.isnan(price_list).any()):
        return initial_cash

    position = 0
    cash = float(initial_cash)

    for i in range(1, len(price_list) - 1):
        if price_list[i] < min(vwap_list[i], lower_bound[i]):
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * price_list[i] * 0.99999
                # print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str}
at {hour[i]}")

        else:
            if position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * price_list[i] * 1.00001
                # print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")
                position = 0

    if position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * price_list[-1] * 1.00001
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization, sigma_lever-
age_period, sigma_leverage_bound, sigma_target):

    df = pd.read_csv('/data/workspace_files/nasdaq.csv')
    df = df[['Time', 'symbol', 'expiry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum']]

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)

    # (Optional) drop bad rows
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST)
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')

    # Build hour/day AFTER conversion
    df['hour'] = df['Time'].dt.strftime('%H:%M')
```

```
df['day']  = df['Time'].dt.date

# Keep only US RTH 09:30-16:00 ET
market_open, market_close = dt.time(9, 30), dt.time(16, 0)
df = df[df['Time'].dt.time.between(market_open, market_close)]

# Sanity check (optional)
tmin, tmax = df['Time'].dt.time.min(), df['Time'].dt.time.max()

### order by symbol, hour and day
df = df.sort_values(['symbol', 'hour', 'day'])

### price
df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])


### opening price
df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')


# settling price and previous day settling price
daily_settle = (
    df.sort_values(['symbol','day','Time'])
    .groupby(['symbol','day'])['price']
    .last()
    .rename('settl_price')
)

prev_settle = (
    daily_settle.groupby(level='symbol')
                .shift(1)
                .rename('settl_price_previous')
)

df = df.join(daily_settle, on=['symbol','day'])
df = df.join(prev_settle,   on=['symbol','day'])


### move since opening
df["move"] = np.where(
    df["open_price"] > 0,
    np.abs(df["price"] / df["open_price"] - 1),
    np.nan
)


### compute mean of move over the last period
```

```
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
        .transform(lambda x: x.rolling(sigma_leverage_bound, min_periods=sigma_lever-
age_bound).mean())
    )


    ### compute upper bound
    df['upper_bound'] = (df[['open_price', 'settl_price_previous']].max(axis=1)) * (1
 + coef_sigma * df['sigma_observed']) + percentage_penalization * df['open_price']


    ### compute lower bound
    df['lower_bound'] = (df[['open_price', 'settl_price_previous']].min(axis=1)) * (1
 - coef_sigma * df['sigma_observed']) - percentage_penalization * df['open_price']


    ### get return
    df['return'] = df['settl_price']/df['open_price'] - 1


    ### get variance onf returns among the last 14 days
    daily = (df.sort_values(['symbol','day'])
            .groupby(['symbol','day'], as_index=False)['return'].last())

    daily[f'ret_std_14d'] = (
        daily.groupby('symbol')['return']
            .transform(lambda s: s.rolling(sigma_leverage_period, min_periods=sig-
ma_leverage_period).std())
    )

    df = df.merge(daily[['symbol','day', f'ret_std_14d']], on=['sym-
bol','day'], how='left')


    df = df.sort_values(['symbol', 'day', 'hour'])

    results = backtest(df, 1, sigma_target)

    return results
```

```
results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2)
results
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	151938.417164	151938.417164	0.000000	0.000000	False
3739	2024-12-26 00:00:00-05:00	151938.417164	151938.417164	0.000000	0.000000	False
3740	2024-12-27 00:00:00-05:00	151938.417164	152292.471936	354.054773	0.002330	True
3741	2024-12-30 00:00:00-05:00	152292.471936	152138.985754	-153.486182	-0.001008	True
3742	2024-12-31 00:00:00-05:00	152138.985754	151674.693997	-464.291756	-0.003052	True

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1

    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
```

```
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

```
{'total_return': 0.5167469399749711,
 'irr_annual': 0.028442696429518577,
 'vol_annual': 0.0624866449879369,
 'sharpe': 0.4799261828451119,
 'hit_ratio': 0.38054607508532423,
 'mdd': -0.14388989690194934,
 'n_days': 3743,
 'n_traded_days': 1172}
```

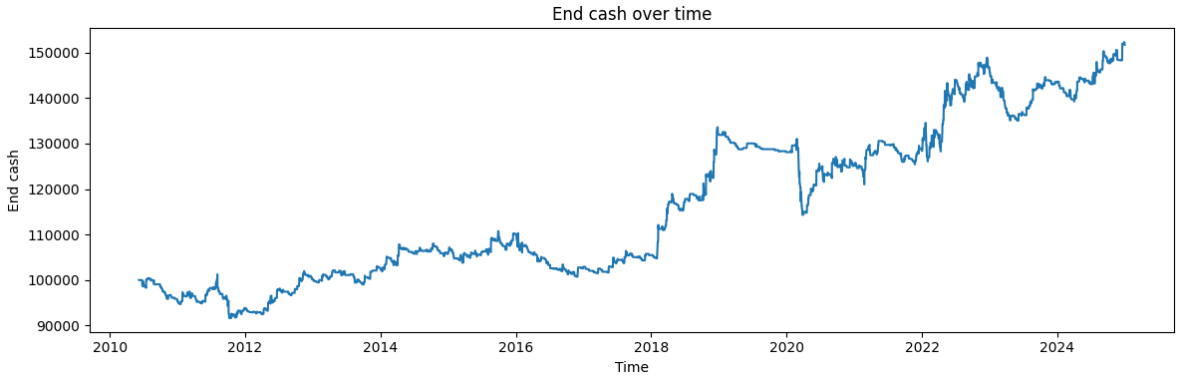
```
s = results.copy()

sp_df = pd.read_csv("/data/workspace_files/SP500.csv")
sp_df['SP500'] = 0
sp_df

# If you have a Time column, make it the datetime index
if "Time" in s.columns:
    s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
    s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

if "observation_date" in sp_df.columns:
    sp_df["observation_date"] = pd.to_datetime(sp_df["observation_date"], errors="co-
erce")
    sp_df = sp_df.dropna(subset=["observation_date"]).sort_values("observation_-
date").set_index("observation_date")

# Plot end_cash over time
plt.figure(figsize=(12,4))
plt.plot(s.date, s["end_cash"].astype(float))
plt.title("End cash over time")
plt.xlabel("Time")
plt.ylabel("End cash")
plt.tight_layout()
plt.show()
```



```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import math

import datetime as dt

import matplotlib.dates as mdates
```

volatility lower threshold

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization,
                                  sigma_leverage_period, sigma_leverage_bound, sig-
ma_target, VIX_treshold):

    # ---------- Load & prepare intraday data ----------
    df = pd.read_csv('/data/workspace_files/nasdaq.csv',
                     usecols=['Time', 'symbol', 'ex-
piry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum'])

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST) and build hour/day on NY time
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date        # <-- keep as datetime.-
date (DO NOT stringify)

    # US RTH 09:30-16:00 ET
    import datetime as dt
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Order & mid price
    df = df.sort_values(['symbol', 'hour', 'day'])
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])

    # Opening price per symbol-day
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')

    # Settle and previous settle (per symbol/day)
    daily_settle = (
```

```
        df.sort_values(['symbol','day','Time'])
          .groupby(['symbol','day'])['price'].last()
          .rename('settl_price')
    )
    prev_settle = (
        daily_settle.groupby(level='symbol').shift(1).rename('settl_price_previous')
    )
    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])

    # Intraday move vs open
    df['move'] = np.where(
        df['open_price'] > 0,
        np.abs(df['price'] / df['open_price'] - 1),
        np.nan
    )

    # Rolling mean of move by (symbol, hour)
    win = int(sigma_leverage_bound) if sigma_leverage_bound is not None else 1
    if win < 1:
        win = 1
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
          .transform(lambda x: x.rolling(win, min_periods=win).mean())
    )

    # Bands
    df['upper_bound'] = df[['open_price','settl_price_previous']].max(axis=1) * (1 +
coef_sigma * df['sigma_observed']) \
                        + percentage_penalization * df['open_price']
    df['lower_bound'] = df[['open_price','settl_price_previous']].min(axis=1) * (1 -
 coef_sigma * df['sigma_observed']) \
                        - percentage_penalization * df['open_price']

    # End-of-day return
    df['return'] = df['settl_price'] / df['open_price'] - 1

    # 14D rolling std of returns per symbol
    daily = (df.sort_values(['symbol','day'])
               .groupby(['symbol','day'], as_index=False)['return'].last())
    daily['ret_std_14d'] = (
        daily.groupby('symbol')['return']
             .transform(lambda s: s.rolling(int(sigma_leverage_period), min_periods=i
nt(sigma_leverage_period)).std())
    )
    df = df.merge(daily[['symbol','day','ret_std_14d']], on=['sym-
bol','day'], how='left')
```

```
    df = df.sort_values(['symbol', 'day', 'hour'])

    # ---------- Load & merge VIX (daily) ----------
    vol_df = pd.read_csv('/data/workspace_files/VIX_History.csv', usecols=
['DATE', 'CLOSE'])
    vol_df = vol_df.rename(columns={'DATE':'day', 'CLOSE':'VIX'})

    # Parse to timezone-naive dates and shift by one day (use yesterday's VIX)
    vol_df['day'] = pd.to_datetime(vol_df['day'], format='%m/%d/%Y', errors='coerce')
.dt.date
    vol_df = vol_df.dropna(subset=['day'])
    vol_df = vol_df.sort_values('day')
    vol_df['VIX'] = pd.to_numeric(vol_df['VIX'], errors='coerce').shift(1)

    # Ensure unique per-day (take last if duplicates)
    vol_df = vol_df.groupby('day', as_index=False).last()

    # Some trading days may not have VIX (holidays). Build a calen-
dar from df['day'] and ffill VIX.
    all_days = pd.DataFrame({'day': sorted(df['day'].unique())})
    vol_df = all_days.merge(vol_df, on='day', how='left').sort_values('day')
    vol_df['VIX'] = vol_df['VIX'].ffill()

    # Merge back (left join; broadcasts daily VIX to intraday rows)
    df = df.merge(vol_df, on='day', how='left')

    # ---------- Run backtest ----------
    results = backtest(df, 1, sigma_target, VIX_treshold)
    return results
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, VIX_treshold, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_target, VIX_treshold)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_tar-
get, VIX_treshold):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    if backtest_df['VIX'].iloc[0] < VIX_treshold:
        return initial_cash

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]

    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()
    price_list   = backtest_df['price'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
ower_bound) == len(price_list) == n) or n == 0:
        return initial_cash
```

```
    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
        np.isnan(lower_bound).any() or np.isnan(price_list).any()):
        return initial_cash

    position = 0
    cash = float(initial_cash)



    for i in range(1, len(price_list) - 1):
        if price_list[i] > max(upper_bound[i], vwap_list[i]):
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * price_list[i] * 1.00001
                # print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")

        elif price_list[i] < min(vwap_list[i], lower_bound[i]):
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * price_list[i] * 0.99999
                # print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str}
at {hour[i]}")

        elif price_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * price_list[i] * 0.99999
                # print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {ho
ur[i]}")
                position = 0
            elif position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * price_list[i] * 1.00001
                # print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")
                position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
```

```
        initial_cash += position * price_list[-1] * 0.99999
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * price_list[-1] * 1.00001
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2, 0)
results
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	375872.516755	378089.918236	2217.401481	0.005899	True
3739	2024-12-26 00:00:00-05:00	378089.918236	378089.918236	0.000000	0.000000	False
3740	2024-12-27 00:00:00-05:00	378089.918236	379093.073425	1003.155189	0.002653	True
3741	2024-12-30 00:00:00-05:00	379093.073425	378720.321267	-372.752158	-0.000983	True
3742	2024-12-31 00:00:00-05:00	378720.321267	377592.755574	-1127.565694	-0.002977	True

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1
```

```
    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
```

```
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

```
{'total_return': 2.77592755573733,
 'irr_annual': 0.09357483221743235,
 'vol_annual': 0.08155040941442078,
 'sharpe': 1.1376730719827486,
 'hit_ratio': 0.4264705882352941,
 'mdd': -0.1516056300345067,
 'n_days': 3743,
 'n_traded_days': 2312}
```

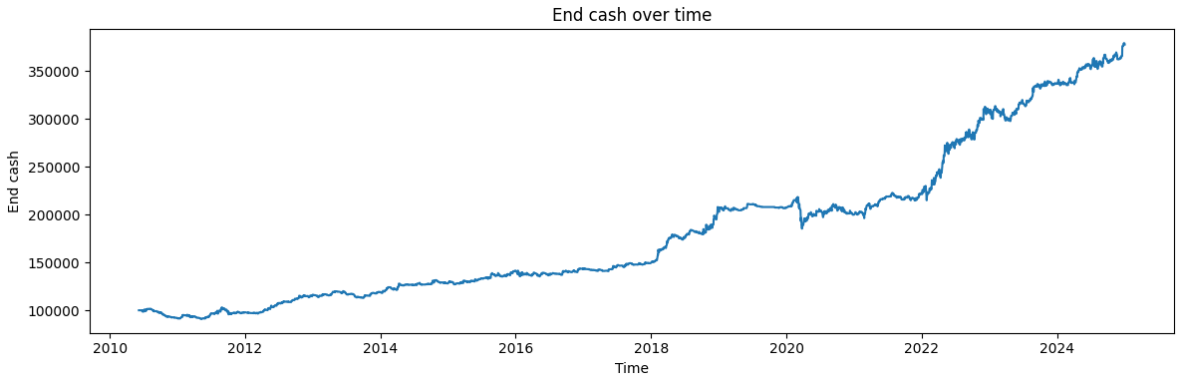
```
s = results.copy()

sp_df = pd.read_csv("/data/workspace_files/SP500.csv")
sp_df['SP500'] = 189095.59434874536 * sp_df['SP500'] / 1938.76
sp_df

# If you have a Time column, make it the datetime index
if "Time" in s.columns:
    s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
    s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

if "observation_date" in sp_df.columns:
    sp_df["observation_date"] = pd.to_datetime(sp_df["observation_date"], errors="co-
erce")
    sp_df = sp_df.dropna(subset=["observation_date"]).sort_values("observation_-
date").set_index("observation_date")

# Plot end_cash over time
plt.figure(figsize=(12,4))
plt.plot(s.date, s["end_cash"].astype(float))
# plt.plot(sp_df['SP500'].astype(float))
plt.plot
plt.title("End cash over time")
plt.xlabel("Time")
plt.ylabel("End cash")
plt.tight_layout()
plt.show()
```

