Continuation Power Flow
Advanced power system analysis

Camille Hamon
camille.hamon@ntnu.no

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"Over the past decade, voltage instability has initiated several severe power system disruptions. Such incidents are likely to increase with increases in transmission line loadings. However, no theoretical or practical body of knowledge is available to meet the needs of power system planners and operators" (**January 1989**, EPRI, "Proceedings: Bulk Power System Voltage Phenomena - Voltage Stability and Security.")

Outline

Agenda

PV and QV curves

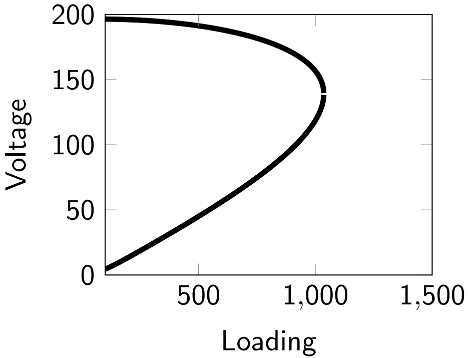
The onion curve

- A lossless two-bus system (single-load infinite-bus systeme) is used for illustration purposes **only**.
- Infinite bus: voltage $= Ee^{j0}$.
- Load bus: load $S = P + jQ$.
- Closed-form solution of the voltage magnitude at the load bus:

$$\frac{V}{E} = \sqrt{\frac{1}{2} - \frac{QX}{E^2} \pm \sqrt{\frac{1}{4} - \left(\frac{XP}{E}\right)^2 - \frac{XQ}{E^2}}} \tag{1}$$

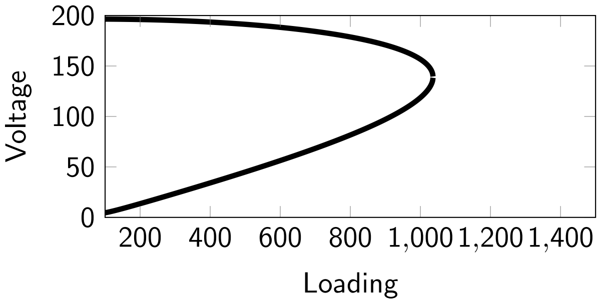
Voltage stability

- Load dynamics are critical to study voltage stability.
- There is a loading point beyond which the system becomes unstable. (What happens beyond this point?)



Voltage stability, cont.

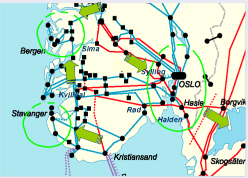
- What does loading mean? What does it mean to *increase the loading*?
- How to get the nose point?
- How to get the rest of the curve?
- Why are we interested in these quantities?
- How is this information typically used in operation and planning?

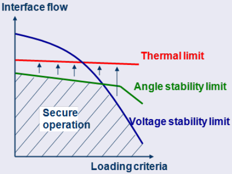


Stability constraints (remember?)

Power system perspective

Typically, stability constraints take the form of active power limits on critical interfaces (=transmission corridors / lines).





Math formulation

$$P_l \leq P_l^{\max}, \ \forall \text{ critical interfaces } l.$$

<u>Note:</u> *same math formulation as thermal limits but fundamentally different constraints!*

Example from Sweden: Using CPF for security management.

- Four price areas separated by bottlenecks (=critical transmission corridors).
- **Security assessment** The TSO monitors the power transfers across the bottlenecks.
- **Security enhancement** The TSO sends re-dispatch orders (increase/decrease production) if necessary.

Price area SE1

Price area SE2

Price area SE3

Price area SE4

What does "monitoring" mean in this context? What do we monitor, and against what?

