Tarea 4.

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1) Identifica el factor integrante adecuado que permita resolver las siguientes ecuaciones diferenciales,

a) $\underbrace{(x^3y^3+1)}_{M}dx + \underbrace{(x^4y^2)}_{N}dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^3y^3+1) = 3x^3y^2$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^4y^2) = 4x^3y^2$$

encontrar factor integrante

caso 1)

$$h(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{4x^3y^2 - 3x^3y^2}{x^3y^3+1} = \frac{x^3y^2}{x^3y^3+1}$$ no depende solo de y

$$g(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{3x^3y^2 - 4x^3y^2}{x^4y^2} = \frac{-x^3y^2}{x^4y^2} = -\frac{1}{x}$$ depende solo de x

entonces $\mu = e^{\int g(x)dx} = e^{\int -\frac{1}{x}dx} = e^{-\ln(x)} = x^{-1} = \frac{1}{x}$

entonces

$$\frac{1}{x}\left[(x^3y^3+1)dx + (x^4y^2)dy\right] = 0$$

$$\underbrace{\left(x^2y^3+\frac{1}{x}\right)}_{M}dx + \underbrace{(x^3y^2)}_{N}dy = 0$$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(x^2y^3+\frac{1}{x}\right) = 3x^2y^2$ $\quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^3y^2) = 3x^2y^2$ ya es exacta

$$\frac{du}{dx} = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \left(x^2y^3+\frac{1}{x}\right)dx + (x^3y^2)dy = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = x^2y^3 + \dfrac{1}{x} \quad \dots (1) \\ \dfrac{\partial u}{\partial y} = x^3y^2 \quad \dots (2) \end{cases}$$

tomamos a la ED(1) y dejamos y constante

$$\Rightarrow \frac{du}{dx} = x^2y^3 + \frac{1}{x} \Rightarrow du = (x^2y^3 + \tfrac{1}{x})dx$$ integramos

$$\int du = \int (x^2y^3 + \tfrac{1}{x})dx \Rightarrow u = \int x^2y^3 dx + \int \tfrac{1}{x} dx + \varphi(y)$$

$$\Rightarrow u = \frac{x^3y^3}{3} + \ln(x) + \varphi(x) \quad \dots (3)$$

derivamos respecto de y

$$\Rightarrow \frac{du}{dy} = \frac{d}{dy}\left(\frac{x^3y^3}{3} + \ln(x) + \varphi(y) \right) = x^3y^2 + \frac{d\varphi(y)}{dy}$$ pero $\frac{du}{dy} = x^3y^2$

$$\Rightarrow \cancel{x^3y^2} = \cancel{x^3y^2} + \frac{d\varphi(y)}{dy} \Rightarrow 0 = \frac{d\varphi(y)}{dy} \Rightarrow d\varphi(y) = 0\,dy$$

integramos $\int d\varphi(y) = \int 0\,dy \Rightarrow \varphi(y) = C$

sustituimos en la ED (3)

$$u = \frac{x^3y^3}{3} + \ln(x) + C = 0$$

$$\frac{x^3y^3}{3} + \ln(x) + c = 0$$

b) $\underbrace{(2t)}_{M}\, ds + \underbrace{s(2+s^2t)}_{N}\, dt$

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial t}(2t) = 2$$

$$\frac{\partial N}{\partial s} = \frac{\partial}{\partial s}(2s+s^3t) = 2+3s^2t$$

encontrar factor integrante

caso 1)

$$h(t) = \frac{1}{M}\left(\frac{\partial N}{\partial s} - \frac{\partial M}{\partial t}\right) = \frac{2+3s^2t-2}{2t} = \frac{3s^2}{2}$$ no depende solo de t

$$g(s) = \frac{1}{N}\left(\frac{\partial M}{\partial t} - \frac{\partial N}{\partial s}\right) = \frac{2-2-3s^2t}{2s+s^3t} = \frac{-3s^2t}{2s+s^3t}$$ no depende solo de t

caso 2)

$$\frac{\partial N}{\partial s} - \frac{\partial M}{\partial t} = n\frac{M}{t} - m\frac{N}{s}$$