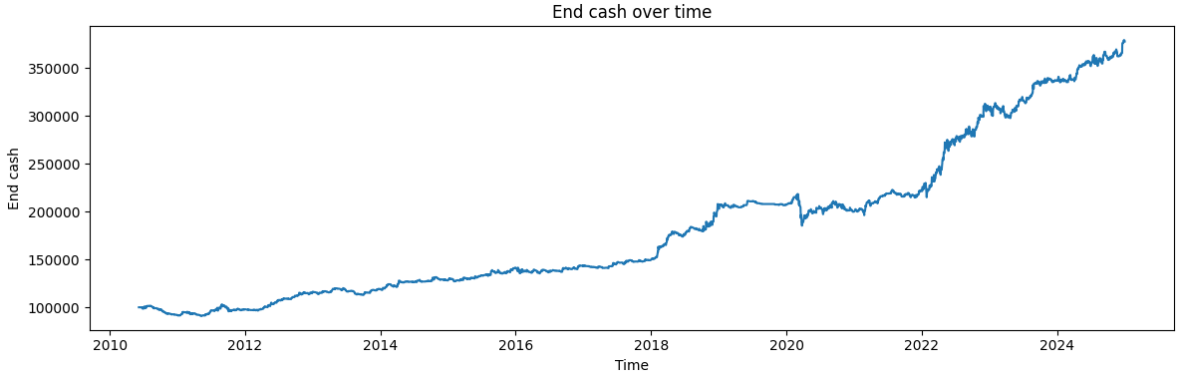


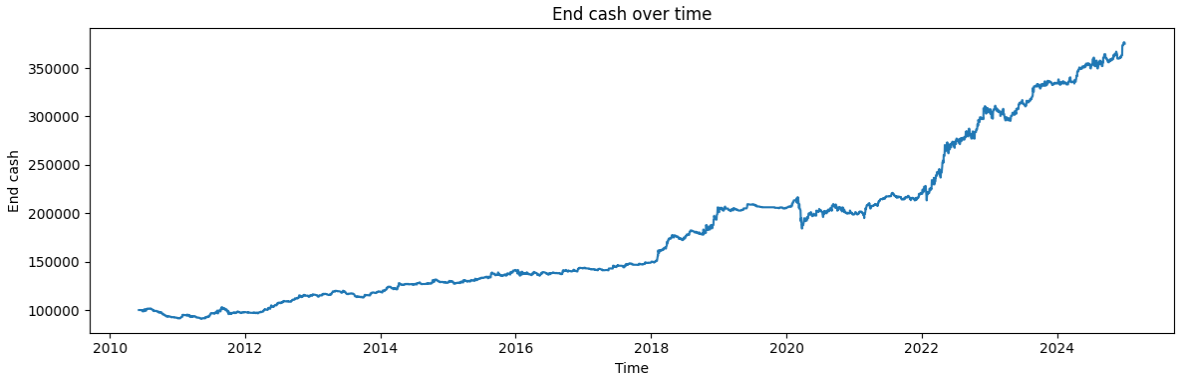
```
for i in [0, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40]:
    results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2, i)
    print(f"Results for VIX treshold: {i}")
    print(evaluate_strategy(results))
    s = results.copy()

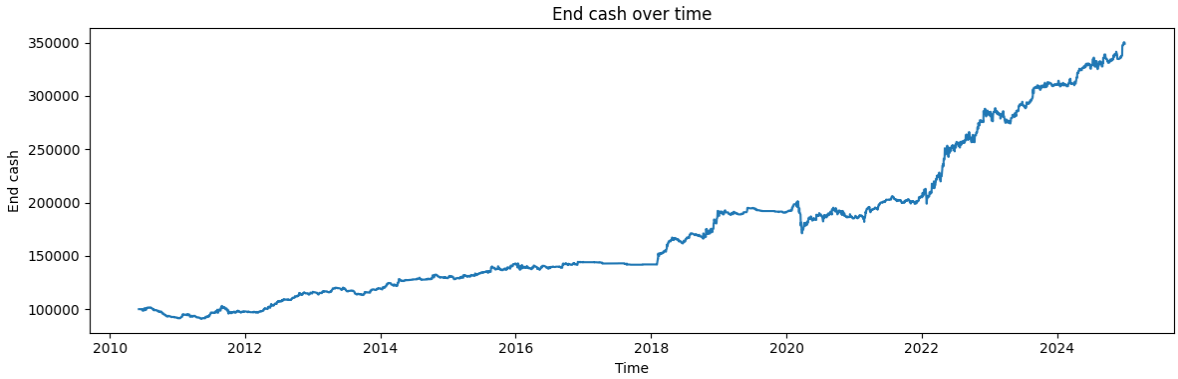
    # If you have a Time column, make it the datetime index
    if "Time" in s.columns:
        s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
        s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

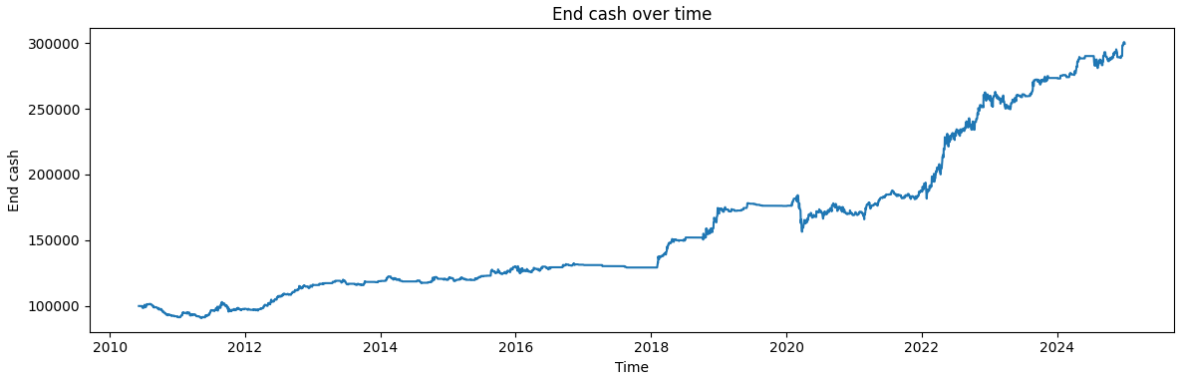
    # Plot end_cash over time
    plt.figure(figsize=(12,4))
    plt.plot(s.date, s["end_cash"].astype(float))
    # plt.plot(sp_df['SP500'].astype(float))
    plt.plot
    plt.title("End cash over time")
    plt.xlabel("Time")
    plt.ylabel("End cash")
    plt.tight_layout()
    plt.show()
```

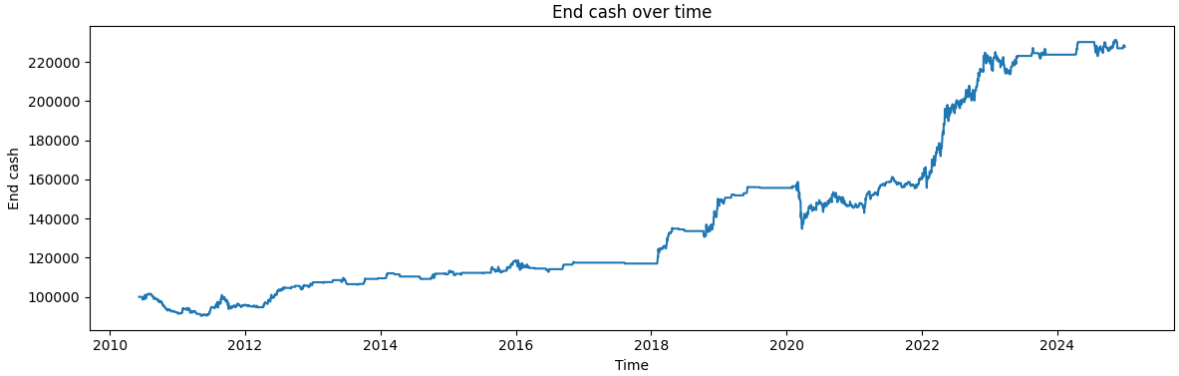
```
Results for VIX treshold: 0
{'total_return': 2.77592755573733, 'irr_annual': 0.09357483221743235, 'vol_annual': 0.
Results for VIX treshold: 10
{'total_return': 2.751834151449839, 'irr_annual': 0.09310363857277038, 'vol_annual': 0
Results for VIX treshold: 12
{'total_return': 2.489806535499853, 'irr_annual': 0.08778850998450594, 'vol_annual': 0
Results for VIX treshold: 14
{'total_return': 1.9949789835623912, 'irr_annual': 0.07664749073291088, 'vol_annual':
Results for VIX treshold: 16
{'total_return': 1.277185841224926, 'irr_annual': 0.05696861889868843, 'vol_annual': 0
Results for VIX treshold: 18
{'total_return': 0.7669102263999552, 'irr_annual': 0.039067788957307226, 'vol_annual':
Results for VIX treshold: 20
{'total_return': 0.2937424366999659, 'irr_annual': 0.017490188763146453, 'vol_annual':
Results for VIX treshold: 22
{'total_return': 0.22778264833747808, 'irr_annual': 0.01391176985138709, 'vol_annual':
Results for VIX treshold: 24
{'total_return': 0.10731443823748843, 'irr_annual': 0.006886626144926522, 'vol_annual'
Results for VIX treshold: 26
{'total_return': -0.056366789300010156, 'irr_annual': -0.00389846441665076, 'vol_annua
```

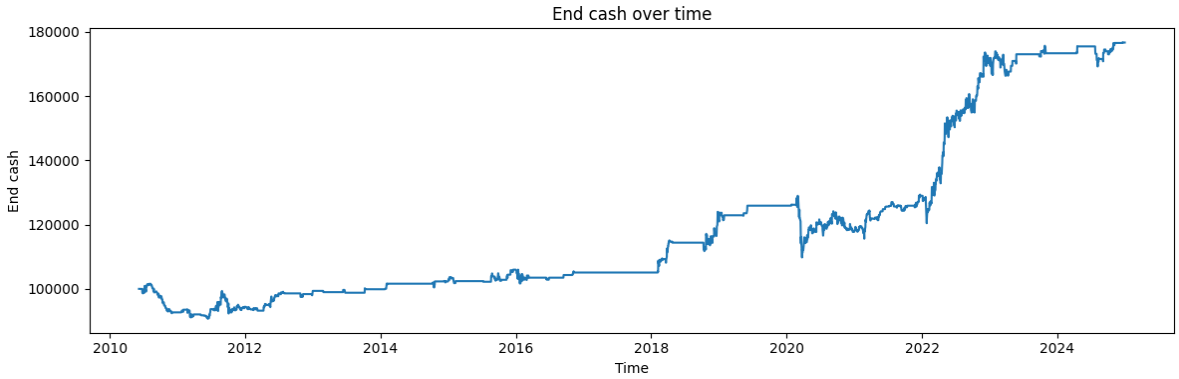

End cash over time
End cash
Time
100000
150000
200000
250000
300000
350000
2010
2012
2014
2016
2018
2020
2022
2024

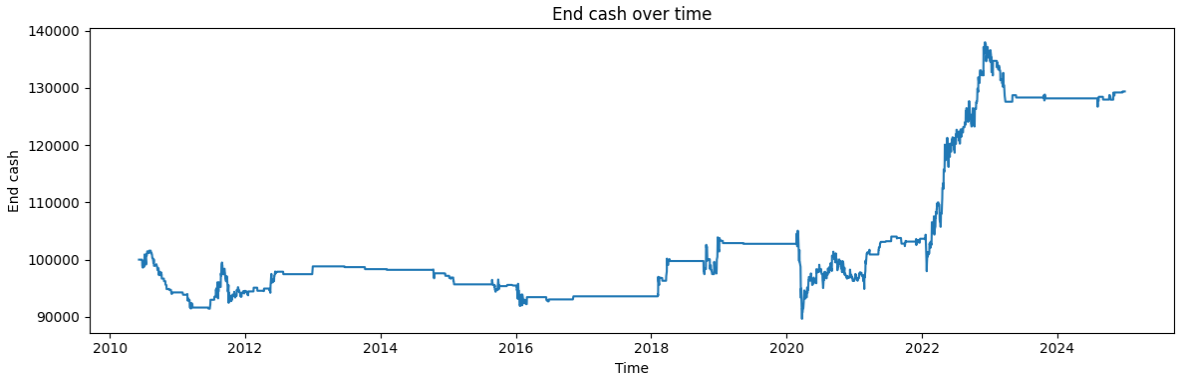

End cash over time
End cash
Time
100000
150000
200000
250000
300000
350000
2010
2012
2014
2016
2018
2020
2022
2024

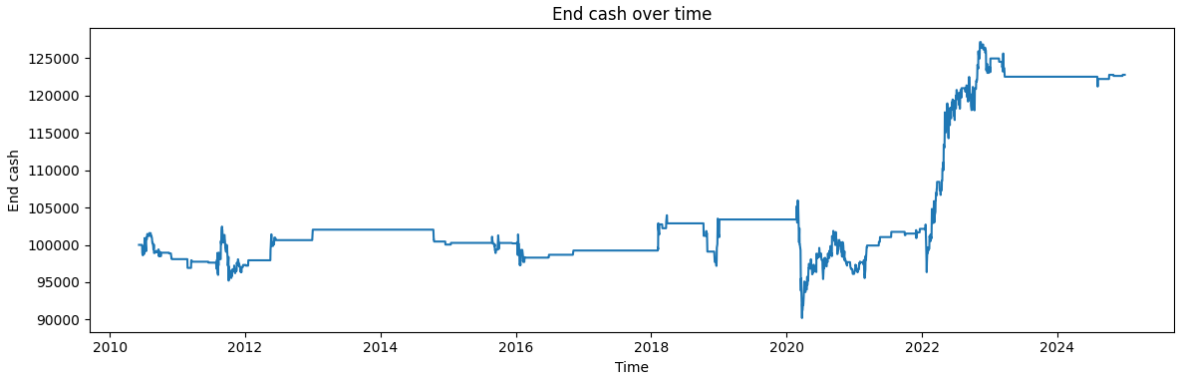

End cash over time
End cash
Time
100000
150000
200000
250000
300000
350000
2010
2012
2014
2016
2018
2020
2022
2024