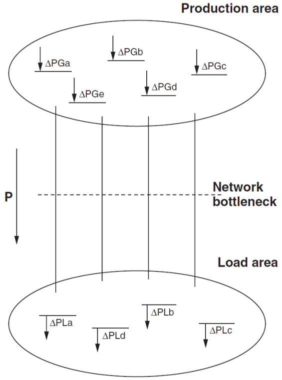
Example from Sweden - 2

Security assessment:

1. A list of contingencies is defined.
2. Every 15 minutes, for each contingency and each bottleneck, transmission limits are computed.
3. The power transfers are monitored and checked against all computed transmission limits.

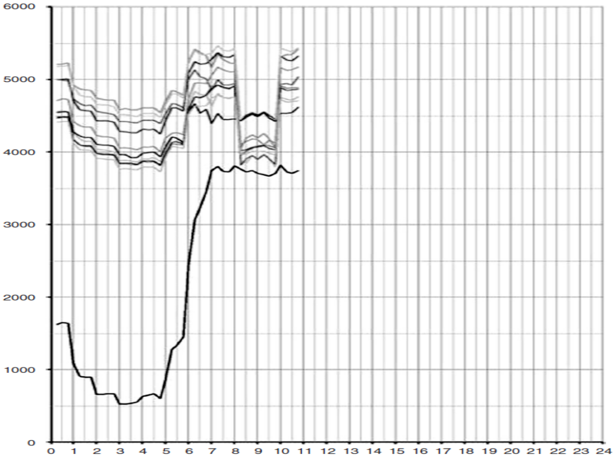
Security enhancement:

4. If the power transfers come close to one of the computed limits, re-dispatch the generation to decrease the power transfers.



How to compute the transmission limits?

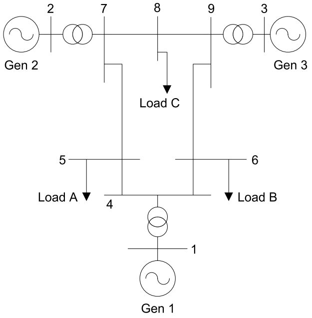
Example from Sweden - 3

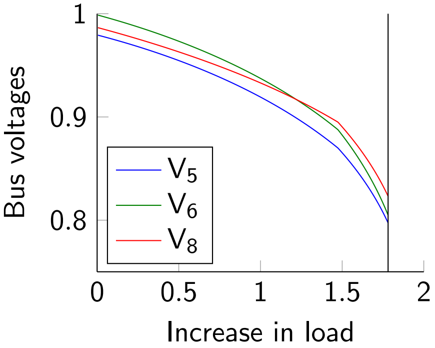


Source: Sandberg, L., & Roudén, K. (1992). *The real-time supervision of transmission capacity in the swedish grid*. In S. C. Savulescu (Ed.), *Real-time stability assessment in modern power system control centers*.

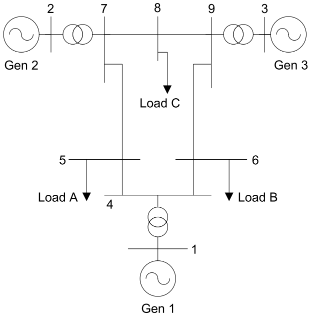
Example of voltage instability - 1

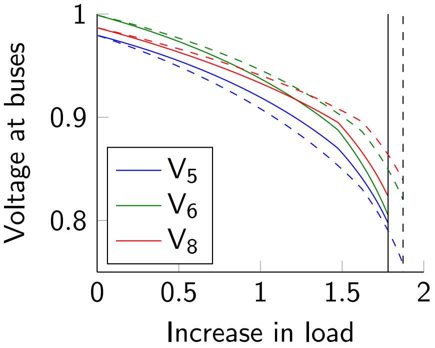
There is a loading point beyond which the system becomes unstable. How to define a loading and a loading increase in power systems?





