Tarea 2

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1) Determina si las siguentes ecuaciones diferenciales son exactas y resuelvelas:

1.1) $x(2x^2+y^2)+y(x^2+2y^2)y'=0$

$\Rightarrow 2x^3+y^2x+(yx^2+2y^3)\frac{dy}{dx}=0 \quad \Rightarrow (yx^2+2y^3)\,dy=-(2x^3+y^2x)\,dx$

$\Rightarrow \underbrace{(2x^3+y^2x)}_{M}dx+\underbrace{(yx^2+2y^3)}_{N}dy=0$

$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(2x^3+y^2x)=2yx$

$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(yx^2+2y^3)=2yx$

son iguales entonces si es exacta

$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy=(2x^3+y^2x)dx+(yx^2+2y^3)dy=0$

$$\begin{cases}\frac{\partial u}{\partial x}=2x^3+y^2x \quad \dots(1)\\ \frac{\partial u}{\partial y}=yx^2+2y^3 \quad \dots(2)\end{cases}$$

tomamos la EC (1) $\frac{\partial u}{\partial x}=2x^3+xy^2$ dejemos y constante

$\Rightarrow \frac{du}{dx}=2x^3+xy^2 \Rightarrow du=(2x^3+xy^2)dx$ integramos

$\int du=\int(2x^3+xy^2)dx \Rightarrow u=\frac{x^4}{2}+\frac{x^2y^2}{2}+\varphi(y)=\frac{x^4+x^2y^2}{2}+\varphi(y)\dots(3)$

derivamos respecto de y

$\Rightarrow \frac{du}{dy}=\frac{d}{dy}\left(\frac{x^4+x^2y^2}{2}+\varphi(y)\right)=x^2y+\frac{d\varphi(y)}{dy}$ pero $\frac{du}{dy}=x^2y+2y^3$

$\Rightarrow \cancel{x^2y}+2y^3=\cancel{x^2y}+\frac{d\varphi(y)}{dy} \Rightarrow 2y^3=\frac{d\varphi(y)}{dy} \Rightarrow d\varphi(y)=2y^3dy$

integramos $\int d\varphi(y)=\int 2y^3dy \Rightarrow \varphi(y)=\frac{y^4}{2}+c$

sustituimos en la ED . (3)

$$u = \frac{x^4 + x^2y^2}{2} + \frac{y^4}{2} + C = 0$$

$$\Rightarrow \frac{x^4 + x^2y^2 + y^2}{2} + c = 0$$

1.2) $\underbrace{(3x^2+6xy^2)}_{M}\,dx + \underbrace{(6x^2y+4y^3)}_{N}\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(3x^2+6xy^2) = 12xy$$

son iguales si es exacta

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(6x^2y+4y^3) = 12xy$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = (3x^2+6xy^2)dx + (6x^2y+4y^3)dy = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = 3x^2+6xy^2 & \ldots(1) \\ \dfrac{\partial u}{\partial y} = 6x^2y+4y^3 & \ldots(2) \end{cases}$$

tomamos la EC (1) $\dfrac{\partial u}{\partial x} = 3x^2+6xy^2$ dejamos y constante

$\Rightarrow \dfrac{du}{dx} = 3x^2+6xy^2 \Rightarrow du = (3x^2+6xy^2)\,dx.$ integramos

$\int du = \int (3x^2+6xy^2)\,dx \Rightarrow u = x^3+3x^2y^2+\varphi(y) \quad \ldots(3)$

derivamos respecto de y

$\Rightarrow \dfrac{du}{dy} = \dfrac{d}{dy}(x^3+3x^2y^2+\varphi(y)) = 6x^2y + \dfrac{d}{dy}\varphi(y)$ pero $\dfrac{du}{dy} = 6x^2y+4y^3$

