Continuation Power Flow
Advanced power system analysis

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(Slides and Matlab files:
`https://github.com/CamilleH/Lecture-on-continuation-power-flow`)

"Over the past decade, voltage instability has initiated several severe power system disruptions. Such incidents are likely to increase with increases in transmission line loadings. However, no theoretical or practical body of knowledge is available to meet the needs of power system planners and operators" (**January 1989**, EPRI, "Proceedings: Bulk Power System Voltage Phenomena - Voltage Stability and Security.")

Outline

the V-P-Q surface

- A lossless two-bus system (single-load infinite-bus systeme) is used for illustration purposes **only**, with one infinite bus (voltage $= Ee^{j0}$), one load bus (load $S = P + jQ$) and one transmission line (resistance = 0, reactance = X).
- Closed-form solution of the voltage magnitude at the load bus:

$$\frac{V}{E} = \sqrt{\frac{1}{2} - \frac{QX}{E^2} \pm \sqrt{\frac{1}{4} - \left(\frac{XP}{E^2}\right)^2 - \frac{XQ}{E^2}}}$$

- The $V - P - Q$ surface gives all operating points in which the grid can operate. Note that we did not make assumptions on the load models (i.e. no assumption on the expressions of P and Q)
- We can make assumptions on the power factor: $Q = \tan\phi \cdot P$
- See example in Matlab (file *study_tanphi.m*).

V-P-Q surface, cont.

- Apart from this very simple two-bus system, we do not know how to get the operating points in closed-form.
- What else can we use to get these points?

V-P-Q surface, cont.

- Apart from this very simple two-bus system, we do not know how to get the operating points in closed-form.
- What else can we use to get these points?
- Power flow computations, continuation power flow computations.

PV and QV curves

Most important questions:

- What are you trying to do when you plot/compute PV and QV curves?

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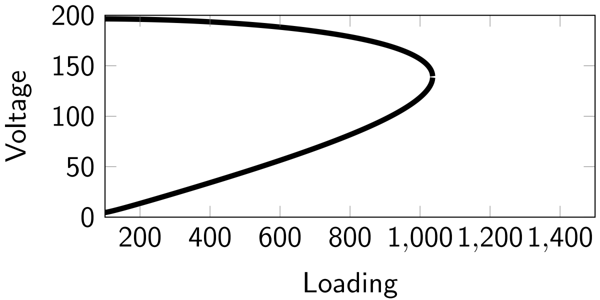
- QV curves: how much reactive compensation is needed at a particular bus? Only reactive power changes at one particular buses are studied.
- See Matlab example (*PV_and_QV_curves.m*).

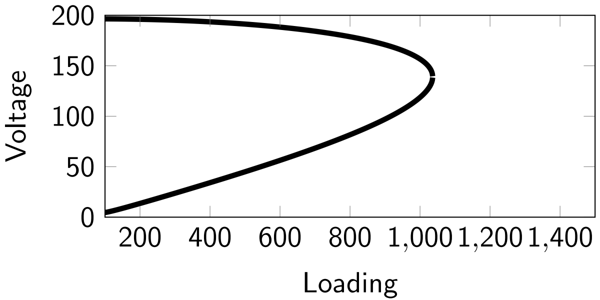
Stability of operating points on the PV curve 1



Consider a constant power load.

- Are the points on the upper part of the PV curve stable? Why?
- Are the points on the lower part of the PV curve stable? Why?

Stability of operating points on the PV curve 2



- What can drive voltage instability? i.e. why would we get to the unstable parts in the first place?

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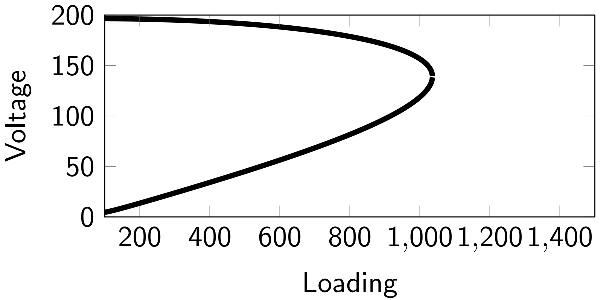
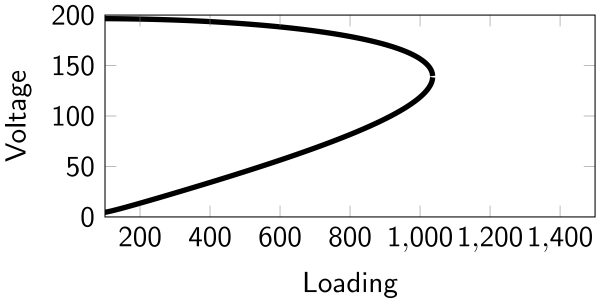


- What can drive voltage instability? i.e. why would we get to the unstable parts in the first place?
- Load build-up (ex: in the morning when people wake up), sudden disturbance (contingency), reactive power limits of generators encountered, . . .