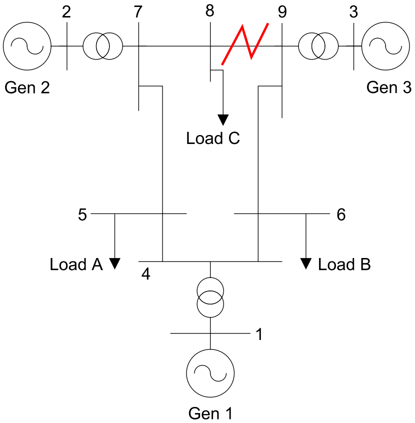
Example of voltage instability - 2

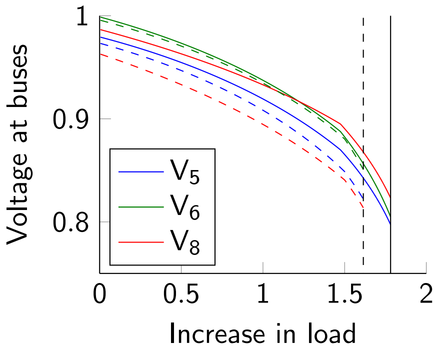




Solid=All loads increase by the same amount
Dashed=Load A increases double as much as the other two.

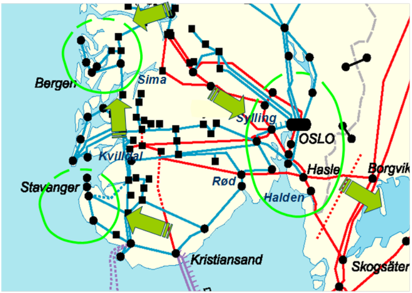
Example of voltage instability - 3

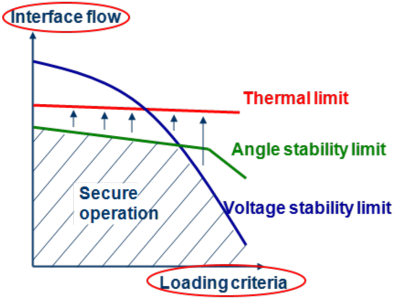




Solid=System intact
Dashed=Fault on line between buses 8 and 9.

Example of voltage instability - 4





Agenda

History

January 1989, EPRI, "Proceedings: Bulk Power System Voltage Phenomena - Voltage Stability and Security."

> *Over the past decade, voltage instability has initiated several severe power system disruptions. Such incidents are likely to increase with increases in transmission line loadings. However, no theoretical or practical body of knowledge is available to meet the needs of power system planners and operators*

History, cont.

Add some historical blackouts.

History, cont.

- **September 1989**: Dobson and Chiang, "Towards a Theory of Voltage Collapse in Electric Power Systems."
- **October 1989**: Ajjarapu and Christy, "The Application of a Locally Parameterized Continuation Technique to the Study of Steady State Voltage Stability."
- **1992**: Ajjarapu and Christy, "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis."
- **1992**: Dobson and Lu, "Voltage Collapse Precipitated by the Immediate Change in Stability When Generator Reactive Power Limits Are Encountered."
- **1998**: Van Cutsem and Vournas, Voltage Stability of Electric Power Systems.
- **2007**: Ajjarapu, Computational Techniques for Voltage Stability Assessment and Control.

And many, many other contributors . . .

Agenda

Newton-Raphson method

- Method to compute x^* such that $f(x^*) = 0$.
- Requires an initial guess.
- Relies on a first-order approximation of f around the current guess x_i:

$$f(x) = f(x_i) + \nabla_x f(x_i)(x - x_i) + \text{ higher order term} \qquad (2)$$

Newton-Raphson method, algorithm

1. Set $x_i = x_0$;
2. **While** $\|f(x_i)\| > \epsilon$, **do**

$$x_{i+1} = x_i - \left(\nabla_x f(x_i)\right)^{-1} f(x_i) \tag{3}$$

3. Hope that you guessed x_0 right so that the method converges.

Agenda

Power flow computations

- **Objective**: ?
- **Method**: Active and reactive power balance at each bus must hold.

