Tarea 3 García Ponce José Camilo

1) Un numero primo es un número entero que no es divisible por otros números, excepto por si mismo y por 1. Formaliza el concepto de ser primo por medio de un predicado, suponiendo que ya tienes otro predica que define divisibilidad

tengamos estos predicado

$P(x) = x$ es primo , $D(x,y) = y$ divide a x

entonces

$$P(x) \iff (x \in \mathbb{N} \wedge [\forall y \in \mathbb{N}, D(x,y) \Rightarrow y=1 \vee y=x] \wedge x>1)$$

2) Señala las variables libres y ligadas de las siguientes formulas

(a) $(\forall x . (\exists y . P_2^2(x,z)) \vee P_2^1(y))$

$F[(\forall x . (\exists y . P_2^2(x,z)) \vee P_2^1(y))]$

$= F[(\exists y . P_2^2(x,z)) \vee P_2^1(y)] - \{x\}$

$= (F[(\exists y . P_2^2(x,z)] \cup F(P_2^1(y))) - \{x\}$

$= [(F(P_2^2(x,z) - \{y\}) \cup \{y\}] - \{x\}$

$= [(\{x,z\} - \{y\}) \cup \{y\}] - \{x\} = \{x,z\} - \{x\} = \{z\}$ variable libre

$B[(\forall x . (\exists y . P_2^2(x,z)) \vee P_2^1(y))]$

$= B[(\exists y . P_2^2(x,z)) \vee P_2^1(y)] \cup \{x\}$

$= (B[(\exists y \; P_2^2(x,z)] \cup B(P_2^1(y))) \cup \{x\}$

$= [(B(P_2^2(x,z) \cup \{y\}) \cup \emptyset] \cup \{x\}$

$= [(\emptyset \cup \{y\}) \cup \emptyset] \cup \{x\} = \{y\} \cup \{x\} = \{x,y\}$ variables ligadas

(b) $(\exists x . (\forall z . (\exists y . P_1^3(x,y,z)))) \Leftrightarrow P_2^3(x_1, y, z_2)$

$F[(\exists x . (\forall z . (\exists y . P_1^3(x,y,z)))) \Leftrightarrow P_2^3(x_1, y, z_2)]$

$= F[(\exists x . (\forall z . (\exists y . P_1^3(x,y,z))))] \cup F(P_2^3(x_1, y, z_2))$

$= (F[(\forall z . (\exists y . P_1^3(x,y,z)))] - \{x\}) \cup \{x_1, y, z_2\}$

$= [(F(\exists y . P_1^3(x,y,z))) - \{z\}) - \{x\}] \cup \{x_1, y, z_2\}$

$= [((F(P_1^3(x,y,z)) - \{y\}) - \{z\}) - \{x\}] \cup \{x_1, y, z_2\}$

$= [((\{x,y,z\} - \{y\}) - \{z\}) - \{x\}] \cup \{x_1, y, z_2\}$

$= [(\{x,z\} - \{z\}) - \{x\}] \cup \{x_1, y, z_2\} = (\{x\} - \{x\}) \cup \{x_1, y, z_2\}$

$= [\emptyset \cup \{x_1, y, z_2\} = \{x_1, y, z_2\}$ variables libres

$B[(\exists x, (\forall z, (\exists y, P_1^3(x,y,z)))) <=> P_2^3(x_1, y, z_2)]$

$= B[(\exists x, (\forall z, (\exists y, P_1^3(x,y,z))))] \cup B(P_2^3(x_1, y, z_2))$

$= (B[(\forall z, (\exists y, P_1^3(x,y,z)))] \cup \{x\}) \cup \emptyset$

$= [(B(\exists y, P_1^3(x,y,z)) \cup \{z\}) \cup \{x\})$

$= [((B(P_1^3(x,y,z)) \cup \{y\}) \cup \{z\}) \cup \{x\})$

$= [((\emptyset \cup \{y\}) \cup \{z\}) \cup \{x\}) = [(\{y\} \cup \{z\}) \cup \{x\}]$