$\Rightarrow \cancel{6x^2y}+4y^3 = \cancel{6x^2y}+\dfrac{d}{dy}\varphi(y) \Rightarrow 4y^3 = \dfrac{d\varphi(y)}{dy} \Rightarrow d\varphi(y) = 4y^3dy$

integramos $\int d\varphi(y) = \int 4y^3dy \Rightarrow \varphi(y) = y^4+c$

sustituimos en la EO (3)

$$u = x^3+3x^2y^2+y^4+c = 0$$

$$\Rightarrow x^3+3x^2y^2+y^4+c = 0$$

1.3) $$\underbrace{\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)}_{M}dx+\underbrace{\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)}_{N}dy=0$$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)=-\frac{xy}{(x^2+y^2)^{\frac{3}{2}}}-\frac{1}{y^2}$$

son iguales entonces si es exacta

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)=-\frac{xy}{(x^2+y^2)^{\frac{3}{2}}}-\frac{1}{y^2}$$

$$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy=\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)dy=0$$

$$\begin{cases}\dfrac{\partial u}{\partial x}=\dfrac{x}{\sqrt{x^2+y^2}}+\dfrac{1}{x}+\dfrac{1}{y} & \dots(1)\\[2ex] \dfrac{\partial u}{\partial y}=\dfrac{y}{\sqrt{x^2+y^2}}+\dfrac{1}{y}-\dfrac{x}{y^2} & \dots(2)\end{cases}$$

tomamos la ED (1) y dejamos a y constante

$$\Rightarrow \frac{du}{dx}=\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\Rightarrow du=\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dx$$ integramos

$$\int du=\int\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dx\Rightarrow u=\int\frac{x}{\sqrt{x^2+y^2}}dx+\int\frac{1}{x}dx+\int\frac{1}{y}dx+\varphi(y)$$

$$\Rightarrow u=\int\frac{x}{\sqrt{x^2+y^2}}dx+\ln(x)+\frac{x}{y}+\varphi(x)$$ sustitucion $w=\sqrt{x^2+y^2}$, $dw=\frac{x}{\sqrt{x^2+y^2}}dx$

$$\Rightarrow u=\int dw+\ln(x)+\frac{x}{y}+\varphi(x)=\sqrt{x^2+y^2}+\ln(x)+\frac{x}{y}+\varphi(x)\quad\dots(3)$$

derivamos respecto de y

$$\Rightarrow\frac{du}{dy}=\frac{d}{dy}\left(\sqrt{x^2+y^2}+\ln(x)+\frac{x}{y}+\varphi(x)\right)=\frac{y}{\sqrt{x^2+y^2}}-\frac{x}{y^2}+\frac{d\varphi(x)}{dy}$$ pero $\frac{du}{dy}=\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}$

$$\Rightarrow\cancel{\frac{y}{\sqrt{x^2+y^2}}}+\frac{1}{y}-\cancel{\frac{x}{y^2}}=\cancel{\frac{y}{\sqrt{x^2+y^2}}}-\cancel{\frac{x}{y^2}}+\frac{d\varphi(x)}{dy}\Rightarrow\frac{1}{y}=\frac{d\varphi(x)}{dy}\Rightarrow d\varphi(x)=\frac{dy}{y}$$

integramos

$$\int d\varphi(y) = \int \frac{1}{y}dy \Rightarrow \varphi(y) = \ln(y) + c$$

sustituimos en la ED .. (3)

$$u = \sqrt{x^2+y^2} + \ln(x) + \ln(y) + c = 0$$

$$\Rightarrow \sqrt{x^2+y^2} + \ln(x) + \ln(y) + c = 0$$

FIRST CLASS

1.4) $\underbrace{\left(3x^2\tan(y)-\frac{2y^3}{x^3}\right)}_{M}dx+\underbrace{\left(x^3\sec^2(y)+4y^3+\frac{3y^2}{x^2}\right)}_{N}dy=0$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}\left(3x^2\tan(y)-\frac{2y^3}{x^3}\right)=3x^2\sec^2(y)-\frac{6y^2}{x^3}$$

son iguales entonces si es exacta

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}\left(x^3\sec^2(y)+4y^3+\frac{3y^2}{x^2}\right)=3x^2\sec^2(y)-\frac{6y^2}{x^3}$$

$$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy=\left(3x^2\tan(y)-\frac{2y^3}{x^3}\right)dx+\left(x^3\sec^2(y)+4y^3+\frac{3y^2}{x^2}\right)dy=0$$