$$2+3s^2t-2 = n\left(\frac{2t}{t}\right) - m\left(\frac{2s+s^3t}{s}\right)$$

$$3s^2t = n(2) - m(2+s^2t)$$

$$3s^2t = 2n - 2m - ms^2t$$ entonces $m=-3$, $n=-3$

por lo tanto el factor integrante es $s^{-3}t^{-3} = \frac{1}{s^3t^3} = \mu$

entonces

$$\frac{1}{s^3t^3}\left[2t\, ds + (2s+s^3t)\, dt\right] = 0$$

$$\underbrace{\left(\frac{2}{s^3t^2}\right)}_{M} ds + \underbrace{\left(\frac{2}{s^2t^3} + \frac{1}{t^2}\right)}_{N} dt = 0$$

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial t}\left(\frac{2}{s^3t^2}\right) = \frac{-4}{s^3t^3} \qquad \frac{\partial N}{\partial s} = \frac{\partial}{\partial s}\left(\frac{2}{s^2t^3}+\frac{1}{t^2}\right) = \frac{-4}{s^3t^3}$$

ya es exacta

$$\frac{du}{ds} = \frac{\partial u}{\partial s}ds + \frac{\partial u}{\partial t}dt = \left(\frac{2}{s^3t^2}\right)ds + \left(\frac{2}{s^2t^3}+\frac{1}{t^2}\right)dt = 0$$

$$\begin{cases} \frac{\partial u}{\partial s} = \frac{2}{s^3t^2} & \ldots(1) \\ \frac{\partial u}{\partial t} = \frac{2}{s^2t^3} + \frac{1}{t^2} & \ldots(2) \end{cases}$$

tomamos (1) y dejamos t constante

$$\Rightarrow \frac{du}{ds} = \frac{2}{s^3t^2} \Rightarrow du = \frac{2}{s^3t^2}ds$$

integramos

$$\int du = \int \frac{2}{s^3t^2}ds \Rightarrow u = \frac{2}{t^2}\int \frac{1}{s^3}ds + \varphi(t) \Rightarrow u = -\frac{1}{t^2s^2} + \varphi(t) \quad \ldots(3)$$

derivamos respecto de t

$$\Rightarrow \frac{du}{dt} = \frac{d}{dt}\left(-\frac{1}{t^2s^2} + \varphi(t)\right) = \frac{2}{s^2t^3} + \frac{d\varphi(t)}{dt}$$

pero $\frac{du}{dt} = \frac{2}{s^2t^3} + \frac{1}{t^2}$

$$\Rightarrow \cancel{\frac{2}{s^2t^3}} + \frac{1}{t^2} = \cancel{\frac{2}{s^2t^3}} + \frac{d\varphi(t)}{dt} \Rightarrow \frac{1}{t^2} = \frac{d\varphi(t)}{dt} \Rightarrow d\varphi(t) = \frac{1}{t^2}dt$$

integramos $\int d\varphi(t) = \int \frac{1}{t^2}dt \Rightarrow \varphi(t) = -\frac{1}{t} + c$

sustituimos en (3)

$$u = -\frac{1}{t^2s^2} - \frac{1}{t} + c = 0$$

$$-\frac{1}{t^2s^2} - \frac{1}{t} + c = 0$$

c) $\underbrace{y(x^4-y^2)}_{M}\,dx + \underbrace{x(x^4+y^2)}_{N}\,dy=0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(yx^4-y^3\right) = x^4-3y^2$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(x^5+y^2x\right) = 5x^4+y^2$$

encontrar factor integrante

caso 1)

$$h(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) = \frac{5x^4+y^2-x^4+3y^2}{yx^4-y^3} = \frac{4x^4+4y^2}{yx^2-y^3}$$ no depende solo de y

$$g(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right) = \frac{x^4-3y^2-5x^4-y^2}{x^5+y^2x} = \frac{-4x^4-4y^2}{x^3+y^2x}$$ no depende solo de y

caso 2)

$$\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y} = n\frac{M}{y} - m\frac{N}{x}$$

$$5x^4+y^2-x^4+3y^2 = n\left(\frac{yx^4-y^3}{y}\right) - m\left(\frac{x^5+y^2x}{x}\right)$$

$$4x^4+4y^2 = n(x^4-y^2) - m(x^4+y^2)$$

$$4x^4+4y^2 = nx^4-ny^2-mx^4-my^2$$ no se puede

caso 3)