$= (\{y,z\} \cup \{x\}) = \{x,y,z\}$ variables ligadas

(c) $(\exists x, P_1^3(x,y,z)) => P_1^1(x)$

$F[(\exists x, P_1^3(x,y,z)) => P_1^1(x)]$

$= F(\exists x, P_1^3(x,y,z)) \cup F(P_1^1(x)]$

$= (F(P_1^3(x,y,z)) - \{x\}) \cup \{x\}$

$= (\{x,y,z\} - \{x\}) \cup \{x\} = \{y,z\} \cup \{x\} = \{x,y,z\}$ variables libres

$B[(\exists x, P_1^3(x,y,z)) => P_1^1(x)]$

$= B(\exists x, P_1^3(x,y,z)) \cup B(P_1^1(x))$

$= (B(P_1^3(x,y,z)) \cup \{x\}) \cup \emptyset$

$= (\emptyset \cup \{x\}) = \{x\}$ variable ligada

(d) $(\forall y, P_1^3(x,y,z)) \wedge (\exists z, P_2^3(x,y,z)) \wedge (\forall x, P_3^3(x,y,z))$

$F[(\forall y, P_1^3(x,y,z)) \wedge (\exists z, P_2^3(x,y,z)) \wedge (\forall x, P_3^3(x,y,z))]$

$= F(\forall y, P_1^3(x,y,z)) \cup F(\exists z, P_2^3(x,y,z)) \cup F(\forall x, P_3^3(x,y,z))$

$= (F(P_1^3(x,y,z)) - \{y\}) \cup (F(P_2^3(x,y,z)) - \{z\}) \cup (F(P_3^3(x,y,z)) - \{x\})$

$= (\{x,y,z\} - \{y\}) \cup (\{x,y,z\} - \{z\}) \cup (\{x,y,z\} - \{x\})$

$= \{x, z\} \cup \{x, y\} \cup \{y, z\} = \{x, y, z\}$ variables libres

$B[(\forall y, P_1^3(x,y,z)) \wedge (\exists z, P_2^3(x,y,z)) \wedge (\forall x, P_3^3(x,y,z))$

$= B(\forall y, P_1^3(x,y,z)) \cup B(\exists z, P_2^3(x,y,z)) \cup B(\forall x, P_3^3(x,y,z))$

$= (B(P_1^3(x,y,z)) \cup \{y\}) \cup (B(P_2^3(x,y,z)) \cup \{z\}) \cup (B(P_3^3(x,y,z)) \cup \{x\})$

$= (\emptyset \cup \{y\}) \cup (\emptyset \cup \{z\}) \cup (\emptyset \cup \{x\})$

$= \{y\} \cup \{z\} \cup \{x\} = \{x, y, z\}$ variables ligadas

FIRST CLASS

regla 1 $(\forall x, \alpha)_{[x:=t]} = (\forall x, \alpha)$

regla 2 $(\forall y, \alpha)_{[x:=t]} = (\forall y, \alpha_{[x:=t]})$

regla 3 $(\forall y, \alpha)_{[x:=t]} = \forall z (\alpha_{[y:=z]})_{[x:=t]}$

(las misma pero para $\exists$)

3) Realiza las siguientes sustituciones

(a) $((\exists x, P_1^3(x,y,z)) \Rightarrow P_1^1(x))_{[z:=f_1^2(x,y)]}$

$((\exists x, P_1^3(x,y,z))_{[z:=f_1^2(x,y)]} \Rightarrow P_1^1(x)_{[z:=f_1^2(x,y)]}$

$(\exists \omega, (P_1^3(x,y,z)_{[x:=\omega]})_{[z:=f_1^2(x,y)]} \Rightarrow P_1^1(x_{[z:=f_1^2(x,y)]})$ regla 3

$(\exists \omega, (P_1^3(x_{[x:=\omega]}, y_{[x:=\omega]}, z_{[x:=\omega]}))_{[z:=f_1^2(x,y)]} \Rightarrow P_1^1(x)$

$(\exists \omega, (P_1^3(\omega,y,z))_{[z:=f_1^2(x,y)]} \Rightarrow P_1^1(x)$

$(\exists \omega, (P_1^3(\omega,y,z)_{[z:=f_1^2(x,y)]}) \Rightarrow P_1^1(x)$ regla 2