$$\begin{cases}\dfrac{\partial u}{\partial x}=3x^2\tan(y)-\dfrac{2y^3}{x^3} & \dots(1)\\[2mm] \dfrac{\partial u}{\partial y}=x^3\sec^2(y)+4y^3+\dfrac{3y^2}{x^2} & \dots(2)\end{cases}$$

tomamos la ED (1) y dejamos a y constante

$$\Rightarrow \frac{du}{dx}=3x^2\tan(y)-\frac{2y^3}{x^3}\Rightarrow du=\left(3x^2\tan(y)-\frac{2y^3}{x^3}\right)dx$$ integramos

$$\int du=\int 3x^2\tan(y)-\frac{2y^3}{x^3}\,dx\Rightarrow u=\int 3x^2\tan(y)\,dx-\int\frac{2y^3}{x^3}dx+\varphi(y)$$

$$\Rightarrow u=x^3\tan(y)+\frac{y^3}{x^2}+\varphi(x)\quad\dots(3)$$

derivamos respecto de y

$$\Rightarrow \frac{du}{dy}=\frac{d}{dy}\left(x^3\tan(y)+\frac{y^3}{x^2}+\varphi(x)\right)=x^3\sec^2(y)+\frac{3y^2}{x^2}+\frac{d\varphi(y)}{dy}$$

pero $\frac{du}{dy}=x^3\sec^2(y)+4y^3+\frac{3y^2}{x^2}$

$$\Rightarrow \cancel{x^3\sec^2(y)}+4y^3+\cancel{\frac{3y^2}{x^2}}=\cancel{x^3\sec^2(y)}+\cancel{\frac{3y^2}{x^2}}+\frac{d\varphi(y)}{dy}\Rightarrow 4y^3=\frac{d\varphi(y)}{dy}$$

$$\Rightarrow d\varphi(y)=4y^3dy$$ integramos $\int d\varphi(y)=\int 4y^3dy$

$$\Rightarrow \varphi(y)=y^4+c$$ sustituimos en la ED (3)

$$u=x^3\tan(y)+\frac{y^3}{x^2}+y^4+c=0\Rightarrow x^3\tan(y)+\frac{y^3}{x^2}+y^4+c=0$$

2) Identifica el factor integrante adecuado que permita resolver las siguientes ecuaciones diferenciales:

2.1) $\underbrace{(x^4\ln(x) - 2xy^3)}_{M}\,dx + \underbrace{(3x^2y^2)}_{N}\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^4\ln(x) - 2xy^3) = -6xy^2$$

no es ED exacta

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(3x^2y^2) = 6xy^2$$

veamos con

$$g(x) = \frac{1}{N}\left(\frac{\partial}{\partial y}M - \frac{\partial}{\partial x}N\right)$$

$$g(x) = \frac{1}{3x^2y^2}(-6xy^2 - 6xy^2) = \frac{-12xy^2}{3x^2y^2} = \frac{-12x}{3x^2} = \frac{-4}{x}$$

entonces $\mu = e^{\int g(x)dx} = e^{\int -\frac{4}{x}dx} = e^{-4\int \frac{1}{x}dx} = e^{-4\ln(x)} = e^{\ln(x)^{-4}} = \frac{1}{x^4}$

multiplicar por μ

$$\frac{1}{x^4}\left[(x^4\ln(x) - 2xy^3)dx + (3x^2y^2)dy\right] = 0$$

$$\Rightarrow \underbrace{\left(\ln(x) - \frac{2y^3}{x^3}\right)}_{M}dx + \underbrace{\left(\frac{3y^2}{x^2}\right)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y}\left(\ln(x) - \frac{2y^3}{x^3}\right) = -\frac{6y^2}{x^3}$$

ya es ED exacta

$$\frac{\partial N}{\partial x}\left(\frac{3y^2}{x^2}\right) = -\frac{6y^2}{x^3}$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \left(\ln(x) - \frac{2y^3}{x^3}\right)dx + \left(\frac{3y^2}{x^2}\right)dy = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \ln(x) - \dfrac{2y^3}{x^3} \quad \ldots (1) \\ \dfrac{\partial u}{\partial y} = \dfrac{3y^2}{x^2} \quad \ldots (2) \end{cases}$$