$$\Delta P = P_g - P_l - P_s(\theta, V) = 0, \tag{4}$$
$$\Delta Q = Q_g - Q_l - Q_s(\theta, V) = 0, \tag{5}$$

- Can be written

$$x = [\theta \ V] \tag{6}$$
$$f(x) = \begin{bmatrix} \Delta P(x) \\ \Delta Q(x) \end{bmatrix} = 0 \tag{7}$$

Power flow computations

- **Objective**: Find voltage magnitudes and angles at all buses, given power injections, voltage set points and angle references at some buses.
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Newton-Raphson iteration for power flows

The power flow equations $f(x) = 0$ can be solved by the Newton-Raphson method. Iterations:

$$x_{i+1} = [\theta_{i+1} \; V_{i+1}] = x_i - J(x_i)^{-1} f(x_i) \tag{8}$$

with (i is for iteration number, *not* bus number)

$$J(x_i) = J([\theta_i \; V_i]) = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} = \begin{bmatrix} f_\theta & f_V \end{bmatrix} \tag{9}$$

So why can't we use power flow computations for obtaining the PV curves?

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So why can't we use power flow computations for obtaining the PV curves?

1. The Jacobian is singular at the nose point $\Rightarrow$ numerical problems as we get close to this point.
2. Convergence of NR method very sensitive to initial conditions $\Rightarrow$ can be difficult to get convergence even away from the nose point.

Agenda

Parametrization of load increase

There is a loading point beyond which the system becomes unstable. How to define a loading and a loading increase in power systems?

- Add one parameter $\lambda \in \mathbb{R}$ to parametrize the load increase process in direction $d \in \mathbb{R}^n$:

$$P_l = P_l^0 + \lambda d \in \mathbb{R}^n \tag{10}$$

- For example, in a three-bus system with two loads at buses 2 and 3, the loads could be increased as follows

$$P_l = \begin{bmatrix} 0 \\ 150 \\ 120 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{11}$$

Parametrization of load increase, cont.

- Reactive power: typically, we assume constant power factor load (but the method can handle other models as well), so

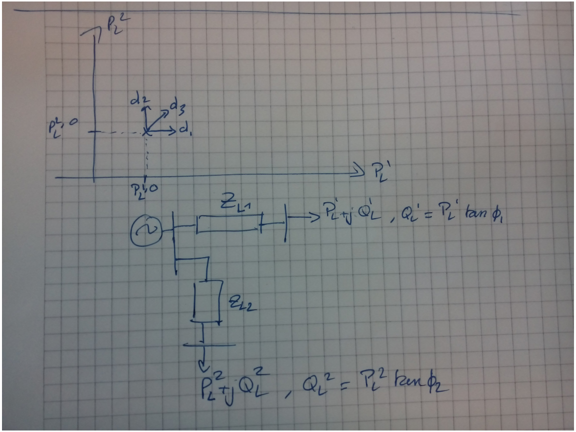
$$Q_l = \text{diag}(\tan\phi)P_l = Q_l^0 + \lambda \cdot \text{diag}(\tan\phi)d \tag{12}$$

- Note that $\text{diag}(\tan\phi)$ is simply a square matrix with the $\tan\phi_i$ on the diagonal
- For example, with $\tan\phi = Q_0/P_0 = 0.5$ for all loads,

$$Q_L = \begin{bmatrix} 0 \\ 75 \\ 60 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} \tag{13}$$

Illustration on the board

Two-dimensional load increase from P_l^0 in direction d.



Note: Important to keep in mind which space ($V - \lambda$, P_l, etc.) is of interest.