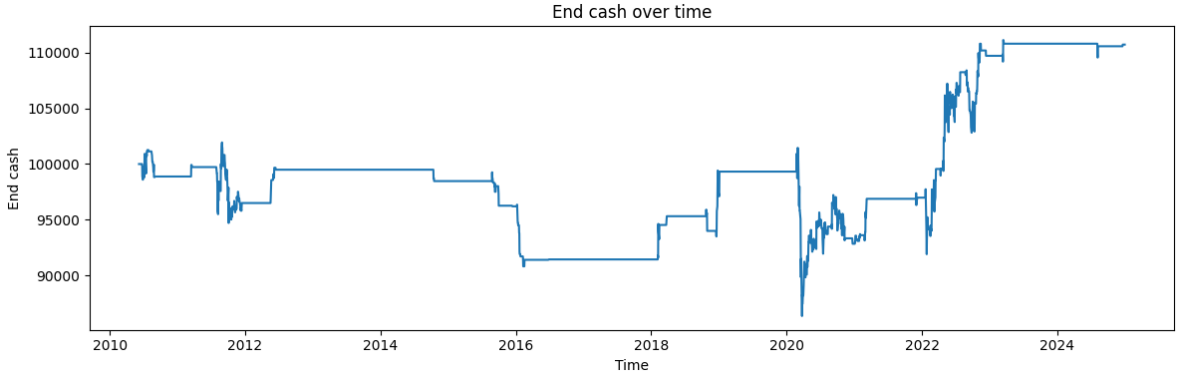

End cash over time
End cash
Time
100000
150000
200000
250000
300000
2010
2012
2014
2016
2018
2020
2022
2024

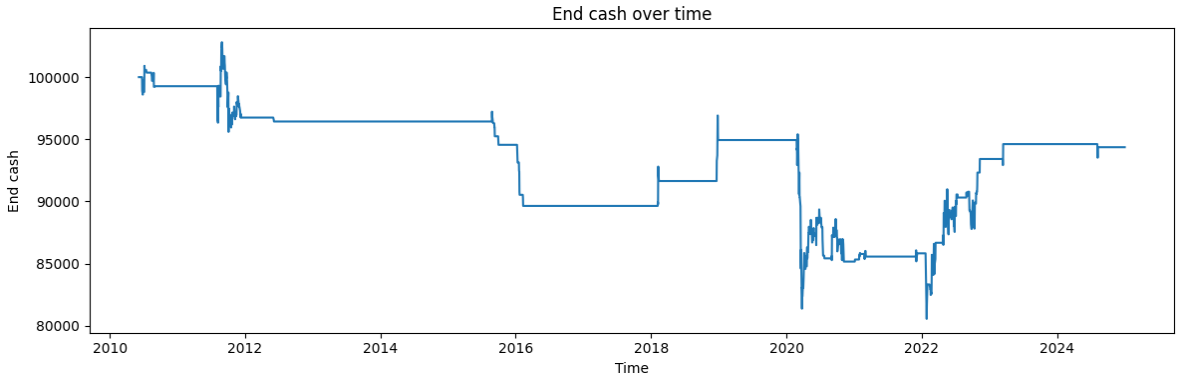

End cash over time
End cash
Time
220000
200000
180000
160000
140000
120000
100000
2010
2012
2014
2016
2018
2020
2022
2024

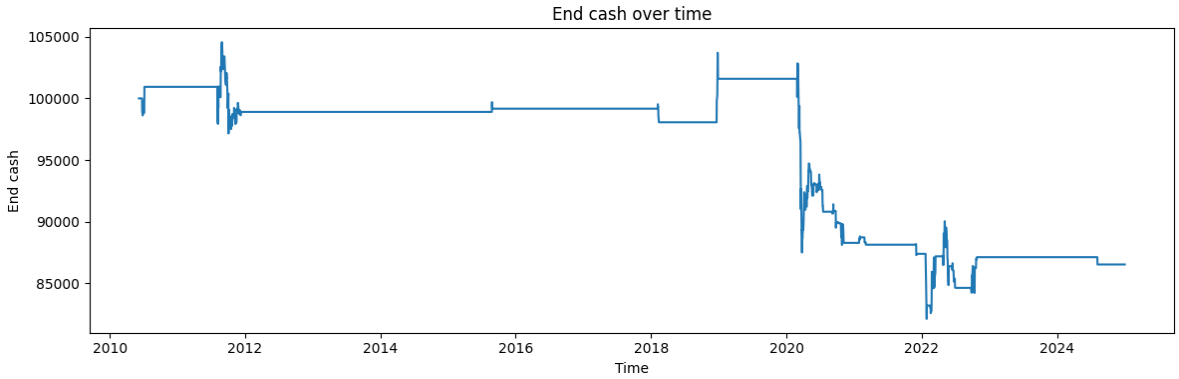

End cash over time
End cash
Time
180000
160000
140000
120000
100000
2010
2012
2014
2016
2018
2020
2022
2024

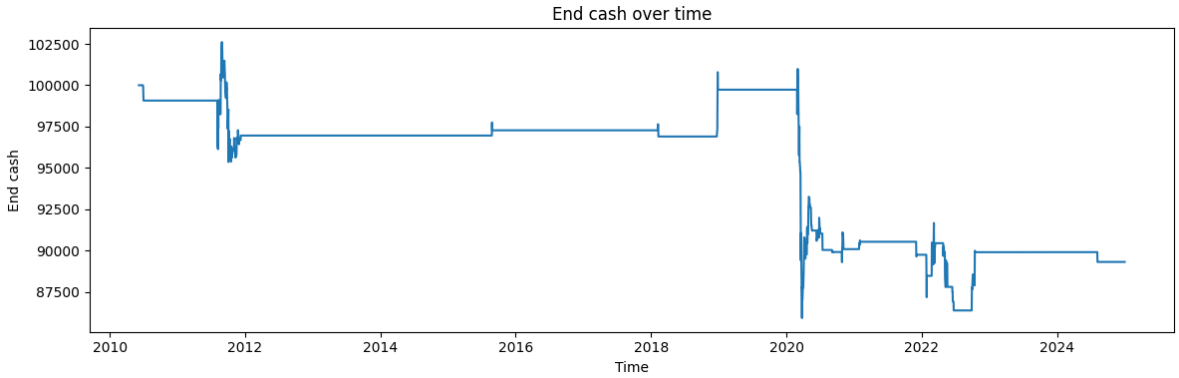

End cash over time
End cash
Time
140000
130000
120000
110000
100000
90000
2010
2012
2014
2016
2018
2020
2022
2024

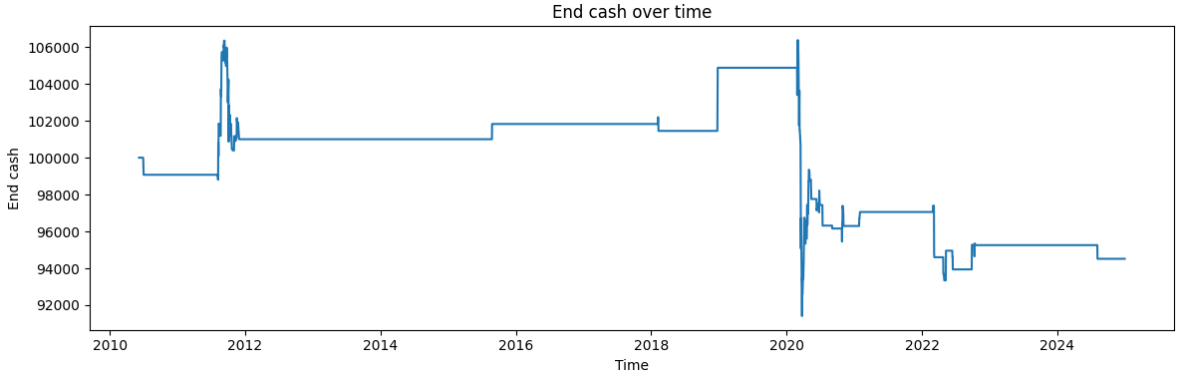

End cash over time
End cash
Time
125000
120000
115000
110000
105000
100000
95000
90000
2010
2012
2014
2016
2018
2020
2022
2024

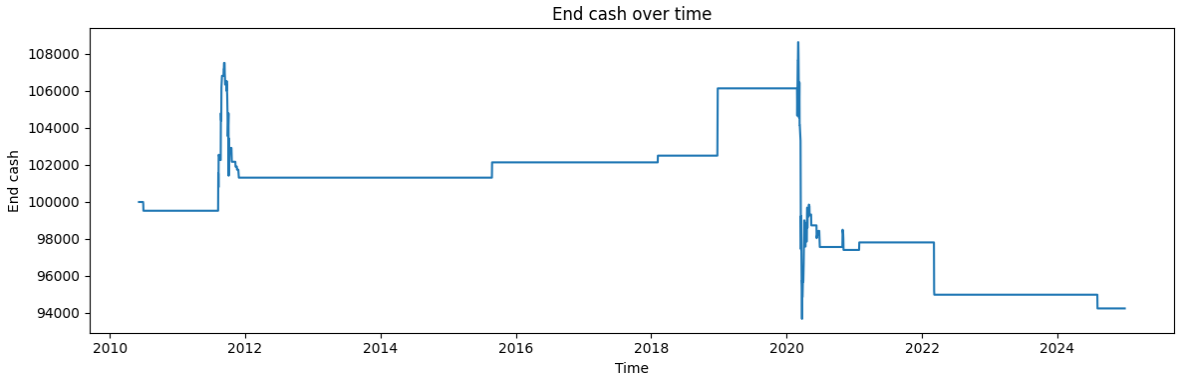

End cash over time
End cash
110000
105000
100000
95000
90000
2010
2012
2014
2016
2018
2020
2022
2024
Time

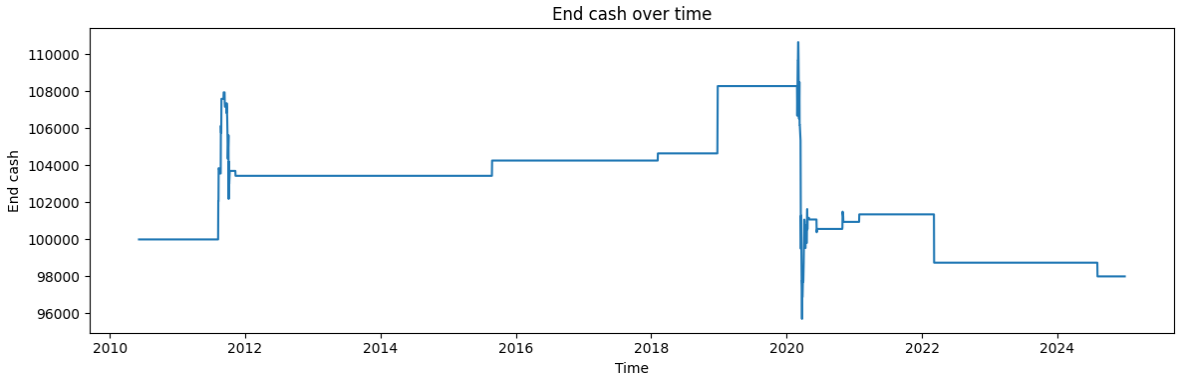

End cash over time
End cash
100000
95000
90000
85000
80000
2010
2012
2014
2016
2018
2020
2022
2024
Time

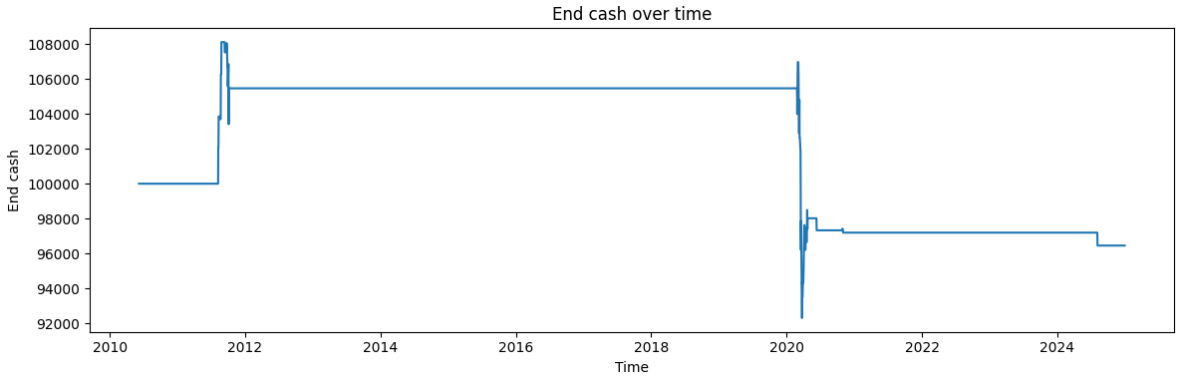

End cash over time
End cash
105000
100000
95000
90000
85000
2010
2012
2014
2016
2018
2020
2022
2024
Time