tomamos a la ED (1) y dejamos a y constante

$$\Rightarrow \frac{du}{dx} = \ln(x) - \frac{2y^3}{x^3} \Rightarrow du = \left(\ln(x) - \frac{2y^3}{x^3}\right) dx$$

integramos

$u = \ln(x)$ $du = \frac{1}{x} dx$

$dv = dx$ $v = x$

$$\int du = \int \ln(x) - \frac{2y^3}{x^3}\, dx \Rightarrow u = \int \ln(x)\,dx - \int \frac{2y^3}{x^3}\, dx + \varphi(y)$$

$$\Rightarrow u = x\ln(x) - \int x \frac{1}{x} dx - 2y^3 \int \frac{1}{x^3} dx + \varphi(y) = x\ln(x) - x + \frac{y^3}{x^2} + \varphi(y) \ldots (3)$$

derivamos respecto de y

$$\Rightarrow \frac{du}{dy} = \frac{d}{dy}\left(x\ln(x) - x + \frac{y^3}{x^2} + \varphi(y)\right) = \frac{3y^2}{x^2} + \frac{d\varphi(y)}{dy}$$

pero $\dfrac{du}{dy} = \dfrac{3y^2}{x^2}$

$$\Rightarrow \cancel{\frac{3y^2}{x^2}} = \cancel{\frac{3y^2}{x^2}} + \frac{d\varphi(y)}{dy} \Rightarrow 0 = \frac{d\varphi(y)}{dy} \Rightarrow d\varphi(y) = 0\,dy$$

integramos $\int d\varphi(y) = \int 0\,dy \Rightarrow \varphi(y) = C$

sustituimos en la ED (3)

$$u = x\ln(x) - x + \frac{y^3}{x^2} + c = 0$$

$$x\ln(x) - x + \frac{y^3}{x^2} + c = 0$$

2.2) $\underbrace{(2xy^2 - 3y^3)}_{M} dx + \underbrace{(7 - 3xy^2)}_{N} dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy^2 - 3y^3) = 4xy - 9y^2$$

no es ED exacta

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(7 - 3xy^2) = -3y^2$$

veamos con

$$g(x) = \frac{1}{N}\left(\frac{\partial}{\partial y}M - \frac{\partial}{\partial x}N\right) = \frac{1}{7-3xy^2}(4xy - 9y^2 + 3y^2) = \frac{4xy - 6y^2}{7 - 3xy^2}$$ no depende solo de x

veamos con

$$h(y) = \frac{1}{M}\left(\frac{\partial}{\partial x}N - \frac{\partial}{\partial y}M\right) = \frac{1}{2xy^2 - 3y^3}(-3y^2 - 4xy + 9y^2) = \frac{-4xy + 6y^2}{2xy^2 - 3y^3} = \frac{-2y(2x - 3y)}{y^2(2x - 3y)}$$

$$= \frac{-2y}{y^2} = \frac{-2}{y}$$

entonces $\mu = e^{\int h(y)dy} = e^{\int -\frac{2}{y}dy} = e^{-2\int \frac{1}{y}dy} = e^{-2\ln(y)} = e^{\ln(y)^{-2}} = \frac{1}{y^2}$

multiplico por μ

$$\frac{1}{y^2}\left[(2xy^2 - 3y^3)dx + (7 - 3xy^2)dy\right] = 0$$

$$\Rightarrow \underbrace{(2x - 3y)}_{M}dx + \underbrace{\left(\frac{7}{y^2} - 3x\right)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y}(2x - 3y) = -3$$

ya es ED exacta

$$\frac{\partial N}{\partial x}\left(\frac{7}{y^2} - 3x\right) = -3$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = (2x - 3y)\,dx + \left(\frac{7}{y^2} - 3x\right)dy = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = 2x - 3y \quad \dots (1) \\ \dfrac{\partial u}{\partial y} = \dfrac{7}{y^2} - 3x \quad \dots (2) \end{cases}$$

tomamos a la ED (1) y dejamos a y constante

$$\Rightarrow \frac{du}{dx} = 2x - 3y \Rightarrow du = (2x - 3y)dx \qquad \text{integramos}$$

$$\int du = \int 2x - 3y \, dx \Rightarrow u = \int 2x dx - \int 3y dx + \varphi(y)$$