$(\exists \omega, (P_1^3(\omega_{[z:=f_1^2(x,y)]}, y_{[z:=f_1^2(x,y)]}, z_{[z:=f_1^2(x,y)]}) \Rightarrow P_1^1(x)$

$(\exists \omega, (P_1^3(\omega, y, f_1^2(x,y))) \Rightarrow P_1^1(x)$

(b) $(((\forall x, P_1^3(x,y,z)) \wedge (\forall y, P_1^3(x,y,z)) \wedge (\exists z, P_1^3(x,y,z)))_{[x:=f_1^2(y,z)]})_{[z:=f_1^2(y,x)]}$

$(((\forall x, P_1^3(x,y,z)) \wedge (\forall y, P_1^3(x,y,z)))_{[x:=f_1^2(y,z)]} \wedge (\exists z, P_1^3(x,y,z))_{[x:=f_1^2(y,z)]})_{[z:=f_1^2(y,x)]}$

regla 3

$(((\forall x, P_1^3(x,y,z))_{[x:=f_1^2(y,z)]} \wedge (\forall y, P_1^3(x,y,z))_{[x:=f_1^2(y,z)]} \wedge (\exists \omega, (P_1^3(x,y,z)_{[z:=\omega]})_{[x:=f_1^2(y,z)]})_{[z:=f_1^2(y,x)]}$

regla 1 regla 3

$(((\forall x, P_1^3(x,y,z)) \wedge (\forall a, (P_1^3(x,y,z)_{[y:=a]})_{[x:=f_1^2(y,z)]} \wedge (\exists \omega, P_1^3(x,y,\omega))_{[x:=f_1^2(y,z)]})_{[z:=f_1^2(y,x)]}$

$(((\forall x, P_1^3(x,y,z)) \wedge (\forall a, P_1^3(x,a,z))_{[x:=f_1^2(y,z)]} \wedge (\exists \omega, P_1^3(x,y,\omega)_{[x:=f_1^2(y,z)]})))_{[z:=f_1^2(y,x)]}$

$$(((\forall x,\ P_i^3(x,y,z)) \wedge (\forall a,\ P_i^3(x,a,z)_{[x:=f_i^2(y,z)]}) \wedge (\exists w,\ P_i^3(f_i^2(y,z),y,w)))_{[z:=f_i^2(y,x)]}$$

$$(((\forall x,\ P_i^3(x,y,z)) \wedge (\forall a,\ P_i^3(f_i^2(y,z),a,z)) \wedge (\exists w,\ P_i^3(f_i^2(y,z),y,w))_{[z:=f_i^2(y,x)]})$$

$$(((\forall x,\ P_i^3(x,y,z)) \wedge (\forall a,\ P_i^3(f_i^2(y,z),a,z)))_{[z:=f_i^2(y,x)]} \wedge (\exists w,\ P_i^3(f_i^2(y,z),y,w)_{[z:=f_i^2(y,x)]}))$$

$$(((\forall x,\ P_i^3(x,y,z)_{[z:=f_i^2(y,x)]} \wedge (\forall a,\ P_i^3(f_i^2(y,z),a,z)_{[z:=f_i^2(y,x)]}) \wedge (\exists w,(P_i^3(f_i^2(y,z),y,w)_{[z:=f_i^2(y,x)]})))$$

regla 2

$$(((\forall x,\ P_i^3(x,y,z)_{[z:=f_i^2(y,x)]} \wedge (\forall a,\ P_i^3(f_i^2(y,z),a,z)_{[z:=f_i^2(y,x)]}) \wedge (\exists w,\ P_i^3(f_i^2(y,f_i^2(y,x)),y,w))))$$

regla 3

$$(((\forall b,(P_i^3(x,y,z)_{[x:=b]})_{[z:=f_i^2(y,x)]} \wedge (\forall a,(P_i^3(f_i^2(y,z),a,z)_{[z:=f_i^2(y,x)]})) \wedge (\exists w,\ P_i^3(f_i^2(y,f_i^2(y,x)),y,w))))$$