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x} = N\,P(x) - M\,Q(y)$$

$$x^4-3y^2-5x^4-y^2 = (x^5+y^2x)P(x) - (yx^4-y^3)Q(y)$$

$$-4x^4-4y^2 = P(x)x^5 + y^2x\,P(x) - yx^4Q(y) + y^3Q(y)$$

propongamos $P(x) = \frac{-4}{x}$ $\quad Q(y) = 0$

sustituimos

$$-4x^4-4y^2 = x^5\left(\frac{-4}{x}\right) + y^2x\left(\frac{-4}{x}\right) - yx^4(0) + y^3(0)$$

$$-4x^4-4y^2 = -4x^4-4y^2$$

entonces el factor integrante es $\mu = e^{\int P(x)dx} e^{\int Q(y)dy}$

$$\mu = e^{\int \frac{-4}{x}dx} e^{\int 0 dy} = e^{-4\ln(x)} e^{0} = e^{-4\ln(x)} = \frac{1}{x^4}$$

entonces

$$\frac{1}{x^4}\left[(yx^4 - y^3)dx + (x^5 + y^2 x)dy\right] = 0$$

$$\underbrace{\left(y - \frac{y^3}{x^4}\right)}_{M} dx + \underbrace{\left(x + \frac{y^2}{x^3}\right)}_{N} dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(y - \frac{y^3}{x^4}\right) = 1 - \frac{3y^2}{x^4} \qquad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(x + \frac{y^2}{x^3}\right) = 1 - \frac{3y^2}{x^4}$$ ya es exacta

$$\frac{du}{dx} = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \left(y - \frac{y^3}{x^4}\right)dx + \left(x + \frac{y^2}{x^3}\right)dy = 0$$

$$\begin{cases} \frac{\partial u}{\partial x} = y - \frac{y^3}{x^4} & \dots (1) \\ \frac{\partial u}{\partial y} = x + \frac{y^2}{x^3} & \dots (2) \end{cases}$$

tomamos (1) y dejamos y constante $\frac{du}{dx} = y - \frac{y^3}{x^4} \Rightarrow du = \left(y - \frac{y^3}{x^4}\right)dx$ integramos

$$\int du = \int y - \frac{y^3}{x^4}\, dx \Rightarrow u = xy + \frac{y^3}{3x^3} + \varphi(y) \quad \dots (3)$$

derivamos respecto de y

$$\Rightarrow \frac{du}{dy} = \frac{d}{dy}\left(xy + \frac{y^3}{3x^3} + \varphi(y)\right) = x + \frac{y^2}{x^3} + \frac{d\varphi(y)}{dy}$$ pero $\frac{du}{dy} = x + \frac{y^2}{x^3}$

$$\Rightarrow x + \frac{y^2}{x^3} = x + \frac{y^2}{x^3} + \frac{d\varphi(y)}{dy} \Rightarrow 0 = \frac{d\varphi(y)}{dy} \Rightarrow d\varphi(y) = 0\,dy$$

integramos $\int d\varphi(y) = \int 0\,dy \Rightarrow \varphi(y) = C$ sustituimos en (3)

$$u = xy + \frac{y^3}{3x^3} + c = 0 \qquad xy + \frac{y^3}{3x^3} + c = 0$$

FIRST CLASS

2) Resuelve las siguientes ecuaciones diferenciales de Ricatti, utilizando la solución particular y_1 indicada

a) $\frac{dy}{dx} = -y^2 + xy + 1$ $\qquad y_1 = x$

$\frac{dy}{dx} + P(x)y + Q(x)y^2 = f(x)$

$Q(x) = +1$

$P(x) = -x$

$F(x) = 1$

proponemos $y = x + z$ $\qquad z = y - x$

derivamos con respecto a x

$\frac{dy}{dx} = \frac{dx}{dx} + \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = 1 + \frac{dz}{dx}$ sustituimos en la FD