$$\Rightarrow u = x^2 - 3yx + \varphi(y) \quad \dots (3)$$

derivamos respecto de y

$$\Rightarrow \frac{du}{dy} = \frac{d}{dy}\left(x^2 - 3yx + \varphi(y)\right) = -3x + \frac{d\varphi(y)}{dy} \qquad \text{pero} \quad \frac{du}{dy} = \frac{7}{y^2} - 3x$$

$$\Rightarrow \frac{7}{y^2} - \cancel{3x} = -\cancel{3x} + \frac{d\varphi(y)}{dy} \Rightarrow \frac{7}{y^2} = \frac{d\varphi(y)}{dy} \Rightarrow d\varphi(y) = \frac{7}{y^2} dy$$

integramos $\quad \int d\varphi(y) = \int \frac{7}{y^2} dy \Rightarrow \varphi(y) = 7 \int \frac{1}{y^2} dy + C$

$$\Rightarrow \varphi(y) = 7\left(-\frac{1}{y}\right) + C \Rightarrow \varphi(y) = -\frac{7}{y} + C$$

sustituimos en la ED (3)

$$u = x^2 - 3yx - \frac{7}{y} + C = 0$$

$$x^2 - 3yx - \frac{7}{y} + C = 0$$

FIRST CLASS

2.3) $\underbrace{(4x^2+3\cos(y))}_{M}dx - \underbrace{x\operatorname{sen}(y)}_{N}dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(4x^2+3\cos(y)) = -3\operatorname{sen}(y)$$ no es ED exacta

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-x\operatorname{sen}(y)) = -\operatorname{sen}(y)$$

veamos con

$$g(x) = \frac{1}{N}\left(\frac{\partial}{\partial y}M - \frac{\partial}{\partial x}N\right) = \frac{1}{-x\operatorname{sen}(y)}(-3\operatorname{sen}(y)+\operatorname{sen}(y)) = \frac{-2\operatorname{sen}(y)}{-x\operatorname{sen}(y)} = \frac{2}{x}$$

entonces $\mu = e^{\int g(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\int \frac{1}{x}dx} = e^{2\ln(x)} = e^{\ln(x)^2} = x^2$

multiplica por μ

$$x^2\left[(4x^2+3\cos(y))dx - x\operatorname{sen}(y)dy\right] = 0$$

$$\Rightarrow \underbrace{(4x^4+3x^2\cos(y))}_{M}dx - \underbrace{x^3\operatorname{sen}(y)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y}(4x^4+3x^2\cos(y)) = -3x^2\operatorname{sen}(y)$$ ya es ED exacta

$$\frac{\partial N}{\partial x}(-x^3\operatorname{sen}(y)) = -3x^2\operatorname{sen}(y)$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = (4x^4+3x^2\cos(y))dx + (-x^3\operatorname{sen}(y))dy = 0$$

$$\begin{cases} \frac{\partial u}{\partial x} = 4x^4+3x^2\cos(y) & \ldots (1) \\ \frac{\partial u}{\partial y} = -x^3\operatorname{sen}(y) & \ldots (2) \end{cases}$$

tomamos a la ED (1) y dejamos a y constante

$$\Rightarrow \frac{du}{dx} = 4x^4+3x^2\cos(y) \Rightarrow du = (4x^4+3x^2\cos(y))dx$$ integramos

$\int du = \int 4x^4 + 3x^2 \cos(y)\, dx \Rightarrow u = \int 4x^4\, dx + \int 3x^2 \cos(y)\, dx + \varphi(y)$

$\Rightarrow u = \frac{4x^5}{5} + x^3 \cos(y) + \varphi(y) \quad \ldots (3)$

derivams respecto de y

$\Rightarrow \frac{du}{dy} = \frac{d}{dy}\left(\frac{4x^5}{5} + x^3\cos(y) + \varphi(y)\right) = -x^3 \operatorname{sen}(y) + \frac{d\varphi(y)}{dy}$ pero $\frac{du}{dy} = -x^3 \operatorname{sen}(y)$