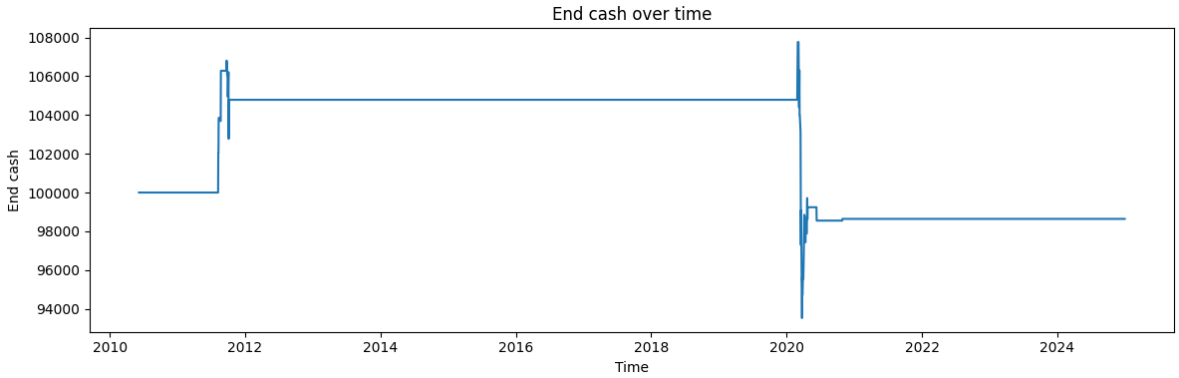

End cash over time
End cash
102500
100000
97500
95000
92500
90000
87500
2010
2012
2014
2016
2018
2020
2022
2024
Time


End cash over time
End cash
106000
104000
102000
100000
98000
96000
94000
92000
2010
2012
2014
2016
2018
2020
2022
2024
Time


End cash over time
End cash
108000
106000
104000
102000
100000
98000
96000
94000
2010
2012
2014
2016
2018
2020
2022
2024
Time


End cash over time
End cash
110000
108000
106000
104000
102000
100000
98000
96000
2010
2012
2014
2016
2018
2020
2022
2024
Time


End cash over time
End cash
108000
106000
104000
102000
100000
98000
96000
94000
92000
2010
2012
2014
2016
2018
2020
2022
2024
Time



Volatility upper threshold

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization,
                                  sigma_leverage_period, sigma_leverage_bound, sig-
ma_target, VIX_treshold):

    # ---------- Load & prepare intraday data ----------
    df = pd.read_csv('/data/workspace_files/nasdaq.csv',
                     usecols=['Time', 'symbol', 'ex-
piry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum'])

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST) and build hour/day on NY time
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date        # <-- keep as datetime.-
date (DO NOT stringify)

    # US RTH 09:30-16:00 ET
    import datetime as dt
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Order & mid price
    df = df.sort_values(['symbol', 'hour', 'day'])
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])

    # Opening price per symbol-day
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')

    # Settle and previous settle (per symbol/day)
```

```
    daily_settle = (
        df.sort_values(['symbol','day','Time'])
          .groupby(['symbol','day'])['price'].last()
          .rename('settl_price')
    )
    prev_settle = (
        daily_settle.groupby(level='symbol').shift(1).rename('settl_price_previous')
    )
    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])

    # Intraday move vs open
    df['move'] = np.where(
        df['open_price'] > 0,
        np.abs(df['price'] / df['open_price'] - 1),
        np.nan
    )

    # Rolling mean of move by (symbol, hour)
    win = int(sigma_leverage_bound) if sigma_leverage_bound is not None else 1
    if win < 1:
        win = 1
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
          .transform(lambda x: x.rolling(win, min_periods=win).mean())
    )

    # Bands
    df['upper_bound'] = df[['open_price','settl_price_previous']].max(axis=1) * (1 +
coef_sigma * df['sigma_observed']) \
                        + percentage_penalization * df['open_price']
    df['lower_bound'] = df[['open_price','settl_price_previous']].min(axis=1) * (1 -
 coef_sigma * df['sigma_observed']) \
                        - percentage_penalization * df['open_price']

    # End-of-day return
    df['return'] = df['settl_price'] / df['open_price'] - 1

    # 14D rolling std of returns per symbol
    daily = (df.sort_values(['symbol','day'])
               .groupby(['symbol','day'], as_index=False)['return'].last())
    daily['ret_std_14d'] = (
        daily.groupby('symbol')['return']
             .transform(lambda s: s.rolling(int(sigma_leverage_period), min_periods=i
nt(sigma_leverage_period)).std())
    )
    df = df.merge(daily[['symbol','day','ret_std_14d']], on=['sym-
```

```
bol','day'], how='left')

    df = df.sort_values(['symbol', 'day', 'hour'])

    # ---------- Load & merge VIX (daily) ----------
    vol_df = pd.read_csv('/data/workspace_files/VIX_History.csv', usecols=
['DATE', 'CLOSE'])
    vol_df = vol_df.rename(columns={'DATE':'day', 'CLOSE':'VIX'})

    # Parse to timezone-naive dates and shift by one day (use yesterday's VIX)
    vol_df['day'] = pd.to_datetime(vol_df['day'], format='%m/%d/%Y', errors='coerce')
.dt.date
    vol_df = vol_df.dropna(subset=['day'])
    vol_df = vol_df.sort_values('day')
    vol_df['VIX'] = pd.to_numeric(vol_df['VIX'], errors='coerce').shift(1)

    # Ensure unique per-day (take last if duplicates)
    vol_df = vol_df.groupby('day', as_index=False).last()

    # Some trading days may not have VIX (holidays). Build a calen-
dar from df['day'] and ffill VIX.
    all_days = pd.DataFrame({'day': sorted(df['day'].unique())})
    vol_df = all_days.merge(vol_df, on='day', how='left').sort_values('day')
    vol_df['VIX'] = vol_df['VIX'].ffill()

    # Merge back (left join; broadcasts daily VIX to intraday rows)
    df = df.merge(vol_df, on='day', how='left')

    # ---------- Run backtest ----------
    results = backtest(df, 1, sigma_target, VIX_treshold)
    return results
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, VIX_treshold, initial_cash=10
0_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_tar-
get, VIX_treshold)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_tar-
get, VIX_treshold):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    # if backtest_df['VIX'].iloc[0] >= VIX_treshold:
    #     return initial_cash

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]

    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()
    price_list   = backtest_df['price'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
ower_bound) == len(price_list) == n) or n == 0:
        return initial_cash
```

```
    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
        np.isnan(lower_bound).any() or np.isnan(price_list).any()):
        return initial_cash

    position = 0
    cash = float(initial_cash)



    for i in range(1, len(price_list) - 1):
        if price_list[i] > max(upper_bound[i], vwap_list[i]):
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * price_list[i] * 1.00001
                # print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")

        elif price_list[i] < min(vwap_list[i], lower_bound[i]):
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * price_list[i] * 0.99999
                # print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str}
at {hour[i]}")

        elif price_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * price_list[i] * 0.99999
                # print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {ho
ur[i]}")
                position = 0
            elif position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * price_list[i] * 1.00001
                # print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")
                position = 0

    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
```

```
        initial_cash += position * price_list[-1] * 0.99999
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * price_list[-1] * 1.00001
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2, np.inf)
results
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	375872.516755	378089.918236	2217.401481	0.005899	True
3739	2024-12-26 00:00:00-05:00	378089.918236	378089.918236	0.000000	0.000000	False
3740	2024-12-27 00:00:00-05:00	378089.918236	379093.073425	1003.155189	0.002653	True
3741	2024-12-30 00:00:00-05:00	379093.073425	378720.321267	-372.752158	-0.000983	True
3742	2024-12-31 00:00:00-05:00	378720.321267	377592.755574	-1127.565694	-0.002977	True

3743 rows × 6 columns

```
def evaluate_strategy(perf: pd.DataFrame, risk_free_rate: float = 0.0) -> dict:
    # Basic checks and ordering
    needed = {'date','start_cash','end_cash','pnl','return','traded'}
    missing = needed.difference(perf.columns)
    if missing:
        raise ValueError(f"Missing required columns: {sorted(missing)}")

    df = perf.copy().sort_values('date').reset_index(drop=True)

    # Total return (equity curve start->end)
    total_return = df["end_cash"].iloc[-1] / df["start_cash"].iloc[0] - 1
```

```
    # Daily returns (drop NaNs/infs quietly)
    daily_returns = pd.to_numeric(df["return"], errors="coerce").replace([np.inf, -
np.inf], np.nan).dropna()
    n_obs = len(daily_returns)

    # Geometric mean daily return (IRR over observed daily returns)
    if n_obs > 0:
        gross = (1.0 + daily_returns.values)
        irr_daily = np.prod(gross) ** (1.0 / n_obs) - 1.0
        irr_annual = (1.0 + irr_daily) ** 252 - 1.0
    else:
        irr_daily = np.nan
        irr_annual = np.nan

    # Volatility (annualized)
    if n_obs > 1:
        vol_daily = float(daily_returns.std(ddof=1))
        vol_annual = vol_daily * np.sqrt(252.0)
    else:
        vol_daily = np.nan
        vol_annual = np.nan

    # Sharpe (annualized), using annual rf converted to daily
    rf_daily = (1.0 + float(risk_free_rate)) ** (1.0 / 252.0) - 1.0
    if n_obs > 1 and pd.notna(vol_daily) and vol_daily > 0:
        sharpe = ((daily_returns.mean() - rf_daily) / vol_daily) * np.sqrt(252.0)
    else:
        sharpe = np.nan

    # Hit ratio among traded days (your 'traded' flag)
    traded_days = df[df["traded"] == True]
    if len(traded_days) > 0:
        hit_ratio = float((traded_days["pnl"] > 0).mean())
    else:
        hit_ratio = np.nan

    # Max drawdown on the equity curve (end_cash)
    cum_curve = pd.to_numeric(df["end_cash"], errors="coerce")
    rolling_max = cum_curve.cummax()
    drawdowns = (cum_curve - rolling_max) / rolling_max
    mdd = float(drawdowns.min())  # negative number (e.g., -0.18 for -18%)

    return {
        "total_return": float(total_return),
        "irr_annual": float(irr_annual) if pd.notna(irr_annual) else np.nan,
        "vol_annual": float(vol_annual) if pd.notna(vol_annual) else np.nan,
```

```
        "sharpe": float(sharpe) if pd.notna(sharpe) else np.nan,
        "hit_ratio": float(hit_ratio) if pd.notna(hit_ratio) else np.nan,
        "mdd": mdd,
        "n_days": int(len(df)),
        "n_traded_days": int(len(traded_days)),
    }
```

```
evaluate_strategy(results)
```

```
{'total_return': 2.77592755573733,
 'irr_annual': 0.09357483221743235,
 'vol_annual': 0.08155040941442078,
 'sharpe': 1.1376730719827486,
 'hit_ratio': 0.4264705882352941,
 'mdd': -0.1516056300345067,
 'n_days': 3743,
 'n_traded_days': 2312}
```

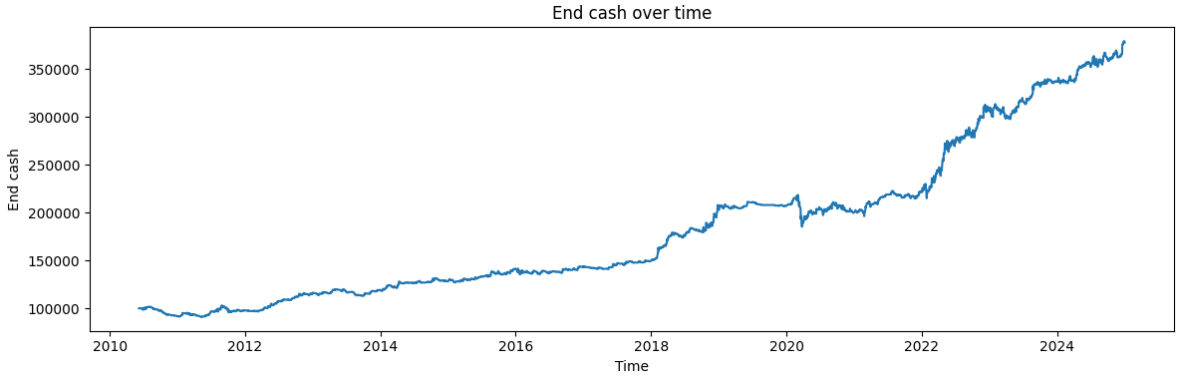
```
s = results.copy()

sp_df = pd.read_csv("/data/workspace_files/SP500.csv")
sp_df['SP500'] = 189095.59434874536 * sp_df['SP500'] / 1938.76
sp_df

# If you have a Time column, make it the datetime index
if "Time" in s.columns:
    s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
    s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

if "observation_date" in sp_df.columns:
    sp_df["observation_date"] = pd.to_datetime(sp_df["observation_date"], errors="co-
erce")
    sp_df = sp_df.dropna(subset=["observation_date"]).sort_values("observation_-
date").set_index("observation_date")

# Plot end_cash over time
plt.figure(figsize=(12,4))
plt.plot(s.date, s["end_cash"].astype(float))
# plt.plot(sp_df['SP500'].astype(float))
plt.plot
plt.title("End cash over time")
plt.xlabel("Time")
plt.ylabel("End cash")
plt.tight_layout()
plt.show()
```