Extending the power flow equations

$$0 = P_g^i - P_l^i - P_s^i(\theta, V), \quad \forall \text{ PV and PQ buses} \tag{14}$$

$$0 = Q_g^i - Q_l^i - Q_s^i(\theta, V), \quad \forall \text{ PQ buses} \tag{15}$$

$$P_l = P_l^0 + \lambda d \in R^n \tag{16}$$

$$Q_l = Q_l^0 + \lambda \cdot \text{diag}(\tan \phi) d \tag{17}$$

	PF	CPF
Equations	(13), (14)	(13), (14), (15), (16)
Variables	$x = [\theta \ V]$	$z = [x \ \lambda] = [\theta \ V \ \lambda]$
Parameters	P_l, $\tan \phi$	P_l^0, $\tan \phi$, d, λ
	(and P_g, θ_{slack}, ...)	(and P_g, θ_{slack}, ...)

Increasing $\lambda \Leftrightarrow$ simulating a load increase in direction d.

CPF: Principles

The CPF is a predictor-corrector process:

1. Start from an operating point $z_i = [\theta_i \ V_i \ \lambda_i]$.
2. Predict what the operating point becomes when loading increases in direction $d \Rightarrow z^p_{i+1}$.
3. Correct the prediction to get a real operating point z_{i+1}.

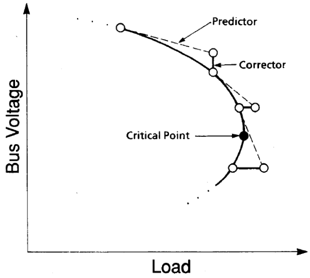


Figure: CPF process, from Ajjarapu and Christy, "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis."

Steps in CPF

Three main steps:

1. Predictor step: From a known operating point z_i (ex: previously corrected), take a step in the direction of the tangent vector t_i.

$$z_{i+1}^{p} = [\theta_{i+1} \ V_{i+1} \ \lambda_{i+1}] = z_i + s \cdot t_i \tag{18}$$

Notes:

- The predicted point does not correspond to a physical operating point.
- It is a mathematical construction to estimate the values of $z = [\theta \ V \ \lambda]$ after taking a step of length s in direction d (in the load space = P_l-space).
- It gives a good initial guess of z before the corrector step.

Steps in CPF, cont.

2. Choosing the continuation parameter: which component in $z = [\theta \ V \ \lambda]$ should we keep constant in the corrector step?
3. Corrector step: Correct the predicted value to a valid operating point (= "project" back onto the PV curve), keeping the continuation parameter constant. Solve:

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \tag{19}$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \tag{20}$$

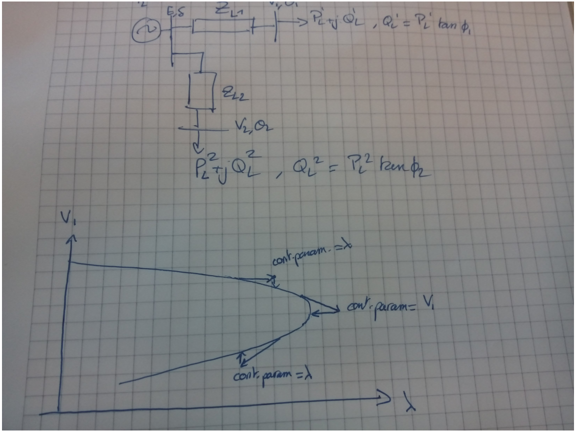
$$0 = z_{i+1}^k - z_{i+1}^{k,p} \tag{21}$$

Equations (18)–(20) solved for $z_{i+1} = [\theta_{i+1} \ V_{i+1} \ \lambda_{i+1}]$ by Newton-Raphson method.

Notes:

- Without (20), the system of equations is underdetermined.
- (20) says how to correct onto the PV curve (keeping λ constant, keeping some V_m constant, ...)