Stability of operating points on the PV curve 3



- What can be consequences of having operating points with low voltages (even if they are stable)?

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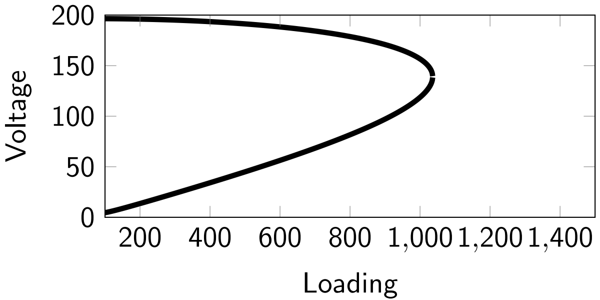
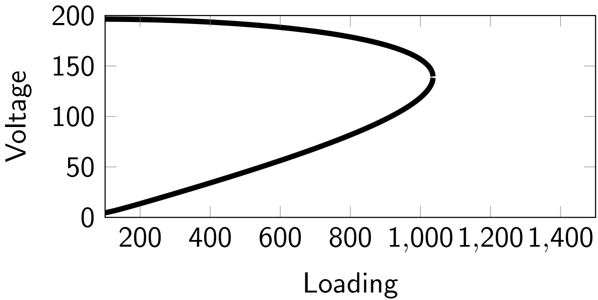


- What can be consequences of having operating points with low voltages (even if they are stable)?
- Protection systems will start disconnecting equipement (generators for example) which will make the system even more constrained..

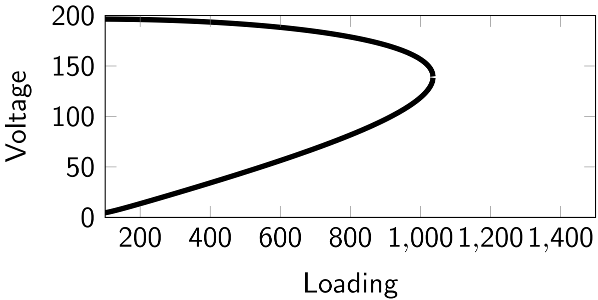
Voltage stability

- There is a loading point beyond which the system cannot supply more power.
- Load dynamics are critical to study voltage stability.
- However, for drawing PV curves, we do not consider load dynamics.
- PV curves show what the grid can supply, irrespective of the load dynamics.



Voltage stability, cont.

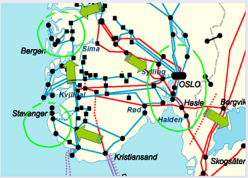
- What does loading mean? What does it mean to *increase the loading*?
- How to get the nose point?
- How to get the rest of the curve?
- Why are we interested in these quantities?
- How is this information typically used in operation and planning?

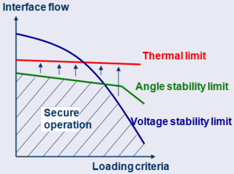


Stability constraints (remember?)

Power system perspective

Typically, stability constraints take the form of active power limits on critical interfaces (=transmission corridors / lines).





Math formulation

$$P_l \leq P_l^{\max}, \; \forall \text{ critical interfaces } l.$$

<u>Note:</u> *same math formulation as thermal limits but fundamentally different constraints!*

Example from Sweden: Using CPF for security management.

- Four price areas separated by bottlenecks (=critical transmission corridors).
- **Security assessment** The TSO monitors the power transfers across the bottlenecks.
- **Security enhancement** The TSO sends re-dispatch orders (increase/decrease production) if necessary.

What does "monitoring" mean in this context? What do we monitor, and against what?

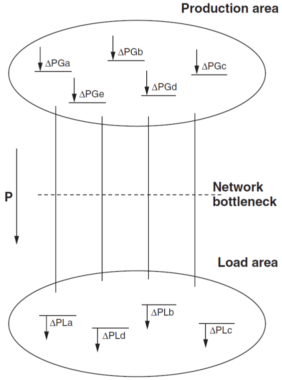
Example from Sweden - 2

Security assessment:

1. A list of contingencies is defined.
2. Every 15 minutes, for each contingency and each bottleneck, transmission limits are computed.
3. The power transfers are monitored and checked against all computed transmission limits.