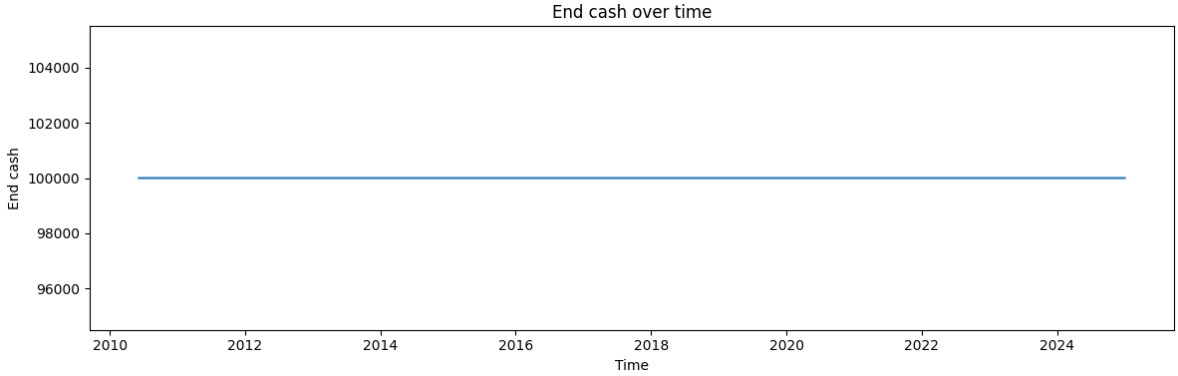


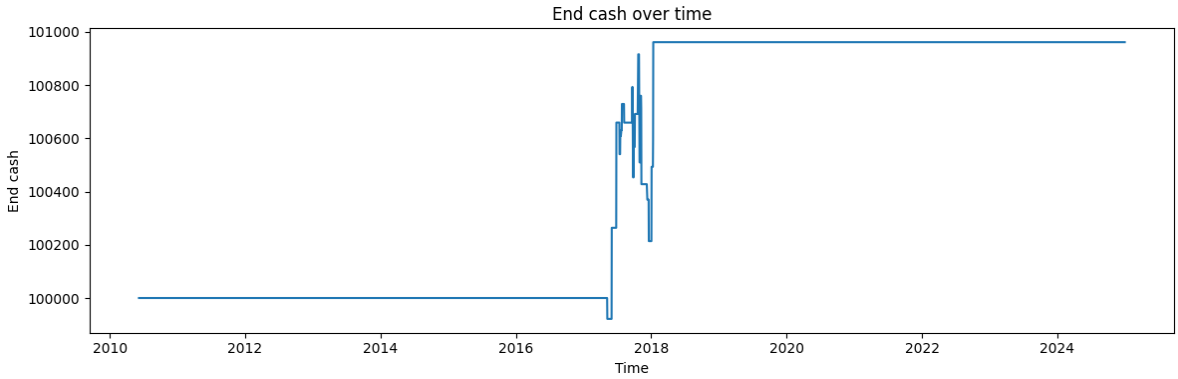
```
for i in [0, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 1000]:
    results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2, i)
    print(f"Results for VIX treshold: {i}")
    print(evaluate_strategy(results))
    s = results.copy()

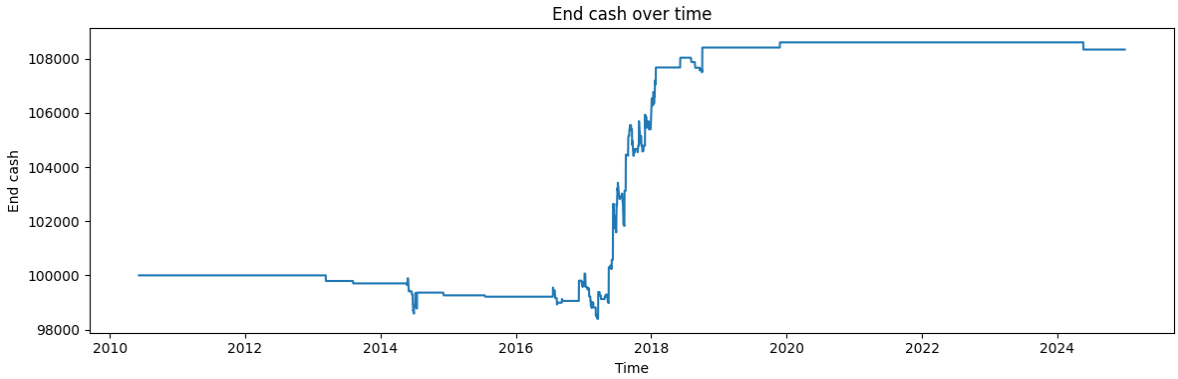
    # If you have a Time column, make it the datetime index
    if "Time" in s.columns:
        s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
        s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

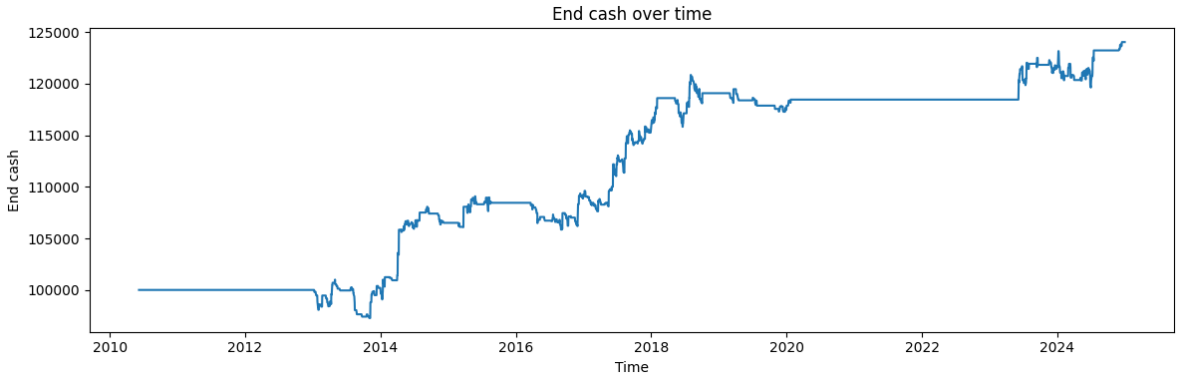
    # Plot end_cash over time
    plt.figure(figsize=(12,4))
    plt.plot(s.date, s["end_cash"].astype(float))
    # plt.plot(sp_df['SP500'].astype(float))
    plt.plot
    plt.title("End cash over time")
    plt.xlabel("Time")
    plt.ylabel("End cash")
    plt.tight_layout()
    plt.show()
```

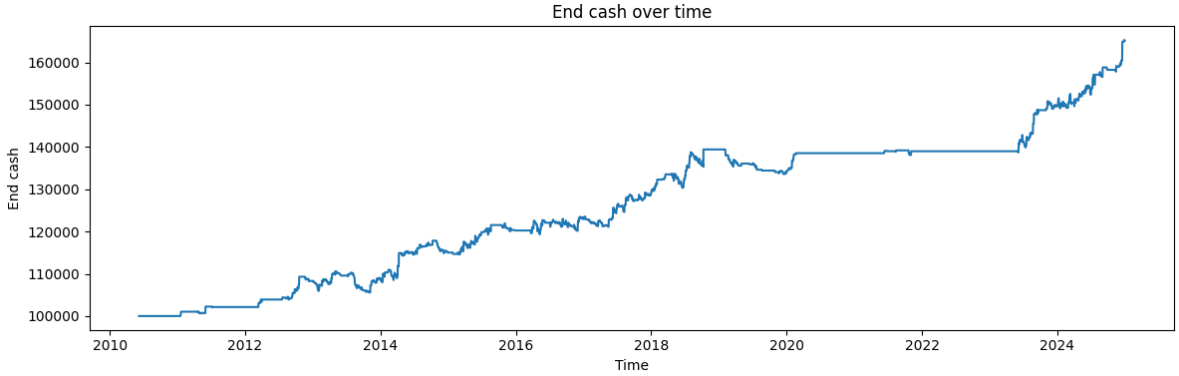
```
Results for VIX treshold: 0
{'total_return': 0.0, 'irr_annual': 0.0, 'vol_annual': 0.0, 'sharpe': nan, 'hit_ratio'
Results for VIX treshold: 10
{'total_return': 0.009614274449999627, 'irr_annual': 0.0006444032529375754, 'vol_annua
Results for VIX treshold: 12
{'total_return': 0.08343621469999829, 'irr_annual': 0.005409903696991236, 'vol_annual'
Results for VIX treshold: 14
{'total_return': 0.24040276408747907, 'irr_annual': 0.014610082210945396, 'vol_annual'
Results for VIX treshold: 16
{'total_return': 0.6513545253124655, 'irr_annual': 0.03434696996786046, 'vol_annual':
Results for VIX treshold: 18
{'total_return': 1.1181737953624444, 'irr_annual': 0.0518300739205928, 'vol_annual': 0
Results for VIX treshold: 20
{'total_return': 1.8601195575874234, 'irr_annual': 0.07331295979257413, 'vol_annual':
Results for VIX treshold: 22
{'total_return': 2.0301186774249054, 'irr_annual': 0.07749334119566931, 'vol_annual':
Results for VIX treshold: 24
{'total_return': 2.365663100974884, 'irr_annual': 0.08513903534387146, 'vol_annual': 0
Results for VIX treshold: 26
{'total_return': 2.977645127512368, 'irr_annual': 0.0974133201406282, 'vol_annual': 0.
```

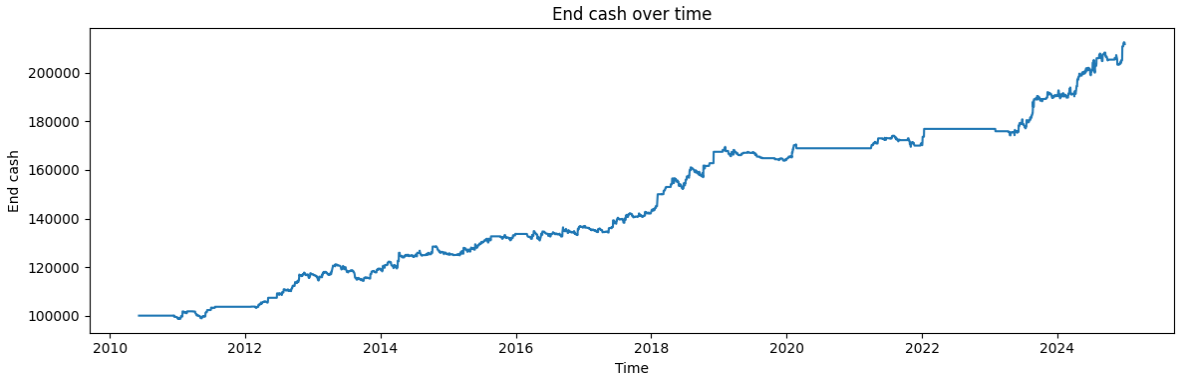

End cash over time
End cash
Time
104000
102000
100000
98000
96000
2010
2012
2014
2016
2018
2020
2022
2024

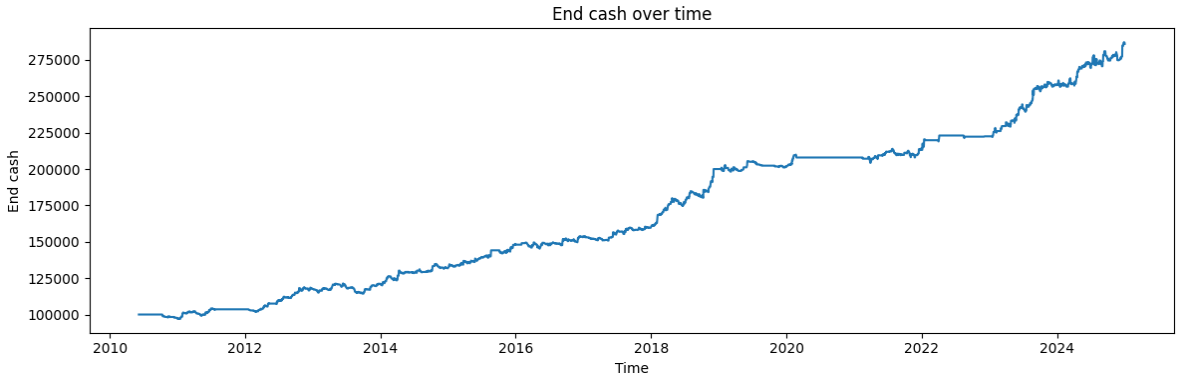

End cash over time
End cash
Time
101000
100800
100600
100400
100200
100000
2010
2012
2014
2016
2018
2020
2022
2024

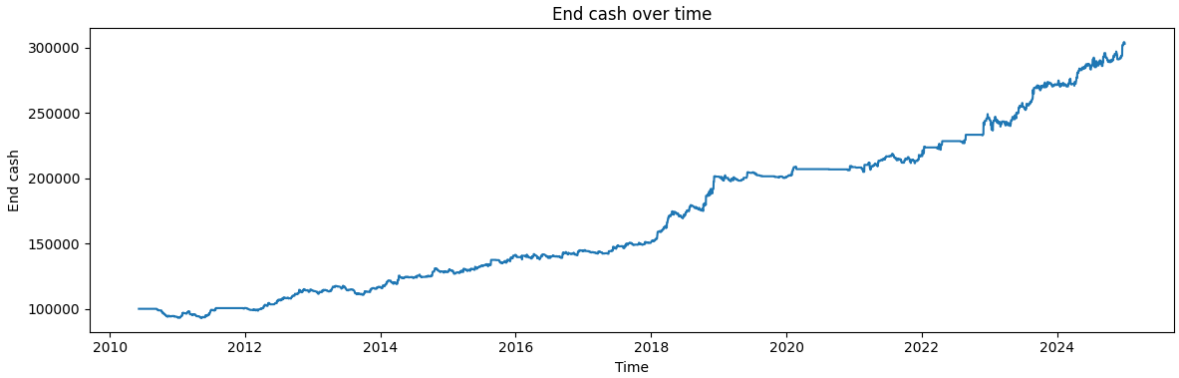

End cash over time
End cash
Time
108000
106000
104000
102000
100000
98000
2010
2012
2014
2016
2018
2020
2022
2024

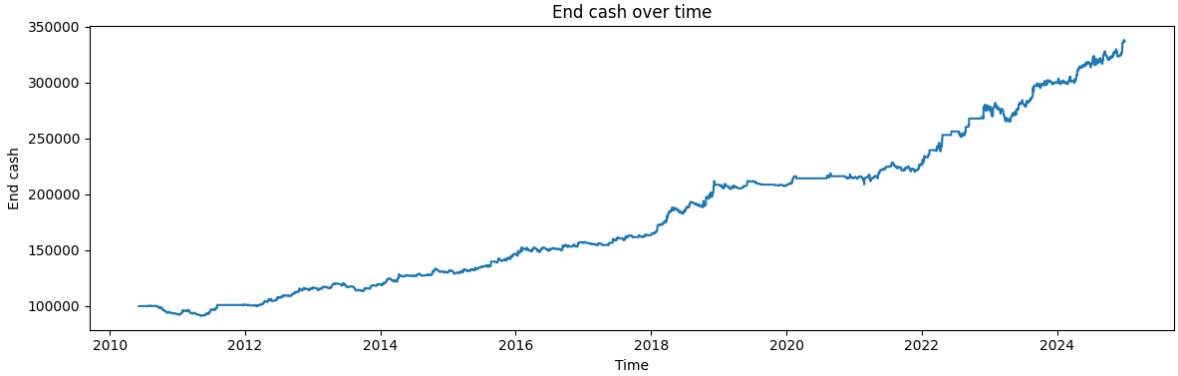

End cash over time
End cash
Time
125000
120000
115000
110000
105000
100000
2010
2012
2014
2016
2018
2020
2022
2024