$\frac{dy}{dx} = -y^2 + xy + 1$

$\Rightarrow 1 + \frac{dz}{dx} = -(x+z)^2 + x(x+z) + 1$

$\Rightarrow \frac{dz}{dx} = -(x^2 + 2xz + z^2) + x^2 + xz = -x^2 - 2xz - z^2 + x^2 + xz$

$\Rightarrow \frac{dz}{dx} = \underbrace{-xz}_{P(x)} \underbrace{- z^2}_{Q(x)} \Rightarrow \frac{dz}{dx} + xz = -z^2$ Bernoulli

cajita herramientas

$w = \frac{z}{z^n} = z^{1-n}$

$\frac{dw}{dx} = (1-n) z^{1-n-1} \frac{dz}{dx}$

$\Rightarrow \frac{dz}{dx} = \frac{1}{1-n} z^n \frac{dw}{dx}$

$w = \frac{z}{z^2} = \frac{1}{z} = z^{-1}$ $\qquad \frac{dz}{dx} = -z^2 \frac{dw}{dx}$

$\frac{dz}{dx} + xz = -z^2 \Rightarrow \frac{dz}{dx}\frac{1}{z^2} + xz\frac{1}{z^2} = -z^2\frac{1}{z^2}$

$\Rightarrow \frac{dz}{dx}\frac{1}{z^2} + x\frac{1}{z} = -1$

sustitumos

FD lineal

$\frac{1}{z^2}\left[-z^2 \frac{dw}{dx}\right] + xw = -1 \Rightarrow -\frac{dw}{dx} + xw = -1 \Rightarrow \frac{dw}{dx} - xw = 1$

sustituimos

cajita deherramientas

$w = ab$

$\frac{dw}{dx} = a\frac{db}{dx} + b\frac{da}{dx}$

$a\frac{db}{dx} + b\frac{da}{dx} - x\,ab = 1$

factoriza a

$$a\Big[\underbrace{\frac{db}{dx} - xb}_{0}\Big] + b\frac{da}{dx} = 1$$

$$\frac{db}{dx} - xb = 0 \Rightarrow \frac{db}{dx} = xb \Rightarrow \frac{db}{b} = x\,dx$$ integramos

$$\int \frac{db}{b} = \int x\,dx$$ (no consideramos c) $$\Rightarrow \ln(b) = \frac{x^2}{2} \Rightarrow e^{\ln(b)} = e^{\frac{x^2}{2}}$$

$$\Rightarrow b = e^{x^2/2}$$

entonces $$a\Big[\cancelto{0}{\frac{db}{dx} + xb}\Big] + b\frac{da}{dx} = 1 \Rightarrow b\frac{da}{dx} = 1 \Rightarrow e^{x^2/2}\frac{da}{dx} = 1$$

$$\Rightarrow da = e^{-x^2/2}\,dx$$ integramos

$$\int da = \int e^{-x^2/2}\,dx + c$$

$$a = \int e^{-x^2/2}\,dx + c$$ es una integral hiperbolica (??)

entonces $w = ab$ $$\left(e^{x^2/2}\right)\left(\int e^{-x^2/2}\,dx + c\right) = w$$

pero $w = \frac{1}{z}$ $$\frac{1}{z} = e^{x^2/2}\int e^{-x^2/2}\,dx + ce^{x^2/2}$$

pero $z = y - x$ entonces

$$\frac{1}{y-x} = e^{x^2/2}\int e^{-x^2/2}\,dx + ce^{x^2/2}$$

b) $\frac{dy}{dx} = e^{2x} + (1-2e^x)y + y^2 \qquad y_1 = e^x$

$$\frac{dy}{dx} - (1-2e^x)y - y^2 = e^{2x}$$

proponemos $y = e^x + z \qquad z = y - e^x$

derivamos con respecto a x

$\frac{dy}{dx} = \frac{d}{dx}e^x + \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = e^x + \frac{dz}{dx}$ sustituimos en la ED

$$\frac{dy}{dx} - (1-2e^x)y - y^2 = e^{2x} \Rightarrow e^x + \frac{dz}{dx} - (1-2e^x)(e^x+z) - (e^x+z)^2 = e^{2x}$$

$$\Rightarrow \cancel{e^x} + \frac{dz}{dx} - z - \cancel{2ze^x} - \cancel{e^x} + 2e^{2x} - e^{2x} - 2ze^x - z^2 = e^{2x}$$