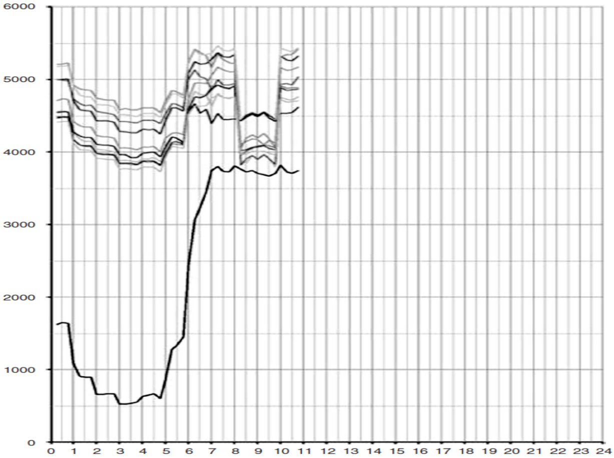
Security enhancement:

4. If the power transfers come close to one of the computed limits, re-dispatch the generation to decrease the power transfers.



How to compute the transmission limits?

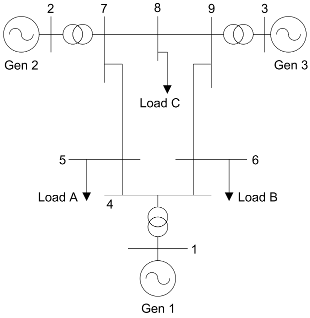
Example from Sweden - 3

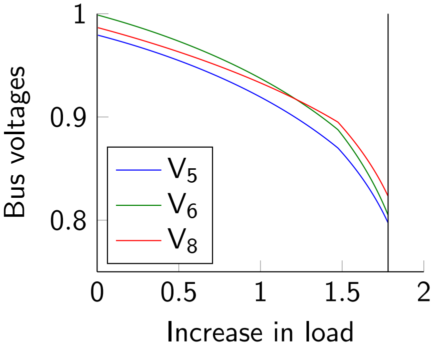


Source: Sandberg, L., & Roudén, K. (1992). *The real-time supervision of transmission capacity in the swedish grid*. In S. C. Savulescu (Ed.), *Real-time stability assessment in modern power system control centers*.

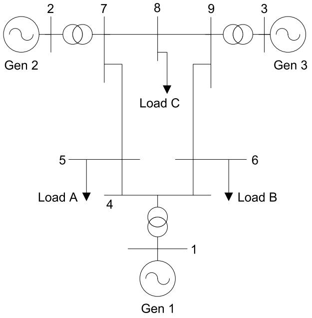
Example of voltage instability - 1

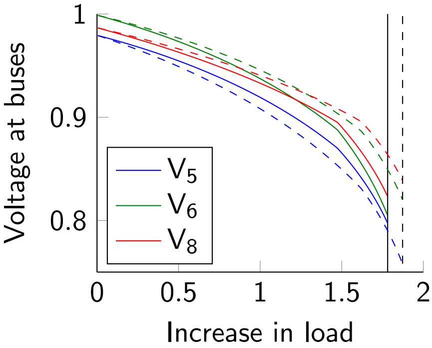
There is a loading point beyond which the system becomes unstable. How to define a loading and a loading increase in power systems?





Example of voltage instability - 2

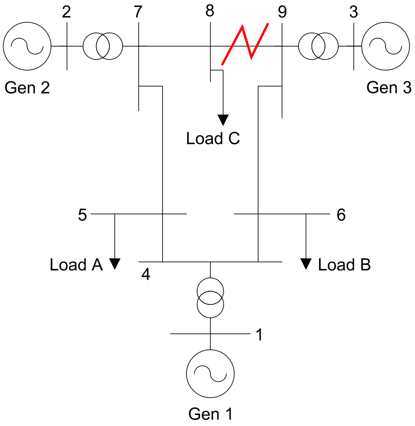


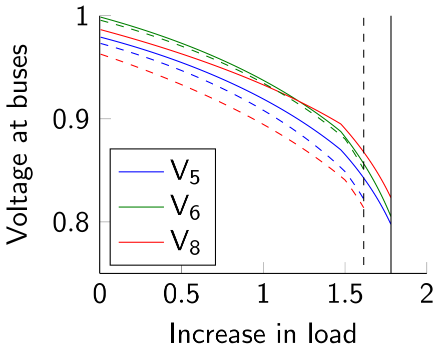


Solid=All loads increase by the same amount

Dashed=Load A increases double as much as the other two.

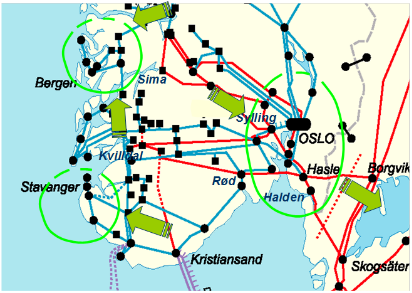
Example of voltage instability - 3

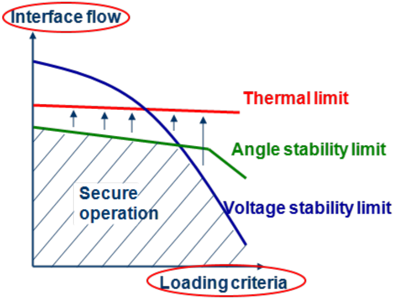




Solid=System intact
Dashed=Fault on line between buses 8 and 9.