regla 2

$$(((\forall b,\ P_i^3(b,y,z))_{[z:=f_i^2(y,x)]} \wedge (\forall a,\ P_i^3(f_i^2(y,f_i^2(y,x)),a,f_i^2(y,x))) \wedge (\exists w,\ P_i^3(f_i^2(y,f_i^2(y,x)),y,w))))$$

regla 2

$$(((\forall b,(P_i^3(b,y,z)_{[z:=f_i^2(y,x)]})) \wedge (\forall a,\ P_i^3(f_i^2(y,f_i^2(y,x)),a,f_i^2(y,x))) \wedge (\exists w,\ P_i^3(f_i^2(y,f_i^2(y,x)),y,w))))$$

$$\underbrace{(((\forall b,\ P_i^3(b,y,f_i^2(y,x))) \wedge (\forall a,\ P_i^3(f_i^2(y,f_i^2(y,x)),a,f_i^2(y,x))) \wedge (\exists w,\ P_i^3(f_i^2(y,f_i^2(y,x)),y,w))))}_{\text{Creo que queda así}}$$

4) Dada la siguiente interpretacion, di si las formulas (a) -(e) son satisfechas, verdaderas, validas (o ninguna de las anteriores):

Universo de numeros racionales $\mathbb{Q}$

$\Psi(c)=0$ $\Psi(x)=1$ $\Psi(y)=-1$

$\phi(f_1^1)=$ sucesor $\phi(f_2^1)=$ predecesor $\phi(f_1^2)=\div$ $\pi(P_1^2)=\leq$

(a) $\forall x, \forall y, \neg(y=0) \Rightarrow \exists z, f_1^2(x,y)=z;$

esta formula es verdadera, ya que para cualquier variable se cumple, pero para toda formula no se cumple, por ejemplo si $f_1^2(x,y)$ es x^y entonces si $x=-1$ y $y=\frac{1}{2}$ tenemos que z no esta en $\mathbb{Q}$

(b) $\forall x, \exists y \; P_1^2(x,y) \Rightarrow P_1^2(x,y)$

esta formula es valida, ya que para cualquier formula/predicado y variable tenemos que se cumple, debido a que si $P_1^2(x,y)$ es verdadero o falso $P_1^2(x,y) \Rightarrow P_1^2(x,y)$ siempre sera verdadero

(c) $\forall x, \forall y, \exists z, P_1^2(x,y) \Rightarrow P_1^2(x,z) \wedge P_1^2(z,y)$

esta formula es verdadera ya que para cualquier variable se cumple, pero para todo predicado no se cumple, por ejemplo si $P_1^2(x,y)$ es $x^2=y$ entonces si $x=2$ y $y=4$ entonces $P_1^2(x,y)$ se cumple, pero no existe z que cumpla que $x^2=z$ y $z^2=y$ ya que $x^2=y$ entonces $x^2=z$ y $z^2=x^2$, pero $x=2$ entonces $z^2=4$ y $z^2=y^2$, pero no se puede cumplir

(d) $c = f_2^1(y) \wedge c = f_1^1(x)$

esta formula no es verdadera, ni valida, ni satisfecha

ya que 0 no es el predecesor de -1 ni es sucesor de 1

(e) $p_1^2(f_2^1(y), c) \wedge P_1^2(c, f_1^1(x))$

esta formula es satisfecha, ya que $-2 \leq 0$ y $0 \leq 2$

pero si $y = 2$ y $x = -2$ entonces $1 \leq 0$ y $0 \leq -1$ lo cual es falso

FIRST CLASS

5) Encuentra un modelo para las siguientes formulas:

$\forall x, (\exists y, P_1^2(y,x))$ $\exists x, \neg P_2^2(x,c) \wedge P_1^2(x,c)$

$\neg \forall x, P_2^2(c, f_1^1(x))$ $\neg \exists x \ P_2^2(x, f_1^1(x))$

El modelo sera:

la siguiente interpretación

Universo de numeros naturales $\mathbb{N}$

$\Psi(c) = 1$ $\Psi(x) = 0$ $\Psi(y) = 0$

$\Phi(f_1^1) =$ sucesor $\Pi(P_1^2) = \leq$ (menor igual) $\Pi(P_2^2) = =$ (igual)

de esta manera las formulas son satisfechas

6) Demuestra los siguientes teoremas de deducción natural

(a) $\vdash_N (\exists x, P_1^1(x) \vee P_2^1(x)) \iff (\exists x, P_1^1(x)) \vee (\exists x, P_2^1(x))$