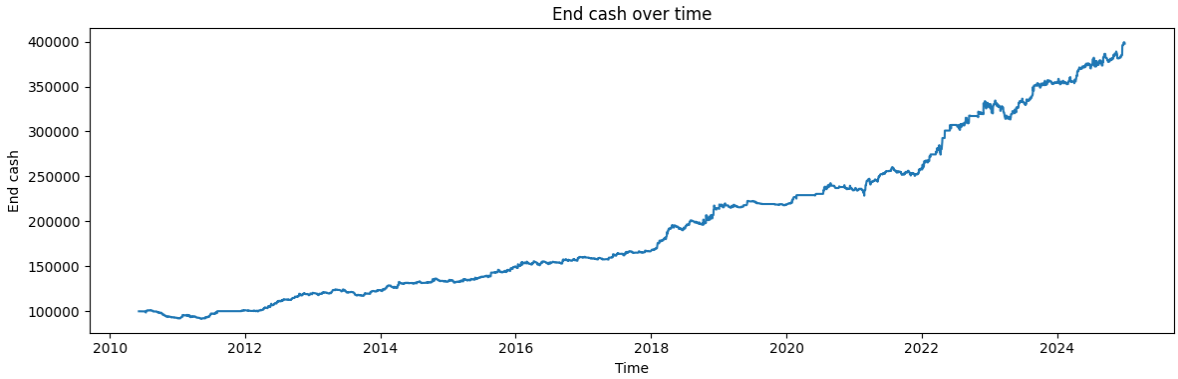

End cash over time
End cash
160000
150000
140000
130000
120000
110000
100000
2010
2012
2014
2016
2018
2020
2022
2024
Time

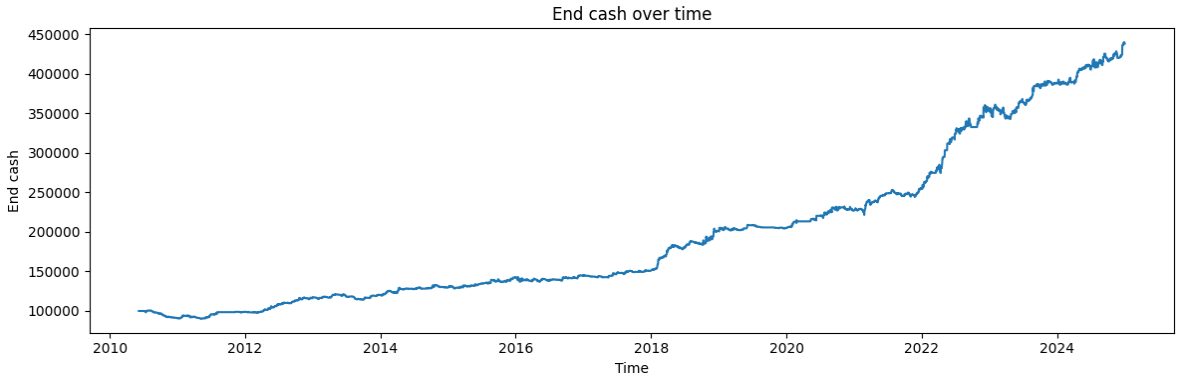

End cash over time
End cash
200000
180000
160000
140000
120000
100000
2010
2012
2014
2016
2018
2020
2022
2024
Time


End cash over time
End cash
275000
250000
225000
200000
175000
150000
125000
100000
2010
2012
2014
2016
2018
2020
2022
2024
Time


End cash over time
End cash
300000
250000
200000
150000
100000
2010
2012
2014
2016
2018
2020
2022
2024
Time


End cash over time
End cash
Time
350000
300000
250000
200000
150000
100000
2010
2012
2014
2016
2018
2020
2022
2024


End cash over time
End cash
Time
400000
350000
300000
250000
200000
150000
100000
2010
2012
2014
2016
2018
2020
2022
2024


End cash over time
End cash
Time
450000
400000
350000
300000
250000
200000
150000
100000
2010
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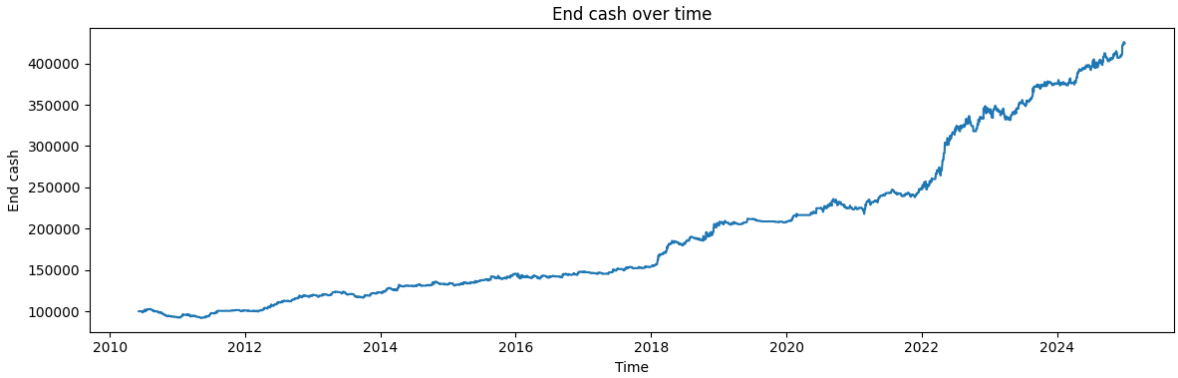
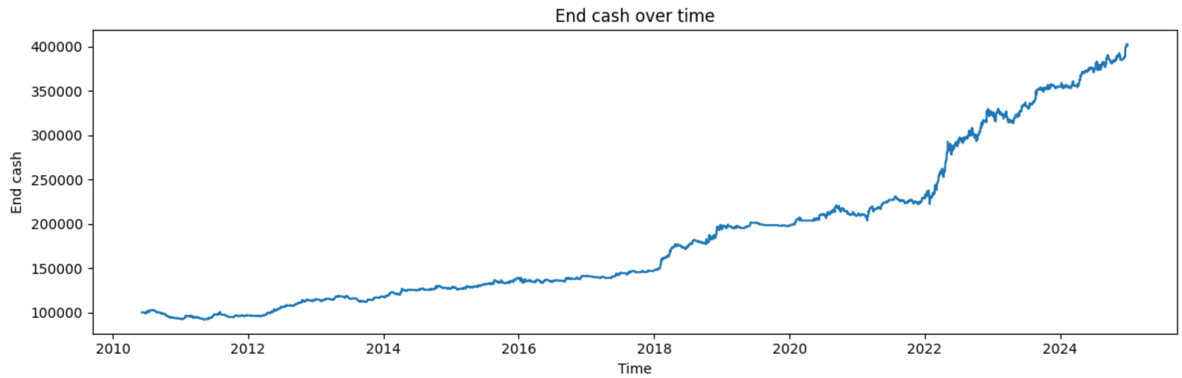
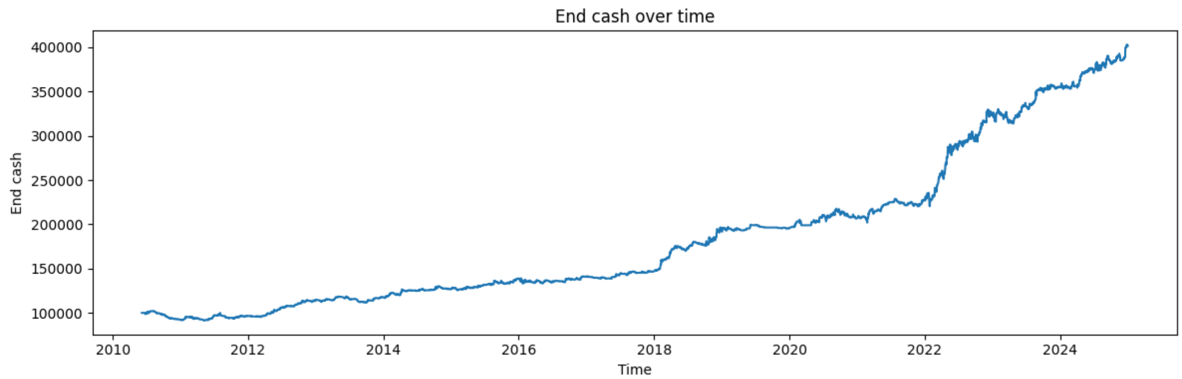
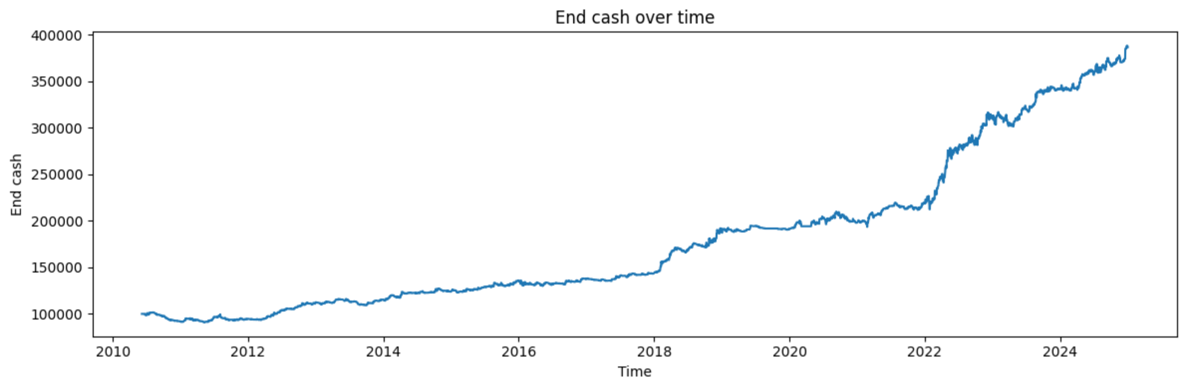
2018
2020
2022
2024


End cash over time
End cash
Time
400000
350000
300000
250000
200000
150000
100000
2010
2012
2014
2016
2018
2020
2022
2024


End cash over time
End cash
Time
400000
350000
300000
250000
200000
150000
100000
2010
2012
2014
2016
2018
2020
2022
2024


End cash over time
End cash
Time
400000
350000
300000
250000
200000
150000
100000
2010
2012
2014
2016
2018
2020
2022
2024


End cash over time
End cash
Time
400000
350000
300000
250000
200000
150000
100000
2010
2012
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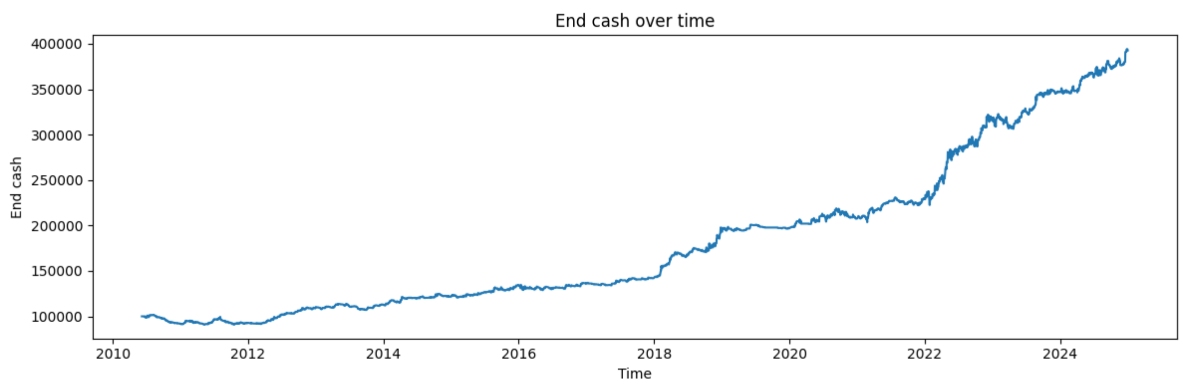
2016
2018
2020
2022
2024


End cash over time
End cash
Time
400000
350000
300000
250000
200000
150000
100000
2010
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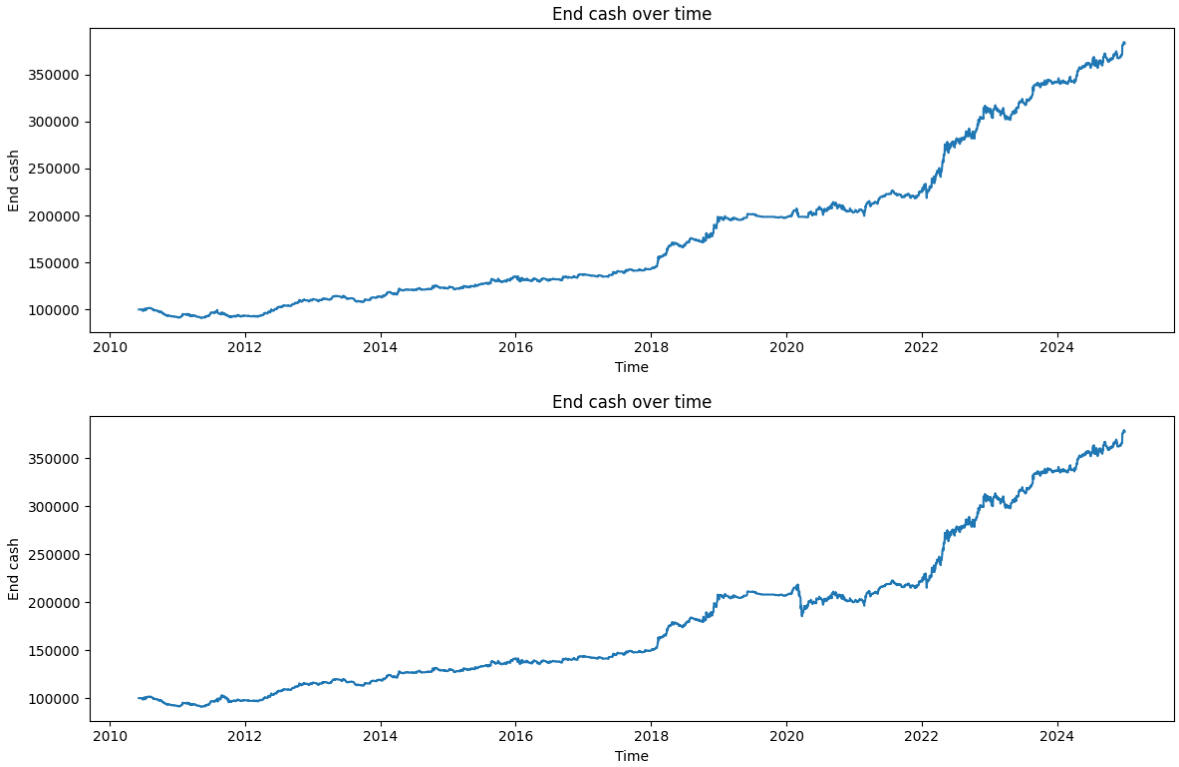
2016
2018
2020
2022
2024