Example of voltage instability - 4





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- I don't know. We need more detailed models of the loads to answer that question.
- What assumptions do we made when drawing PV curves?
- A certain relationship between P and Q for loads, slack buses, generation at all PV buses, . . . (what generators maintain power balance)

Outline

History

Many problem in the 1980s!

Date	Location	Time frame
13 April 1986	Winnipeg, Canada. Nelson River HVDC link	Transient, 1 sec.
30 Nov. 1986	SE Brazil, Paraguay. Itaipu HVDC link	Transient, 2 sec.
17 May 1985	South Florida, USA	Transient, 4 sec.
22 Aug. 1987	Western Tennessee, USA	Transient, 10 sec.
27 Dec. 1983	Sweden	Longer term, 55 sec.
02 Sept. 1982	Florida, USA	Longer term, 1-3 min.
26 Nov. 1982	Florida, USA	Longer term, 1-3 min.
28 Dec. 1982	Florida, USA	Longer term, 1-3 min.
30 Dec. 1982	Florida, USA	Longer term, 1-3 min.
22 Sept. 1977	Jacksonville, Florida	Longer term, few min.
04 Aug. 1982	Belgium	Longer term, 4.5 min.
12 Jan. 1987	Western France	Longer term, 6-7 min.
09 Dec. 1965	Brittany, France	Longer term
10 Nov. 1976	Brittany, France	Longer term
23 July 1987	Tokyo, Japan	Longer term, 20 min.
19 Dec. 1978	France	Longer term, 26 min.
22 Aug. 1970	Japan	Longer term, 30 min.

From D. T. Duong, 2016, “Online Voltage Stability Monitoring and Coordinated Secondary Voltage Control”

History, cont.

January 1989, EPRI, "Proceedings: Bulk Power System Voltage Phenomena - Voltage Stability and Security."

> *Over the past decade, voltage instability has initiated several severe power system disruptions. Such incidents are likely to increase with increases in transmission line loadings. However, no theoretical or practical body of knowledge is available to meet the needs of power system planners and operators*

History, cont.

- **September 1989**: Dobson and Chiang, "Towards a Theory of Voltage Collapse in Electric Power Systems."
- **October 1989**: Ajjarapu and Christy, "The Application of a Locally Parameterized Continuation Technique to the Study of Steady State Voltage Stability."
- **1992**: Ajjarapu and Christy, "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis."
- **1992**: Dobson and Lu, "Voltage Collapse Precipitated by the Immediate Change in Stability When Generator Reactive Power Limits Are Encountered."
- **1998**: Van Cutsem and Vournas, Voltage Stability of Electric Power Systems.
- **2007**: Ajjarapu, Computational Techniques for Voltage Stability Assessment and Control.

And many, many other contributors . . .

Outline

Why do we need it?

- The Newton-Raphson method is a method to compute x^* such that $f(x^*) = 0$.
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- Power flows boil down to computing voltage angles and magnitudes such that power flow equations are fulfilled.
- We'll see that it comes handy for CPF computations as well.

Newton-Raphson method

- Method to compute x^* such that $f(x^*) = 0$.
- Requires an initial guess.
- Relies on a first-order approximation of f around the current guess x_i:

$$f(x) = f(x_i) + \nabla_x f(x_i)(x - x_i) + \text{ higher order term} \tag{1}$$

Newton-Raphson method, algorithm

1. Set $x_i = x_0$;
2. **While** $\|f(x_i)\| > \epsilon$, **do**

$$x_{i+1} = x_i - \left(\nabla_x f(x_i)\right)^{-1} f(x_i)$$
$$i \leftarrow i + 1$$

3. Hope that you guessed x_0 right so that the method converges.

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Question: Can you think of one case in which this method will not work?

Outline

Power flow computations

- **Objective**: ?
- **Method**: Active and reactive power balance at each bus must hold.

$$\Delta P = P_g - P_l - P_s(\theta, V) = 0, \quad (2)$$

$$\Delta Q = Q_g - Q_l - Q_s(\theta, V) = 0, \quad (3)$$

- Can be written

$$x = [\theta \ V] \quad (4)$$

$$f(x) = \begin{bmatrix} \Delta P(x) \\ \Delta Q(x) \end{bmatrix} = 0 \quad (5)$$

Power flow computations

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Newton-Raphson iteration for power flows

The power flow equations $f(x) = 0$ can be solved by the Newton-Raphson method. Iterations:

$$x_{i+1} = [\theta_{i+1} \; V_{i+1}] = x_i - J(x_i)^{-1} f(x_i) \tag{6}$$

with (i is for iteration number, *not* bus number)

$$J(x_i) = J([\theta_i \; V_i]) = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} = \begin{bmatrix} f_\theta & f_V \end{bmatrix} \tag{7}$$

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So assume that you are an operator and the power flow computations fail, what can you conclude?