$\Rightarrow \frac{dz}{dx} - z + e^{2x} - z^2 = e^{2x} \Rightarrow \frac{dz}{dx} = \underbrace{z^2}_{Q(x)} + \underbrace{z}_{P(x)}$ Bernoulli

cajita de herramientas

$w = z^{1-n}$

$\frac{dz}{dx} = \frac{1}{1-n} z^n \frac{dw}{dx}$

$w = \frac{1}{z} \qquad \frac{dz}{dx} = -z^2 \frac{dw}{dx}$

$$\frac{dz}{dx} - z = z^2 \Rightarrow \frac{dz}{dx}\frac{1}{z^2} - z\frac{1}{z^2} = z^2\frac{1}{z^2} \Rightarrow \frac{dz}{dx}\frac{1}{z^2} - \frac{1}{z} = 1$$

sustituimos

$$\frac{1}{z^2}\left[-z^2\frac{dw}{dx}\right] - w = 1 \Rightarrow -\frac{dw}{dx} - w = 1 \Rightarrow \frac{dw}{dx} + w = -1$$

ED lineal

cajita de herramientas

$w = ab$

$\frac{dw}{dx} = a\frac{db}{dx} + b\frac{da}{dx}$

sustituimos

$$a\frac{db}{dx} + b\frac{da}{dx} + ab = -1$$

factorizo a

$$\underbrace{a\left[\frac{db}{dx} + b\right]}_{0} + b\frac{da}{dx} = -1$$

$\frac{db}{dx} + b = 0 \Rightarrow \frac{db}{dx} = -b \Rightarrow \frac{db}{b} = -dx$ integramos

$\int \frac{db}{b} = -\int dx$ (no consideramos a c) $\Rightarrow \ln(b) = -x \Rightarrow e^{\ln(b)} = e^{-x}$

$\Rightarrow b = \frac{1}{e^x}$

entonces $a\left[\cancel{\frac{db}{dx} + b}\right] + b\frac{da}{dx} = -1 \Rightarrow b\frac{da}{dx} = -1 \Rightarrow \frac{1}{e^x}\frac{da}{dx} = -1$

$\Rightarrow da = -e^x dx$ integramos

$\int da = \int -e^x dx \Rightarrow a = -\int e^x dx + c \Rightarrow a = -e^x + c$

entonces

$w = ab \qquad w = (-e^x + c)\left(\frac{1}{e^x}\right) = \frac{-e^x + c}{e^x}$

pero $w = \frac{1}{z} \qquad \frac{1}{z} = \frac{-e^x + c}{e^x}$

pero $z = y - e^x \qquad \frac{1}{y - e^x} = \frac{-e^x + c}{e^x}$

$\Rightarrow y - e^x = \frac{e^x}{-e^x + c} \Rightarrow y = \frac{e^x}{-e^x + c} + e^x$

c) $\frac{dy}{dx} = -\frac{4}{x^2} - \frac{y}{x} + y^2$ $\qquad y_1 = \frac{2}{x}$

$$\frac{dy}{dx} + \frac{1}{x}y - y^2 = -\frac{4}{x^2}$$

proponemos $y = \frac{2}{x} + z$ $\qquad z = y - \frac{2}{x}$

derivamos con respecto a x

$\frac{dy}{dx} = \frac{d}{dx}\frac{2}{x} + \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = -\frac{2}{x^2} + \frac{dz}{dx}$ sustituimos en la EU

$$\frac{dy}{dx} + \frac{1}{x}y - y^2 = \frac{-4}{x^2} \Rightarrow -\frac{2}{x^2} + \frac{dz}{dx} + \frac{1}{x}\left(\frac{2}{x}+z\right) - \left(\frac{2}{x}+z\right)^2 = \frac{-4}{x^2}$$

$$\Rightarrow \cancel{\frac{-2}{x^2}} + \frac{dz}{dx} + \cancel{\frac{2}{x^2}} + \frac{z}{x} - \cancel{\frac{4}{x^2}} - \frac{4z}{x} - z^2 = \cancel{\frac{-4}{x^2}}$$