Illustration on the board



Predictor step, details

From a known operating point z_i (ex: previously corrected), take a step in the direction of the tangent vector t_i.

$$z_{i+1}^{p} = [\theta_{i+1} \; V_{i+1} \; \lambda_{i+1}] = z_i + s \cdot t_i \tag{22}$$

Question: How is defined the tangent vector?
More general question: How to compute *one* tangent vector to a surface described by $g(u) = 0$?
Notes:

- t_i is the tangent vector to the operating point $z_i = [\theta_i \; V_i \; \lambda_i]$ defined by the power flow equations $f(z_i) = 0$.
- Notice the space we are talking about. It is the z-space, not only the (V, λ)-space
- Where is the direction d here?

Computing a tangent vector

Equation of the form

$$g(u_0) = 0 \tag{23}$$

Any vector t in the null space of the Jacobian $J(u_0)$ is a tangent vector to g at u_0, i.e. any vector $u \neq 0$ such that

$$J(x_0)t = 0. \tag{24}$$

Intuition,

$$g(u_0 + t) = g(u_0) + J(u_0)t + \text{ higher order terms} \tag{25}$$

The set of all $u_0 + t$ such that $J(u_0)t = 0$ is the tangent plane of g at u_0.

See also `http://mathworld.wolfram.com/SubmanifoldTangentSpace.html`

Tangent vector to power flow equations

Let $z_i = [\theta_i \ V_i \ \lambda_i]$ be an operating point, i.e.

$$\Delta P(z_i) = 0 \tag{26}$$

$$\Delta Q(z_i) = 0 \tag{27}$$

Any vector t such that

$$J(z_i)t = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \end{bmatrix} t = 0 \tag{28}$$

is a tangent vector.

Note: We cannot just solve $J(z_i)t = 0$ for t to find t (i.e. (27) is a necessary but not sufficient condition to get *the* t we are looking for). Why?

Tangent vector to power flow equations

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Note: We cannot just solve $J(z_i)t = 0$ for t to find t (i.e. (27) is a necessary but not sufficient condition to get *the* t we are looking for). Why? Ex: $t = 0$ satisfies (27).

Tangent vector to power flow equations, cont.

- We need one more equation to specify which tangent vector, among all possible satisfying (27), we want to obtain.
- Typically, we want one component k in v to be equal to one, so

$$t_k = \pm 1 \tag{29}$$

Combining everything:

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} t = \pm e_{2n+1} \tag{30}$$

where e_j is the j-th column of the identity matrix I_{2n+1}.

Tangent vector, summary

Combining everything:

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} t = Bt = \pm e_{2n+1} \tag{31}$$

where e_j is the j-th column of the identity matrix I_{2n+1}, and B is just the matrix on the left.

Solving (30): Simply use the inverse of matrix on the left: $t = B^{-1}(\pm e_{2n+1})$.

Interpretation: By following t, the k-th component in z changes with 1 or -1, the other ones change with t_j, $j \neq k$, where these t_j are computed from (30) to make sure that the step we take is in the tangent plane around the current operating point z_i.

Question: Where is the direction d?

Tangent vector, example

Matlab example.

Choosing the continuation parameter

- Previously: predictor step gave t and $z_{i+1}^p = z_i + s \cdot t$
- Problem: z_{i+1}^p does not satisfy power flow equations (it satisfies them only to the first order, in fact).
- Now: correct the prediction to get an operating point satisfying the power flow equations.
- Remember, in the next step, the correction step, some component of $z = [\theta \ V \ \lambda]$ will be kept constant:

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \tag{32}$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \tag{33}$$

$$0 = z_{i+1}^k - z_{i+1}^{k,p} \tag{34}$$

Intuition:

- we want to have control over the component that varies the fastest when taking a step.
- How to choose among all θ, V and λ the one we want to keep constant?

Choosing the continuation parameter, cont.