1. The maximum loadability limit (=nose point) has been reached, or
2. There were numerical issues with the power flow computations.

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Parametrization of load increase

There is a loading point beyond which the system cannot more power. How to define a loading and a loading increase in power systems?

- Add one parameter $\lambda \in \mathbb{R}$ to parametrize the load increase process in direction $d \in \mathbb{R}^n$:

$$P_l = P_l^0 + \lambda d \in \mathbb{R}^n \tag{8}$$

- For example, in a three-bus system with two loads at buses 2 and 3, the loads could be increased as follows

$$P_l = \begin{bmatrix} 0 \\ 150 \\ 120 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{9}$$

Parametrization of load increase, cont.

- Reactive power: typically, we assume constant power factor load (but the method can handle other models as well), so

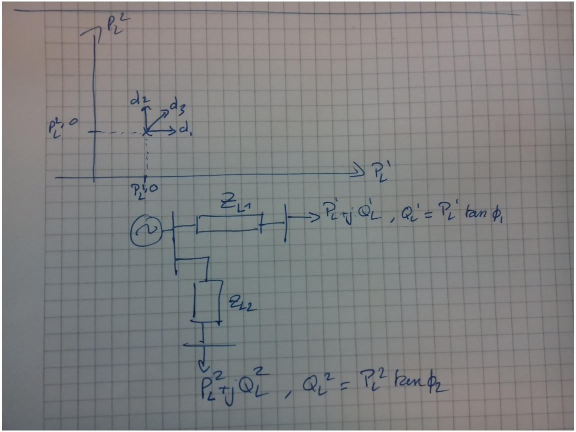
$$Q_l = \mathrm{diag}(\tan\phi) \cdot P_l = Q_l^0 + \lambda \cdot \mathrm{diag}(\tan\phi) \cdot d \qquad (10)$$

- For example, with $\tan\phi = Q_0/P_0 = 0.5$ for all loads,

$$Q_L = \begin{bmatrix} 0 \\ 75 \\ 60 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} \qquad (11)$$

Illustration on the board

Two-dimensional load increase from P_l^0 in direction d.



Note: Important to keep in mind which space ($V - \lambda$, P_l, etc.) is of interest.

Extending the power flow equations

$$0 = P_g^i - P_l^i - P_s^i(\theta, V), \quad \forall \text{ PV and PQ buses} \tag{12}$$

$$0 = Q_g^i - Q_l^i - Q_s^i(\theta, V), \quad \forall \text{ PQ buses} \tag{13}$$

$$P_l = P_l^0 + \lambda d \in R^n \tag{14}$$

$$Q_l = Q_l^0 + \lambda \cdot \mathrm{diag}(\tan\phi) \cdot d \tag{15}$$

	PF	CPF
Equations	(12), (13)	(12), (13), (14), (15)
Variables	$x = [\theta \ V]$	$z = [x \ \lambda] = [\theta \ V \ \lambda]$
Parameters	P_l, $\tan\phi$	P_l^0, $\tan\phi$, d, λ
	(and P_g, θ_{slack}, ...)	(and P_g, θ_{slack}, ...)

Increasing $\lambda \Leftrightarrow$ simulating a load increase in direction d.

Outline

CPF: Principles

The CPF is a predictor-corrector process:

1. Start from an operating point $z_i = [\theta_i \ V_i \ \lambda_i]$.
2. Predict what the operating point becomes when loading increases in direction $d \Rightarrow z_{i+1}^{p}$.
3. Correct the prediction to get a real operating point z_{i+1}.

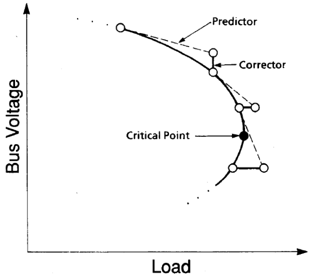


Figure: CPF process, from Ajjarapu and Christy, 1992, "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis."

Steps in CPF

Three main steps:

1. Predictor step: From a known operating point z_i (ex: previously corrected), take a step of length s in the direction of the tangent vector t_i.

$$z_{i+1}^p = [\theta_{i+1} \; V_{i+1} \; \lambda_{i+1}] = z_i + s \cdot t_i \tag{16}$$

Notes:

- The predicted point does **not** correspond to a physical operating point.
- It is a mathematical construction to estimate the values of $z = [\theta \; V \; \lambda]$ after taking a step of length s in direction d (in the load space $= P_l$-space).
- It gives a good initial guess of z before the corrector step.