$\Rightarrow \frac{dz}{dx} + \frac{3}{x}z - z^2 = 0 \Rightarrow \frac{dz}{dx} = \underbrace{\frac{3}{x}}_{P(x)}z + \underbrace{z^2}_{Q(x)}$ Bernoulli

cajita de herramientas

$w = z^{1-n}$

$\frac{dz}{dx} = \frac{1}{1-n}z^n\frac{dw}{dx}$

$w = \frac{1}{z}$ $\qquad \frac{dz}{dx} = -z^2\frac{dw}{dx}$

$$\frac{dz}{dx} - \frac{3}{x}z = z^2 \Rightarrow \frac{dz}{dx}\frac{1}{z^2} - \frac{3}{x}z\frac{1}{z^2} = z^2\frac{1}{z^2} \Rightarrow \frac{dz}{dx}\frac{1}{z^2} - \frac{3}{x}\frac{1}{z} = 1$$

sustituimos

$\frac{1}{z^2}\left[-z^2\frac{dw}{dx}\right] - \frac{3}{x}w = 1 \Rightarrow -\frac{dw}{dx} - \frac{3}{x}w = 1 \Rightarrow \frac{dw}{dx} + \frac{3}{x}w = -1$ ED lineal

cajita de herramientas

$w = ab$

$\frac{dw}{dx} = a\frac{db}{dx} + b\frac{da}{dx}$

sustituimos

$$a\frac{db}{dx} + b\frac{da}{dx} + \frac{3}{x}ab = -1$$

factorizar a

$$a\underbrace{\left[\frac{db}{dx} + \frac{3}{x}b\right]}_{0} + b\frac{da}{dx} = -1$$

$$\frac{db}{dx} + \frac{3}{x}b = 0 \Rightarrow \frac{db}{dx} = -\frac{3}{x}b \Rightarrow \frac{db}{b} = -\frac{3}{x}dx$$ integramos

$$\int \frac{db}{b} = -\int \frac{3}{x}dx$$ (no consideramos a c) $$\Rightarrow \ln(b) = -3\ln(x) \Rightarrow e^{\ln(b)} = e^{-3\ln(x)}$$

$$\Rightarrow e^{\ln(b)} = e^{\ln(x)^{-3}} \Rightarrow b = \frac{1}{x^3}$$

entonces $$a\left[\cancel{\frac{db}{dx} + \frac{3}{x}b}\right] + b\frac{da}{dx} = -1 \Rightarrow b\frac{da}{dx} = -1 \Rightarrow \frac{1}{x^3}\frac{da}{dx} = -1$$

$$\Rightarrow da = -x^3 dx$$ integramos

$$\int da = \int -x^3 dx \Rightarrow a = -\int x^3 dx \Rightarrow a = -\frac{x^4}{4} + c$$

entonces

$w = ab$ $$w = \left(\frac{1}{x^3}\right)\left(-\frac{x^4}{4} + c\right) = \frac{-x^4 + 4c}{4x^3}$$

pero $w = \frac{1}{z}$ $$\frac{1}{z} = \frac{-x^4 + 4c}{4x^3}$$

pero $z = y - \frac{2}{x}$ $$\frac{1}{y - \frac{2}{x}} = \frac{-x^4 + 4c}{4x^3}$$

$$\Rightarrow y - \frac{2}{x} = \frac{4x^3}{-x^4 + 4c} \Rightarrow y = \frac{4x^3}{-x^4 + 4c} + \frac{2}{x}$$