Volatility window

```
def backtest_with_hyperparameters(coef_sigma, percentage_penalization,
                                  sigma_leverage_period, sigma_leverage_bound, sig-
ma_target, VIX_treshold_upper, VIX_treshold_lower):

    # ---------- Load & prepare intraday data ----------
    df = pd.read_csv('/data/workspace_files/nasdaq.csv',
                     usecols=['Time', 'symbol', 'ex-
piry_order', 'bid_px_00', 'ask_px_00', 'vwap_cum'])

    df['Time'] = pd.to_datetime(df['Time'], errors='coerce', utc=True)
    df = df.dropna(subset=['Time'])

    # Convert UTC -> New York (handles DST) and build hour/day on NY time
    df['Time'] = df['Time'].dt.tz_convert('America/New_York')
    df['hour'] = df['Time'].dt.strftime('%H:%M')
    df['day']  = df['Time'].dt.date        # <-- keep as datetime.-
date (DO NOT stringify)

    # US RTH 09:30-16:00 ET
    import datetime as dt
    market_open, market_close = dt.time(9, 30), dt.time(16, 0)
```

```
    df = df[df['Time'].dt.time.between(market_open, market_close)]

    # Order & mid price
    df = df.sort_values(['symbol', 'hour', 'day'])
    df['price'] = 0.5 * (df['bid_px_00'] + df['ask_px_00'])

    # Opening price per symbol-day
    df['open_price'] = df.groupby(['symbol', 'day'])['price'].transform('first')

    # Settle and previous settle (per symbol/day)
    daily_settle = (
        df.sort_values(['symbol','day','Time'])
          .groupby(['symbol','day'])['price'].last()
          .rename('settl_price')
    )
    prev_settle = (
        daily_settle.groupby(level='symbol').shift(1).rename('settl_price_previous')
    )
    df = df.join(daily_settle, on=['symbol','day'])
    df = df.join(prev_settle,   on=['symbol','day'])

    # Intraday move vs open
    df['move'] = np.where(
        df['open_price'] > 0,
        np.abs(df['price'] / df['open_price'] - 1),
        np.nan
    )

    # Rolling mean of move by (symbol, hour)
    win = int(sigma_leverage_bound) if sigma_leverage_bound is not None else 1
    if win < 1:
        win = 1
    df['sigma_observed'] = (
        df.groupby(['symbol', 'hour'])['move']
          .transform(lambda x: x.rolling(win, min_periods=win).mean())
    )

    # Bands
    df['upper_bound'] = df[['open_price','settl_price_previous']].max(axis=1) * (1 +
coef_sigma * df['sigma_observed']) \
                        + percentage_penalization * df['open_price']
    df['lower_bound'] = df[['open_price','settl_price_previous']].min(axis=1) * (1 -
 coef_sigma * df['sigma_observed']) \
                        - percentage_penalization * df['open_price']

    # End-of-day return
    df['return'] = df['settl_price'] / df['open_price'] - 1
```

```
    # 14D rolling std of returns per symbol
    daily = (df.sort_values(['symbol','day'])
               .groupby(['symbol','day'], as_index=False)['return'].last())
    daily['ret_std_14d'] = (
        daily.groupby('symbol')['return']
             .transform(lambda s: s.rolling(int(sigma_leverage_period), min_periods=i
nt(sigma_leverage_period)).std())
    )
    df = df.merge(daily[['symbol','day','ret_std_14d']], on=['sym-
bol','day'], how='left')

    df = df.sort_values(['symbol', 'day', 'hour'])

    # ---------- Load & merge VIX (daily) ----------
    vol_df = pd.read_csv('/data/workspace_files/VIX_History.csv', usecols=
['DATE', 'CLOSE'])
    vol_df = vol_df.rename(columns={'DATE':'day', 'CLOSE':'VIX'})

    # Parse to timezone-naive dates and shift by one day (use yesterday's VIX)
    vol_df['day'] = pd.to_datetime(vol_df['day'], format='%m/%d/%Y', errors='coerce')
.dt.date
    vol_df = vol_df.dropna(subset=['day'])
    vol_df = vol_df.sort_values('day')
    vol_df['VIX'] = pd.to_numeric(vol_df['VIX'], errors='coerce').shift(1)

    # Ensure unique per-day (take last if duplicates)
    vol_df = vol_df.groupby('day', as_index=False).last()

    # Some trading days may not have VIX (holidays). Build a calen-
dar from df['day'] and ffill VIX.
    all_days = pd.DataFrame({'day': sorted(df['day'].unique())})
    vol_df = all_days.merge(vol_df, on='day', how='left').sort_values('day')
    vol_df['VIX'] = vol_df['VIX'].ffill()

    # Merge back (left join; broadcasts daily VIX to intraday rows)
    df = df.merge(vol_df, on='day', how='left')

    # ---------- Run backtest ----------
    results = backtest(df, 1, sigma_target, VIX_treshold_upper, VIX_treshold_lower)
    return results
```

```
def backtest(backtest_df, rank_to_expiry, sigma_target, VIX_treshold_up-
per, VIX_treshold_lower, initial_cash=100_000):
    df1 = backtest_df.copy()

    # Ensure Time is datetime and non-null
    df1['Time'] = pd.to_datetime(df1['Time'], errors='coerce')
    df1 = df1.dropna(subset=['Time'])

    # Unique trading days (normalized to midnight) in ascending order
    unique_days = (
        df1['Time'].dt.normalize()
          .drop_duplicates()
          .sort_values()
          .to_list()
    )

    results = []
    cash = initial_cash

    for day_ts in unique_days:
        start_cash = cash
        # print(f"Running backtest for {day_ts.date()}")

        # keep your existing signature for backtest_one_day
        cash = backtest_one_day(df1, day_ts, rank_to_expiry, start_cash, sigma_tar-
get, VIX_treshold_upper, VIX_treshold_lower)
        # print(f"Cash after {day_ts.date()}: {cash}\n")

        pnl = cash - start_cash
        results.append({
            "date": day_ts,  # normalized pandas Timestamp
            "start_cash": start_cash,
            "end_cash": cash,
            "pnl": pnl,
            "return": (pnl / start_cash) if start_cash else np.nan,
            "traded": (pnl != 0),
        })

    return (pd.DataFrame(results)
              .sort_values("date")
              .reset_index(drop=True))

# backtest(df, rank_to_expiry = 1, sigma_target=0.02, initial_cash=100_000)
```

```
def backtest_one_day(df, day_str, rank_to_expiry, initial_cash, sigma_tar-
get, VIX_treshold_upper, VIX_treshold_lower):
    day = pd.to_datetime(day_str).date()

    backtest_df = df[df['expiry_order'] == rank_to_expiry]
    backtest_df = (backtest_df.loc[(backtest_df['day'] == day)]
                              .sort_values('Time')
                              .copy())
    backtest_df = backtest_df.sort_values('hour')

    if backtest_df['VIX'].iloc[0] < VIX_treshold_upper:
        return initial_cash

    if backtest_df['VIX'].iloc[0] > VIX_treshold_lower:
        return initial_cash

    # --- basic guards ---
    if backtest_df.empty:
        return initial_cash

    # Pull scalars safely
    ret_std = backtest_df['ret_std_14d'].iloc[0]
    open_price = backtest_df['open_price'].iloc[0]

    # Validate inputs used in floor(); avoid NaN / inf / nonpositive std or price
    if (not np.isfinite(ret_std)) or (ret_std <= 0) or (not np.isfinite(open_price))
or (open_price <= 0):
        total_shares_to_hold = 0
    else:
        lev = 1  # cap leverage at 4x
        sizing = initial_cash * lev / open_price
        total_shares_to_hold = int(np.floor(sizing)) if np.isfinite(sizing) and siz-
ing > 0 else 0

    # Extract arrays
    bid_list     = backtest_df['bid_px_00'].to_numpy()
    ask_list     = backtest_df['ask_px_00'].to_numpy()
    upper_bound  = backtest_df['upper_bound'].to_numpy()
    lower_bound  = backtest_df['lower_bound'].to_numpy()
    vwap_list    = backtest_df['vwap_cum'].to_numpy()
    hour         = backtest_df['hour'].to_numpy()
    price_list   = backtest_df['price'].to_numpy()

    # Length / NaN checks
    n = len(bid_list)
    if not (len(ask_list) == len(vwap_list) == len(hour) == len(upper_bound) == len(l
```

```
ower_bound) == len(price_list) == n) or n == 0:
        return initial_cash

    # IMPORTANT: your previous NaN check added arrays together (elementwise sum).
    # Do proper per-array NaN checks:
    if (np.isnan(bid_list).any() or np.isnan(ask_list).any() or
        np.isnan(vwap_list).any() or np.isnan(upper_bound).any() or
        np.isnan(lower_bound).any() or np.isnan(price_list).any()):
        return initial_cash

    position = 0
    cash = float(initial_cash)



    for i in range(1, len(price_list) - 1):
        if price_list[i] > max(upper_bound[i], vwap_list[i]):
            # we must be long and buy more than what we already have
            if total_shares_to_hold > position:
                shares_to_buy = total_shares_to_hold - position
                position += shares_to_buy
                initial_cash -= shares_to_buy * price_list[i] * 1.00001
                # print(f"Bought {shares_to_buy} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")

        elif price_list[i] < min(vwap_list[i], lower_bound[i]):
            # we must be short and sell more than what we already have
            if -total_shares_to_hold < position:
                shares_to_sell = position + total_shares_to_hold
                position -= shares_to_sell
                initial_cash += shares_to_sell * price_list[i] * 0.99999
                # print(f"Sold {shares_to_sell} shares at {bid_list[i]} on {day_str}
at {hour[i]}")

        elif price_list[i] >= min(vwap_list[i], lower_bound[i]) and max(up-
per_bound[i], vwap_list[i]) >= ask_list[i]:
            # we must close our position
            if position > 0: # we are long so we need to sell
                initial_cash += position * price_list[i] * 0.99999
                # print(f"Sold {position} shares at {bid_list[i]} on {day_str} at {ho
ur[i]}")
                position = 0
            elif position < 0: # we are short so we need to buy
                initial_cash -= abs(position) * price_list[i] * 1.00001
                # print(f"Bought {abs(position)} shares at {ask_list[i]} on {day_str}
 at {hour[i]}")
                position = 0
```

```
    # closing daily position at last time point
    if position > 0: # we are long so we need to sell
        initial_cash += position * price_list[-1] * 0.99999
        position = 0

    elif position < 0: # we are short so we need to buy
        initial_cash -= abs(position) * price_list[-1] * 1.00001
        position = 0

    return initial_cash

# backtest_one_day(df, "2011-10-10", 1, 100000)
```

```
results = backtest_with_hyperparameters(0.5, 0.002, 14, 14, 0.2, 0, 100)
results
```

	date	start_cash	end_cash	pnl	return	traded
0	2010-06-07 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
1	2010-06-08 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
2	2010-06-09 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
3	2010-06-10 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
4	2010-06-11 00:00:00-04:00	100000.000000	100000.000000	0.000000	0.000000	False
...	...	...	...	...	...	...
3738	2024-12-24 00:00:00-05:00	375872.516755	378089.918236	2217.401481	0.005899	True
3739	2024-12-26 00:00:00-05:00	378089.918236	378089.918236	0.000000	0.000000	False
3740	2024-12-27 00:00:00-05:00	378089.918236	379093.073425	1003.155189	0.002653	True
3741	2024-12-30 00:00:00-05:00	379093.073425	378720.321267	-372.752158	-0.000983	True
3742	2024-12-31 00:00:00-05:00	378720.321267	377592.755574	-1127.565694	-0.002977	True