Steps in CPF, cont.

2. Choosing the continuation parameter: which component in $z = [\theta \ V \ \lambda]$ should we keep constant in the corrector step?
3. Corrector step: Correct the predicted value to a valid operating point (= "project" back onto the PV curve), keeping the continuation parameter constant. Solve:

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \tag{17}$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \tag{18}$$

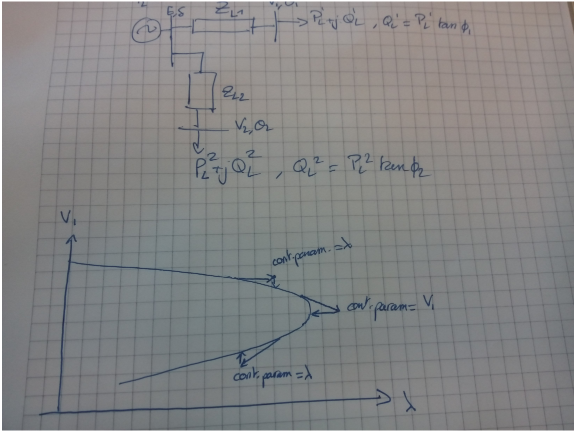
$$0 = z_{i+1}^k - z_{i+1}^{k,p} \tag{19}$$

Equations (17)–(19) solved for $z_{i+1} = [\theta_{i+1} \ V_{i+1} \ \lambda_{i+1}]$ by Newton-Raphson method.

Notes:

- Without (19), the system of equations is underdetermined.
- (19) says how to correct onto the PV curve (keeping λ constant, keeping some V_m constant, ...)

Illustration on the board



Outline

Predictor step, details

From a known operating point z_i (ex: previously corrected), take a step in the direction of the tangent vector t_i.

$$z_{i+1}^{p} = [\theta_{i+1} \; V_{i+1} \; \lambda_{i+1}] = z_i + s \cdot t_i \tag{20}$$

Question: How is defined the tangent vector?
More general question: How to compute *one* tangent vector to a surface described by $g(u) = 0$?
Notes:

- t_i is the tangent vector to the operating point $z_i = [\theta_i \; V_i \; \lambda_i]$ defined by the power flow equations $f(z_i) = 0$.
- Notice the space we are talking about. It is the z-space, not only the (V, λ)-space
- Where is the direction d here?

Computing a tangent vector

Equation of the form

$$g(u_0) = 0 \tag{21}$$

Any vector t in the null space of the Jacobian $J(u_0)$ is a tangent vector to g at u_0, i.e. any vector $u \neq 0$ such that

$$J(x_0)t = 0. \tag{22}$$

Intuition,

$$g(u_0 + t) = g(u_0) + J(u_0)t + \text{ higher order terms} \tag{23}$$

The set of all $u_0 + t$ such that $J(u_0)t = 0$ is the tangent plane of g at u_0.

See also `http://mathworld.wolfram.com/SubmanifoldTangentSpace.html`

Tangent vector to power flow equations

Let $z_i = [\theta_i \ V_i \ \lambda_i]$ be an operating point, i.e.

$$\Delta P(z_i) = 0 \tag{24}$$
$$\Delta Q(z_i) = 0 \tag{25}$$

Any vector t such that

$$J(z_i)t = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \end{bmatrix} t = 0 \tag{26}$$

is a tangent vector.
Note: We cannot just solve $J(z_i)t = 0$ for t to find t (i.e. (26) is a necessary but not sufficient condition to get *the* t we are looking for). Why?

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Tangent vector to power flow equations, cont.

- We need one more equation to specify which tangent vector, among all possible satisfying (26), we want to obtain.
- Typically, we want one component k in t to be equal to one, so

$$t_k = \begin{cases} 1 & \text{if we take a step towards} + \lambda \\ -1 & \text{if we take a step towards} - \lambda \text{ or } - V_k \end{cases} \tag{27}$$

Combining everything:

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} t = \pm e_{2n+1} \tag{28}$$

where e_j is the j-th column of the identity matrix I_j.

Tangent vector, summary

Combining everything:

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} t = Bt = \pm e_{2n+1} \tag{29}$$

where e_j is the j-th column of the identity matrix I_j, and B is just the matrix on the left.

Solving (29): Simply use the inverse of matrix on the left: $t = B^{-1}(\pm e_{2n+1})$.

Interpretation: By following t, the k-th component in z changes with 1 or -1, the other ones change with t_j, $j \neq k$, where these t_j are computed from (29) to make sure that the step we take is in the tangent plane around the current operating point z_i.

Question: Where is the direction d?