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} t = \pm e_{2n+1} \tag{35}$$

- Remember: Each component in t is associated with either one bus voltage angle θ_j, one bus voltage magnitude V_j or the loading λ.
- The components in the tangent vector t indicates how all variables θ, V and λ vary when we take a step.
- We choose k corresponding to the component with maximum variation:

$$k = \arg\max\{|t_1|, \ldots, |t_{2n+1}|\} \tag{36}$$

Correction step, details

- Previously:
  1. z_{i+1}^{p}: predicted value = approximation of the operating point when taking a step from z_i in direction p.
  2. k: the variable, among all θ, V and λ, that is kept constant during the correction step.
- Corrector step "easy": we just need to solve:

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \quad (37)$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \quad (38)$$

$$0 = z_{i+1}^{k} - z_{i+1}^{k,p} \quad (39)$$

Question: How to solve this?

Correction step, details

- Previously:
  1. z^p_{i+1}: predicted value = approximation of the operating point when taking a step from z_i in direction p.
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$$0 = z^k_{i+1} - z^{k,p}_{i+1} \tag{39}$$

Question: How to solve this? using Newton-Raphson method

Corrector step with Newton-Raphson

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \tag{40}$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \tag{41}$$

$$0 = z_{i+1}^{k,p} - z_{i+1}^{k} \tag{42}$$

- Newton-Raphson with z_{i+1}^{p} as initial guess
- j-th iteration in Newton-Raphson:

$$z_{i+1,j+1} = -J_{\mathrm{aug}}(z_{i+1,j})^{-1} f(z_{i+1,j}) \tag{43}$$

where the Jacobian of (39)–(41) is

$$J_{\mathrm{aug}}(z_{i+1,j}) = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} \tag{44}$$

Note: Easy to mix up all indices (i,k,j)!

Around the nose point

Remember: the maximum loadability point is characterized by singularity of the power flow Jacobian J (i.e. $\det J = 0$):

$$J = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} \tag{45}$$

Note that the Jacobian in the corrector process is

$$J_{\text{aug}} = \begin{bmatrix} J & \frac{\partial \Delta P}{\partial \lambda_i} \\ & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T \end{bmatrix} \tag{46}$$

If we choose λ as the continuation parameter, $\det J_{\text{aug}} = \det J$. Close to the nose point, J_{aug} would also be close to singular.

Importance of the continuation parameter

The continuation parameter helps in two different ways

1. Predictor step: $t_k = \pm 1$ helps controlling the fastest changing variable when we take a step.
2. Corrector step: Choosing another continuation parameter than λ ensures that the corrector step is numerically stable (Jacobian in the Newton-Raphson process not close to singular)

Why does it help?

So, why does it help?
Remember, problems with using PF calculations to trace the PV curve:

1. PF convergence very sensitive to initial conditions $\Rightarrow$ CPF uses a predictor step to get a good initial (=predicted) value.
2. The PF Jacobian is singular at the nose point $\Rightarrow$ numerical problems as we get close to this point $\Rightarrow$ Continuation parameter is set to ensure nonsingularity of the Jacobian matrix.

Agenda

Demo

Matlab demo.

What we haven't talked about

- Reactive power limits.
- How to choose the step size s.
- How to interpret and use the tangent vector.
- Other load models.
- Incorporating load dynamics, generator dynamics, etc

Inspiration

- Voltage stability phenomena explained by bifurcation theory (dynamical system). Nose point = saddle-node bifurcation.
- Continuation methods come from that field.
- Applied by Werner Rheinboldt in the field of structural mechanics.
- The work of Rheinboldt inspired Ajjarapu and Christy to adapt the method to the study of voltage stability.

Take-home messages

- Understand the Newton-Raphson method and how to use it.
- The principles of and intuition behind CPF are simple.
- The maths are important. Link intuition to maths! Important to be able to write down intuitions as mathematical formulations.
- Keep track of what has a physical meaning and what is just math (predictor step, for example, is a mathematical construction, although the tangent vector does have a physical interpretation)
- Keep track of indices, variables, Jacobians . . . Easy to get lost!
- Always make sure you understand the physics (i.e. what are we talking about, what is going on in the system, what is reactive power, . . .)
- When encountering a new problem, study other fields.