1	$\exists x \; P_1^1(x) \vee P_2^1(x)$	hip
2	$P_1^1(n) \vee P_2^1(n)$ E∃n	hip
3	$P_1^1(n)$	hip
4	$\exists x \; P_1^1(x)$	I∃ 3
5	$\exists x \; P_1^1(x) \vee \exists x P_2^1(x)$	I∨ 4
6	$P_2^1(n)$	hip
7	$\exists x \; P_2^1(x)$	I∃ 6
8	$\exists x \; P_1^1(x) \vee \exists x P_2^1(x)$	I∨ 7
9	$\exists x P_1^1(x) \vee \exists x P_2^1(x)$	E∨ 2, 3-5, 6-8
10	$(\exists x \; P_1^1(x)) \vee (\exists x \; P_2^1(x))$	E∃ 1, 2-9
11	$(\exists x, P_1^1(x) \vee P_2^1(x)) \Rightarrow (\exists x, P_1^1(x)) \vee (\exists x, P_2^1(x))$	I⇒ 1-10
12	$\exists x, P_1^1(x) \vee \exists x, P_2^1(x)$	hip
13	$\exists x \; P_1^1(x)$	hip
14	$P_1^1(m)$ E∃m	hip
15	$P_1^1(m) \vee P_2^1(m)$	I∨ 14
16	$\exists x \, , P_1^1(x) \vee P_2^1(x)$	I∃ 15
17	$\exists x \, , P_1^1(x) \vee P_2^1(x)$	E∃ 13, 14-16
18	$\exists x \; P_2^1(x)$	hip
19	$P_2^1(k)$ E∃k	hip
20	$P_1^1(k) \vee P_2^1(k)$	I∨ 19

21	$\exists x, P^1_1(x) \vee P^1_2(x)$	$I\exists$ 20
22	$\exists x, P^1_1(x) \vee P^1_2(x)$	$E\exists$ 18, 19 – 21
23	$\exists x, P^1_1(x) \vee P^1_2(x)$	$E\vee$ 12, 13 – 17, 18 – 22
24	$(\exists x, P^1_1(x)) \vee (\exists x, P^1_2(x)) \Rightarrow (\exists x, P^1_1(x) \vee P^1_2(x))$	$I\Rightarrow$ 12 – 23
25	$(\exists x, P^1_1(x) \vee P^1_2(x)) \Leftrightarrow (\exists x, P^1_1(x)) \vee (\exists x, P^1_2(x))$	$I\Leftrightarrow$ 11, 24

(b) $\forall x, (\exists y, P^1_1(x) \Rightarrow P^1_2(y)) \vdash_N \neg(\exists x, (\forall y, P^1_1(x) \wedge \neg P^1_2(y)))$

1	$\forall x, (\exists y, P^1_1(x) \Rightarrow P^1_2(y))$	premisa
2	$\exists x, (\forall y, P^1_1(x) \wedge \neg P^1_2(y))$	hip
3	$\forall y, P^1_1(n) \wedge \neg P^1_2(y)$ $E\exists n$	hip
4	$\exists y, P^1_1(n) \Rightarrow P^1_2(y)$	$E\forall$ 1
5	$P^1_1(n) \Rightarrow P^1_2(m)$ $E\exists m$	hip
6	$P^1_1(n) \Rightarrow P^1_2(m)$	$E\exists$ 4, 5
7	$P^1_1(n) \wedge \neg P^1_2(m)$	$E\forall$ 3
8	$P^1_1(n)$	$E\wedge$ 7
9	$\neg P^1_2(m)$	$E\wedge$ 7
10	$P^1_2(m)$	$E\Rightarrow$ 6, 8
11	$\perp$	$I\perp$ 10, 9
12	$\perp$	$E\exists$ 2, 3-11
13	$\neg(\exists x, (\forall y, P^1_1(x) \wedge \neg P^1_2(y)))$	$I\neg$ 2-12