Tangent vector, example

Tangent vector to take a step towards increasing λ (upper part of the PV curve):

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & d \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \text{diag}(\tan \phi) \cdot d \\ 0 & 0 & 1 \end{bmatrix} t = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Tangent vector, example

Tangent vector to take a step towards decreasing V_k (when close to the nose point):

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & d \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \mathrm{diag}(\tan\phi)\cdot d \\ 0 & 1_k & 0 \end{bmatrix} t = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

-1_k means 1 at the k-th element and 0 elsewhere.

Tangent vector, example

Tangent vector to take a step towards decreasing λ (lower part of the PV curve):

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & d \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \operatorname{diag}(\tan\phi) \cdot d \\ 0 & 0 & 1 \end{bmatrix} t = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Tangent vector in Matlab

Matlab example.

Choosing the continuation parameter

- Previously: predictor step gave t and $z_{i+1}^p = z_i + s \cdot t$
- Problem: z_{i+1}^p does not satisfy power flow equations (it satisfies them only to the first order, in fact).
- Now: correct the prediction to get an operating point satisfying the power flow equations.
- Remember, in the next step, the correction step, some component of $z = [\theta \ V \ \lambda]$ will be kept constant:

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \tag{30}$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \tag{31}$$

$$0 = z_{i+1}^k - z_{i+1}^{k,p} \tag{32}$$

Intuition:

- we want to have control over the component that varies the fastest when taking a step.
- How to choose among all θ, V and λ the one we want to keep constant?

Choosing the continuation parameter, cont.

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} t = \pm e_{2n+1} \quad (33)$$

- Remember: Each component in t is associated with either one bus voltage angle θ_j, one bus voltage magnitude V_j or the loading λ.
- The components in the tangent vector t indicates how all variables θ, V and λ vary when we take a step.
- We choose k corresponding to the component with maximum variation:

$$k = \arg\max\{|t_1|, \ldots, |t_{2n+1}|\} \quad (34)$$

Correction step, details

- Previously:
  1. z_{i+1}^p: predicted value = approximation of the operating point when taking a step from z_i in direction p.
  2. k: the variable, among all θ, V and λ, that is kept constant during the correction step.
- Corrector step "easy": we just need to solve:

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \tag{35}$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \tag{36}$$

$$0 = z_{i+1}^k - z_{i+1}^{k,p} \tag{37}$$

to get $z_{i+1} = [\theta_{i+1} \ V_{i+1} \ \lambda_{i+1}]$.
Question: How to solve this?

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to get $z_{i+1} = [\theta_{i+1} \; V_{i+1} \; \lambda_{i+1}]$.

Question: How to solve this? using Newton-Raphson method

Corrector step with Newton-Raphson

$$0 = P_g - (P_l^0 + \lambda_{i+1} d) - P_s(\theta_{i+1}, V_{i+1}), \quad (38)$$

$$0 = Q_g - (Q_l^0 + \lambda_{i+1} \cdot \mathrm{diag}(\tan\phi) d) - Q_s(\theta_{i+1}, V_{i+1}), \quad (39)$$

$$0 = z_{i+1}^{k,p} - z_{i+1}^{k} \quad (40)$$

- Newton-Raphson with z_{i+1}^{p} as initial guess
- j-th iteration in Newton-Raphson:

$$z_{i+1,j+1} = z_{i+1,j} - J_{\mathrm{aug}}(z_{i+1,j})^{-1} f(z_{i+1,j}) \quad (41)$$

where the Jacobian of (38)–(40) is

$$J_{\mathrm{aug}}(z_{i+1,j}) = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial \lambda} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T & \end{bmatrix} \quad (42)$$

Note: Easy to mix up all indices (i,k,j)!

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- It is certainly **not** a valid operating point. It is a "first-order" estimate of what the system state will be.

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- What does the predicted point correspond to physically?
- It is certainly **not** a valid operating point. It is a "first-order" estimate of what the system state will be.
- What information does the tangent vector carry?
- Its elements show how the voltage angles, magnitudes and loading vary when taking a step (toward different loading, different voltages, . . .)

Question, cont.

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- Two things: (1) How the prediction step is made (step in loading, or step in voltage) and (2) What the correction step keeps constant (the additional equation compared to power flow equations)

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- Why do we want to switch the continuation parameter?

Question, cont.

- What does the continuation parameter control?
- Two things: (1) How the prediction step is made (step in loading, or step in voltage) and (2) What the correction step keeps constant (the additional equation compared to power flow equations)
- Why do we want to switch the continuation parameter?
- Two things: Switching to voltage allows us to (1) make finer guesses in the prediction step (we control how much voltages decrease) and (2) obtain solutions in the correction step when the predicted loading is beyond the nose point.