d) Muestra que la ecuacion diferencial de Riccati $\left(\frac{dy}{dx} = P(x) + Q(x)y + R(x)y^2\right)$ se reduce a una ecuacion diferencial de Bernoulli $\left(\frac{dy}{dx} + P(x)y = f(x)y^n\right)$ con $n=2$ mediante la sustitucion $y = y_1 + u$

tenemos que de Riccati

$$\frac{dy}{dx} = P(x) + Q(x)y + R(x)y^2 \Rightarrow \frac{dy}{dx} - Q(x)y - R(x)y^2 = P(x)$$

proponemos sustitucion

$$y = y_1 + u$$ donde y_1 es una solucion particular.

derivamos $\frac{dy}{dx} = \frac{dy_1}{dx} + \frac{du}{dx}$ y sustitumos en la ED

$$\Rightarrow \frac{dy_1}{dx} + \frac{du}{dx} - Q(x)\left[y_1 + u\right] - R(x)\left[y_1 + u\right]^2 = P(x)$$

$$\Rightarrow \frac{dy_1}{dx} + \frac{dz}{dx} - Q(x)y_1 - Q(x)u + R(x)y_1^2 - R(x)u^2 - 2R(x)y_1 u = P(x)$$

pero y_1 es solucion $\left[\frac{dy_1}{dx} - Q(x)y_1 - R(x)y_1^2 = P(x)\right]$

entonces $\frac{dz}{dx} - Q(x)u - R(x)u^2 - 2R(x)y_1 u = 0$

$$\Rightarrow \frac{dz}{dx} + \left[-Q(x) - 2R(x)y_1\right]u = R(x)u^2$$ con $n=2$

una ED de Bernoulli

e) $y' = (y-x)^2 + 1$ $\qquad y_1(x) = x \qquad y(0) = \frac{1}{2}$

$$\frac{dy}{dx} = y^2 - 2xy + x^2 + 1$$

$$\frac{dy}{dx} + 2xy - y^2 = x^2 + 1$$

proponemos $\quad y = x + z \qquad z = y - x$

derivamos con respecto a x

$$\frac{dy}{dx} = \frac{dx}{dx} + \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = 1 + \frac{dz}{dx}$$ sustituimos en la ED

$$\frac{dy}{dx} + 2xy - y^2 = x^2 + 1 \Rightarrow 1 + \frac{dz}{dx} + 2x(x+z) - (x+z)^2 = x^2 + 1$$

$$\Rightarrow \cancel{1} + \frac{dz}{dx} \cancel{+2x^2} \cancel{+2xz} \cancel{-x^2} - 2xz - z^2 = \cancel{x^2+1}$$

$$\Rightarrow \frac{dz}{dx} - z^2 = 0 \qquad \text{Bernoulli}$$

cajita de herramientas

$w = z^{1-n}$

$\frac{dz}{dx} = \frac{1}{1-n} z^n \frac{dw}{dx}$

$$w = \frac{1}{z} \qquad \frac{dz}{dx} = -z^2 \frac{dw}{dx}$$

$$\frac{dz}{dx} = z^2 \Rightarrow \frac{dz}{dx}\frac{1}{z^2} = z^2 \frac{1}{z^2} \Rightarrow \frac{dz}{dx}\frac{1}{z^2} = 1$$

sustituimos

$$\frac{1}{z^2}\left[-z^2 \frac{dw}{dx}\right] = 1 \Rightarrow -\frac{dw}{dx} = 1 \Rightarrow \frac{dw}{dx} = -1$$ var separadas

$\Rightarrow dw = -dx$. integremos $\int dw = -\int dx \Rightarrow w = -x + c$

pero $w = \frac{1}{z}$ $\qquad \frac{1}{z} = -x + c \Rightarrow z = \frac{1}{-x+c}$, pero $z = y - x$

$\Rightarrow y = \frac{1}{-x+c} + x$, pero $y(x=0) = \frac{1}{2} \Rightarrow y(0) = \frac{1}{-0+c} + 0 = \frac{1}{2}$

entonces $c = 2$ por lo tanto $y = \frac{1}{-x+2} + x$