3743 rows × 6 columns

```
evaluate_strategy(results)
```

```
{'total_return': 2.77592755573733,
 'irr_annual': 0.09357483221743235,
 'vol_annual': 0.08155040941442078,
 'sharpe': 1.1376730719827486,
 'hit_ratio': 0.4264705882352941,
 'mdd': -0.1516056300345067,
 'n_days': 3743,
 'n_traded_days': 2312}
```

```
for i in [(0, 5), (5, 10), (10, 15), (15, 20), (20, 25), (25, 30), (30,35), (35, 40),
 (40, 45)]:
    results = backtest_with_hyperparameters(0.1, 0.005, 10, 14, 0.1, i[0], i[1])
    print(f"Results for VIX treshold: {i}")
    print(evaluate_strategy(results))
    s = results.copy()

    sp_df = pd.read_csv("/data/workspace_files/SP500.csv")
    sp_df['SP500'] = 189095.59434874536 * sp_df['SP500'] / 1938.76
    sp_df

    # If you have a Time column, make it the datetime index
    if "Time" in s.columns:
        s["Time"] = pd.to_datetime(s["Time"], errors="coerce")
        s = s.dropna(subset=["Time"]).sort_values("Time").set_index("Time")

    if "observation_date" in sp_df.columns:
        sp_df["observation_date"] = pd.to_datetime(sp_df["observation_date"], errors=
"coerce")
        sp_df = sp_df.dropna(subset=["observation_date"]).sort_values("observation_-
date").set_index("observation_date")

    # Plot end_cash over time
    plt.figure(figsize=(12,4))
    plt.plot(s.date, s["end_cash"].astype(float))
    # plt.plot(sp_df['SP500'].astype(float))
    plt.plot
    plt.title("End cash over time")
    plt.xlabel("Time")
    plt.ylabel("End cash")
    plt.tight_layout()
    plt.show()
```

```
Results for VIX treshold: (0, 5)
{'total_return': 0.0, 'irr_annual': 0.0, 'vol_annual': 0.0, 'sharpe': nan, 'hit_ratio'
Results for VIX treshold: (5, 10)
{'total_return': 0.0012502379874994585, 'irr_annual': 8.412407976909897e-05, 'vol_annu
Results for VIX treshold: (10, 15)
{'total_return': 0.20735934481248353, 'irr_annual': 0.012767370639904962, 'vol_annual'
Results for VIX treshold: (15, 20)
{'total_return': 0.8131469220749734, 'irr_annual': 0.04087643370479754, 'vol_annual':
Results for VIX treshold: (20, 25)
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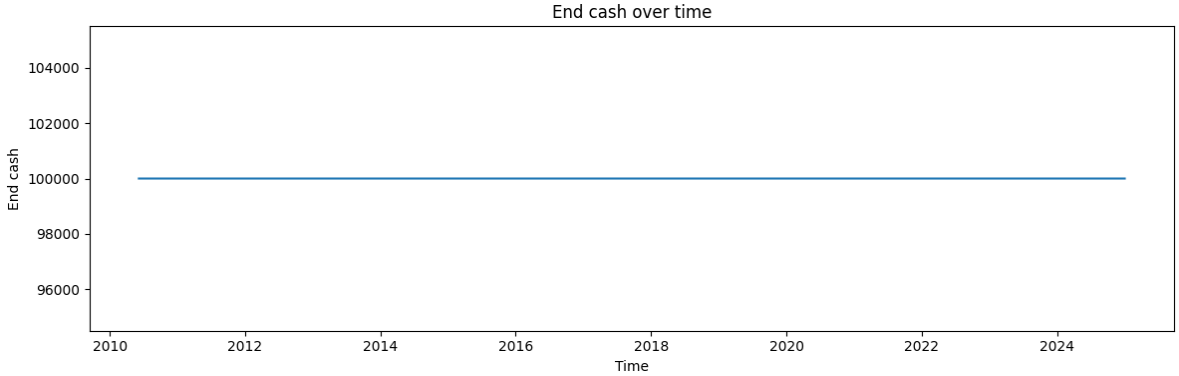

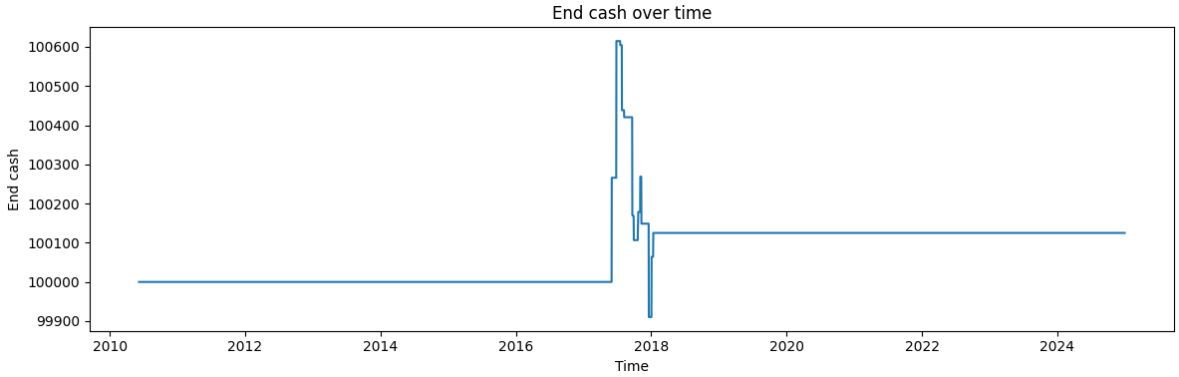

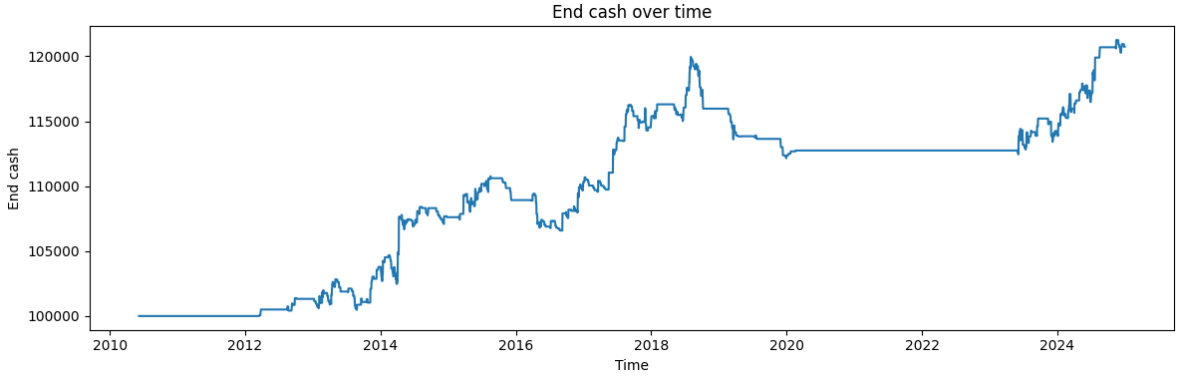
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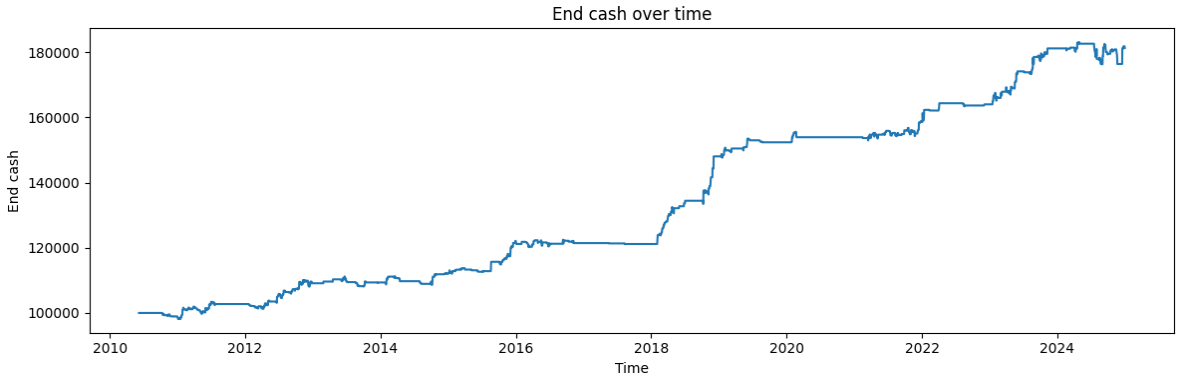
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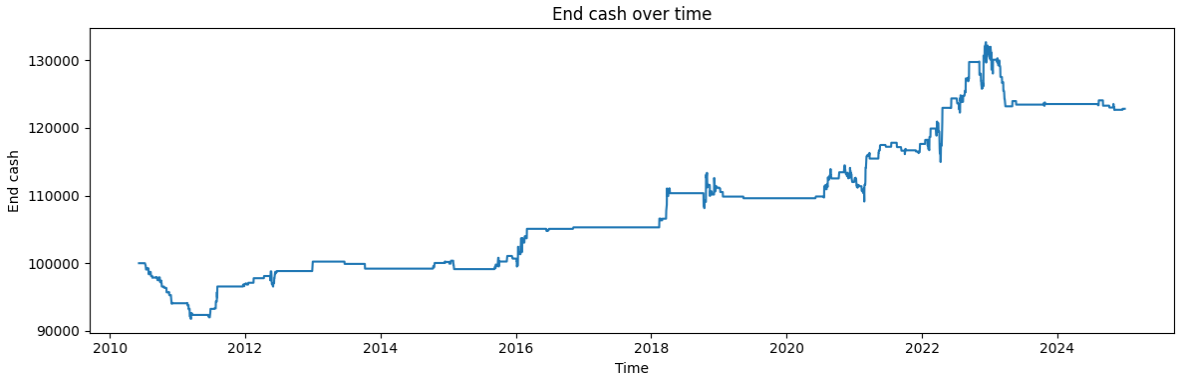
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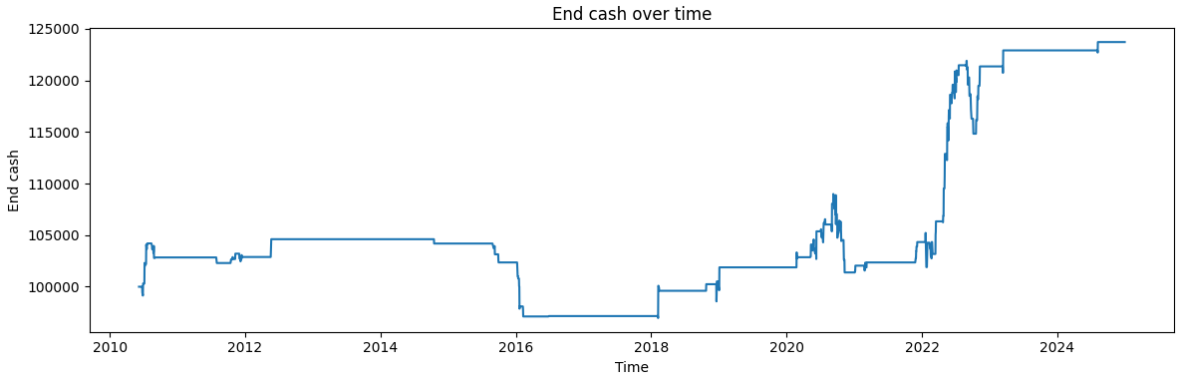
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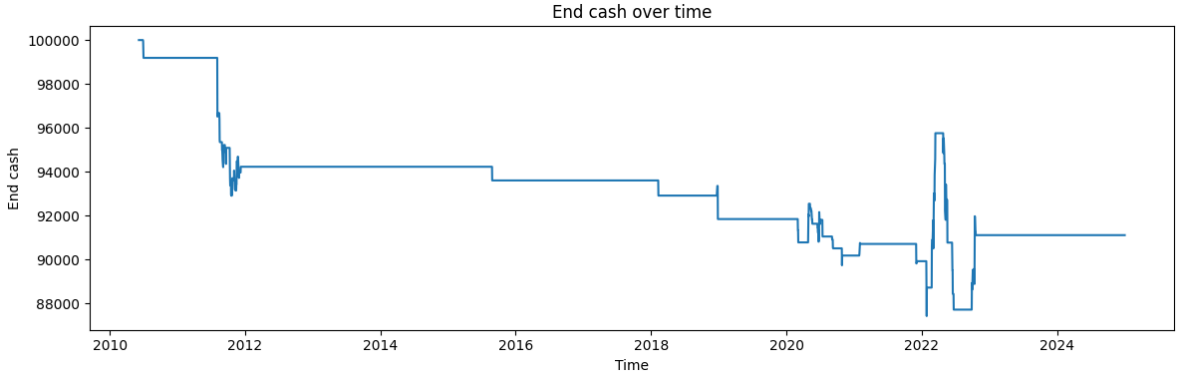
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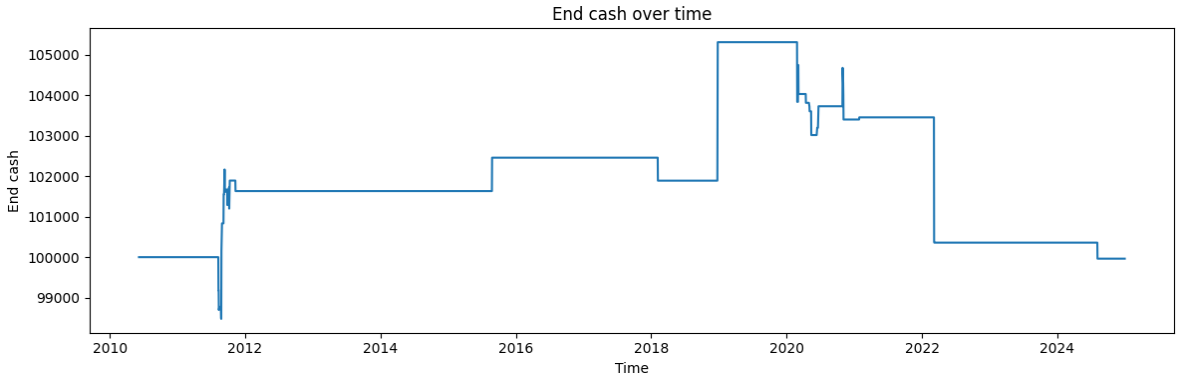
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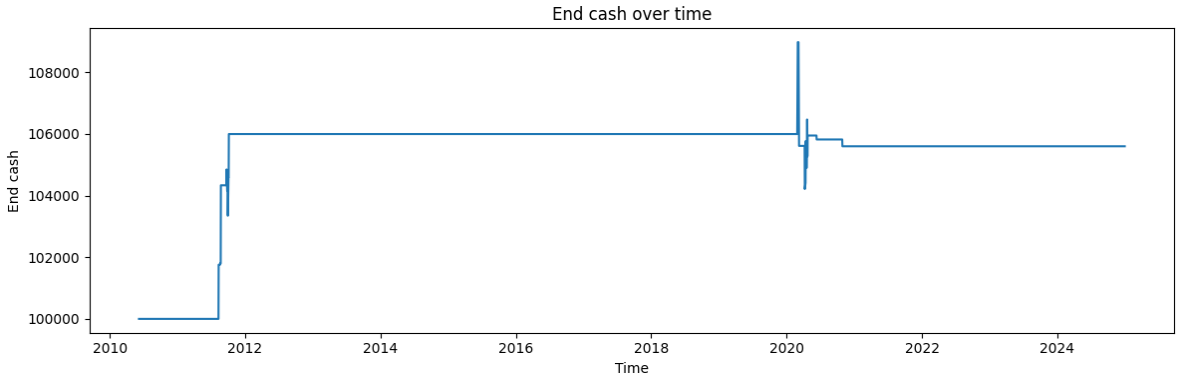
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