Outline

Around the nose point

Remember: the maximum loadability point is characterized by singularity of the power flow Jacobian J (i.e. $\det J = 0$):

$$J = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} \tag{43}$$

Note that the Jacobian in the corrector process is

$$J_{\text{aug}} = \begin{bmatrix} J & \frac{\partial \Delta P}{\partial \lambda_i} \\ & \frac{\partial \Delta Q}{\partial \lambda} \\ & e_k^T \end{bmatrix} \tag{44}$$

If we choose λ as the continuation parameter, $\det J_{\text{aug}} = \det J$. Close to the nose point, J_{aug} would also be close to singular.

Importance of the continuation parameter

The continuation parameter helps in two different ways

1. Predictor step: $t_k = \pm 1$ helps control the fastest changing variable when we take a step.
2. Corrector step: Choosing another continuation parameter than λ ensures that the corrector step is numerically stable (Jacobian in the Newton-Raphson process not close to singular)

Why does it help?

So, why does it help?
Remember, problems with using PF calculations to trace the PV curve:

1. PF convergence very sensitive to initial conditions $\Rightarrow$ CPF uses a predictor step to get a good initial (=predicted) value.
2. The PF Jacobian is singular at the nose point $\Rightarrow$ numerical problems as we get close to this point $\Rightarrow$ Continuation parameter is set to ensure nonsingularity of the Jacobian matrix.

Outline

Demo

Matlab demo.

What we haven't talked about

- Reactive power limits.
- How to choose the step size s.
- Other load models.
- Incorporating load dynamics, generator dynamics, etc

Inspiration

- Voltage stability phenomena explained by bifurcation theory (dynamical system). Nose point = saddle-node bifurcation.
- Continuation methods come from that field.
- Applied by Werner Rheinboldt in the field of structural mechanics.
- The work of Rheinboldt inspired Ajjarapu and Christy to adapt the method to the study of voltage stability.

Take-home messages

- Understand the Newton-Raphson method and how to use it.
- The principles and intuition behind CPF are simple.
- The maths are important. Link intuition to maths! Important to be able to write down intuitions as mathematical formulations.
- Keep track of what has a physical meaning and what is just math (predictor step, for example, is a mathematical construction, although the tangent vector does have a physical interpretation)
- Keep track of indices, variables, Jacobians . . . Easy to get lost!
- Always make sure you understand the physics (i.e. what are we talking about, what is going on in the system, what is reactive power, . . .)
- When encountering a new problem, study other fields.

Load models

- The active and reactive power consumptions of the loads depend voltage and demand.
- Load demand and active and reactive power consumptions are different quantities.

$$P = P(z, V)$$
$$Q = Q(z, V)$$

Load models - Individual loads

- Individual loads can be modelled by the exponential load model.
- See example in Matlab (file *study_load_models.m*).

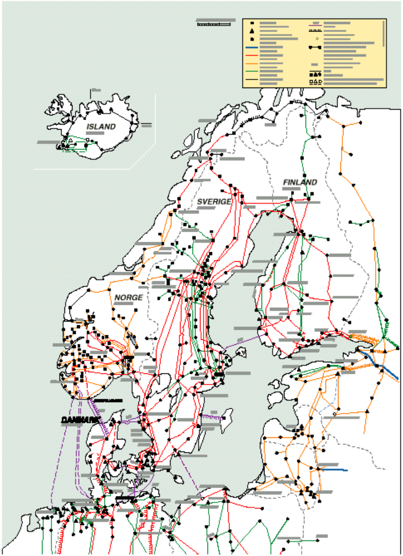
$$P = zP_0\left(\frac{V}{V_0}\right)^{\alpha},$$

$$Q = zQ_0\left(\frac{V}{V_0}\right)^{\beta}$$

Load	α	β
incandescent lamps	1.54	-
room air conditioner	0.5	2.5
furnace fan	0.08	1.6
battery charger	2.59	4.06

(Van Cutsem and Vournas, 1998)

Load models - aggregate loads



- Where are your incandescent lamps, electric heating, ...?

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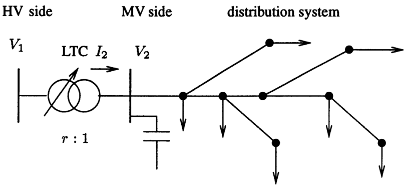

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- See from the high-voltage network, each "load" is actually one substation, i.e. an aggregation of loads at lower voltage levels beyond the transformer between high and lower voltage levels.

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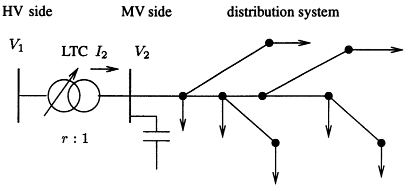

- Where are your incandescent lamps, electric heating, ...?
- See from the high-voltage network, each “load” is actually one substation, i.e. an aggregation of loads at lower voltage levels beyond the transformer between high and lower voltage levels.
- The parameters of the models for aggregate loads are obtained by measurements.