$\Rightarrow \cancel{-x^3 \operatorname{sen}(y)} = \cancel{-x^3 \operatorname{sen}(y)} + \frac{d\varphi(y)}{dy} \Rightarrow 0 = \frac{d\varphi(y)}{dy} \Rightarrow d\varphi(y) = 0\, dy$

integramos $\int d\varphi(y) = \int 0\, dy \Rightarrow \varphi(y) = C$

sustituimos en la ED (3)

$u = \frac{4x^5}{5} + x^3\cos(y) + C = 0$

$\frac{4x^5}{5} + x^3\cos(y) + C = 0$

FIRST CLASS

2.4) $(1+\frac{e^y}{x})\,dx - (x+3e^y)\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(1+\frac{e^y}{x}\right) = \frac{e^y}{x}$$

no es ED exacta

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(-x-3e^y\right) = -1$$

veamos con

$$g(x) = \frac{1}{N}\left(\frac{\partial}{\partial y}M - \frac{\partial}{\partial x}N\right) = \frac{1}{-x-3e^y}\left(\frac{e^y}{x}+1\right) = -\frac{e^y+x}{x^2+3e^y x}$$

no depende solo de x

veamos con

$$h(y) = \frac{1}{M}\left(\frac{\partial}{\partial x}N - \frac{\partial}{\partial y}M\right) = \frac{1}{1+\frac{e^y}{x}}\left(-1-\frac{e^y}{x}\right) = \frac{1}{\cancel{1+\frac{e^y}{x}}}\cdot -\cancel{\left(1+\frac{e^y}{x}\right)} = -1$$

entonces $\mu = e^{\int h(y)dy} = e^{-\int 1 dy} = e^{-y} = \frac{1}{e^y}$

multiplicar por μ

$$\frac{1}{e^y}\left[\left(1+\frac{e^y}{x}\right)dx + \left(-x-3e^y\right)dy\right] = 0$$

$$\Rightarrow \underbrace{\left(\frac{1}{e^y}+\frac{1}{x}\right)}_{M} dx + \underbrace{\left(\frac{-x}{e^y}-3\right)}_{N} dy = 0$$

$$\frac{\partial M}{\partial y}\left(\frac{1}{e^y}+\frac{1}{x}\right) = -\frac{1}{e^y}$$

ya es ED exacta

$$\frac{\partial N}{\partial x}\left(\frac{-x}{e^y}-3\right) = \frac{-1}{e^y}$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \left(\frac{1}{e^y}+\frac{1}{x}\right)dx + \left(-\frac{x}{e^y}-3\right)dy = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{1}{e^y}+\dfrac{1}{x} \quad \ldots (1) \\[2ex] \dfrac{\partial u}{\partial y} = \dfrac{-x}{e^y} - 3 \quad \ldots (2) \end{cases}$$

tomamos a la EO (1) y dejamos a y constante

$$\Rightarrow \frac{du}{dx} = \frac{1}{e^y} + \frac{1}{x} \Rightarrow du = \left(\frac{1}{e^y} + \frac{1}{x}\right)dx$$ integramos

$$\int du = \int \frac{1}{e^y} + \frac{1}{x}\, dx \Rightarrow u = \int \frac{1}{e^y} dx + \int \frac{1}{x} dx + \varphi(y)$$

$$\Rightarrow u = \frac{x}{e^y} + \ln(x) + \varphi(y) \quad \ldots (3)$$

derivamos respecto de y

$$\Rightarrow \frac{du}{dy} = \frac{d}{dy}\left(\frac{x}{e^y} + \ln(x) + \varphi(y)\right) = -\frac{x}{e^y} + \frac{d\varphi(y)}{dy}$$ pero $\frac{du}{dy} = \frac{-x}{e^y} - 3$

$$\Rightarrow -\frac{x}{e^y} - 3 = \cancel{-\frac{x}{e^y}} + \frac{d\varphi(y)}{dy} \Rightarrow -3 = \frac{d\varphi(y)}{dy} \Rightarrow d\varphi(y) = -3\,dy$$

Integramos $\int d\varphi(y) = \int -3\,dy \Rightarrow \varphi(y) = -3y + C$

sustituimos en la EO (3)

$$u = \frac{x}{e^y} + \ln(x) - 3y + c = 0$$

$$\frac{x}{e^y} + \ln(x) - 3y + c = 0$$