Tarea 5

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1) Integra las siguientes ecuaciones diferenciales simples de segundo orden que se reducen a ecuaciones de primer orden

(a) $xy'' + y' + x = 0$

$$y'' = \frac{-x - y'}{x}$$ Metodo 1

cambio de variable

$$\frac{dy}{dx} = p \quad \text{entonces} \quad \frac{d^2y}{dx^2} = \frac{dp}{dx}$$

sustituimos

$$x\frac{dp}{dx} + p + x = 0$$

dividimos por x $\quad \frac{dp}{dx} + \frac{p}{x} = -1$ $\quad$ ED lineal

proponemos $\quad p = uv \quad$ con $\quad \frac{dp}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

sustituimos $\quad u\frac{dv}{dx} + v\frac{du}{dx} + \frac{uv}{x} = -1$

factorizar u $\quad u\underbrace{\left[\frac{dv}{dx} + \frac{v}{x}\right]}_{0} + v\frac{du}{dx} = -1$

$$\frac{dv}{dx} + \frac{v}{x} = 0 \Rightarrow \frac{dv}{dx} = \frac{-v}{x} \Rightarrow \frac{dv}{v} = -\frac{dx}{x}$$ integramos

$$\int\frac{dv}{v} = -\int\frac{dx}{x} \Rightarrow \ln(v) = -\ln(x) \Rightarrow e^{\ln(v)} = e^{-\ln(x)} \Rightarrow v = \frac{1}{x}$$

$$u\left[\cancel{\frac{dv}{dx} + \frac{v}{x}}\right] + v\frac{du}{dx} = -1$$

$$\frac{1}{x}\frac{du}{dx} = -1 \Rightarrow \frac{du}{dx} = -x \Rightarrow du = -x\,dx$$

integramos

$$\int du = -\int x\,dx + C_1 \Rightarrow u = -\frac{x^2}{2} + C_1$$

entonces

$$p = \left(\frac{1}{x}\right)\left(-\frac{x^2}{2} + C_1\right) = -\frac{x}{2} + \frac{C_1}{x}$$

pero $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{x}{2} + \frac{C_1}{x} \Rightarrow dy = \left(-\frac{x}{2} + \frac{C_1}{x}\right) dx$$

integramos

$$\int dy = \int -\frac{x}{2} + \frac{C_1}{x}\,dx = -\int \frac{x}{2}dx + C_1\int \frac{1}{x}dx + C_2$$

$$\Rightarrow y = -\frac{x^2}{4} + C_1 \ln(x) + C_2$$

(b) $y y'' = (y')^2$ Metodo 2

cambio de variable

$\frac{dy}{dx} = p$ entons $\frac{d^2y}{dx^2} = \frac{dp}{dy} p$

sustituims

$y \frac{dp}{dy} p = p^2 \Rightarrow y \frac{dp}{dy} = p \Rightarrow \frac{dp}{p} = \frac{dy}{y}$ integrams

$\int \frac{dp}{p} = \int \frac{dy}{y} + C_1 \Rightarrow \ln(p) = \ln(y) + C_1 \Rightarrow e^{\ln(p)} = e^{\ln(y)+C_1}$

$\Rightarrow p = C_1 y$

pero $p = \frac{dy}{dx} \Rightarrow dx = \frac{dy}{p}$

integrams $\int dx = \int \frac{dy}{C_1 y} + C_2 \Rightarrow x = \frac{1}{C_1} \ln(y) + C_2$

(c) $(1+x^2)y'' - 2xy' = 0$ con $y(0)=0$ y $y'(0)=3$

Metodo 1

cambio de variable

$$\frac{dy}{dx} = p \quad \text{entonces} \quad \frac{d^2y}{dx^2} = \frac{dp}{dx}$$

sustituimos

$$(1+x^2)\frac{dp}{dx} - 2xp = 0 \Rightarrow \frac{dp}{dx} = \frac{2xp}{x^2+1} \Rightarrow \frac{dp}{p} = \frac{2x}{x^2+1}dx$$

integramos $\int \frac{dp}{p} = \int \frac{2x}{x^2+1}dx + C_1$ sustituimos $u = x^2+1$

$du = 2x\,dx$

$$\Rightarrow \ln(p) = \int \frac{1}{u}du \Rightarrow \ln(p) = \ln(u) = \ln(x^2+1) + C_1 \Rightarrow e^{\ln(p)} = e^{\ln(x^2+1)+C_1}$$

$$\Rightarrow p = C_1(x^2+1)$$

pero $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = C_1(x^2+1) \Rightarrow dy = C_1(x^2+1)\,dx$$ integramos

$$y = \int dy = C_1\int x^2+1\,dx = C_1\left[\int x^2dx + \int dx\right] + C_2 = C_1\frac{x^3}{3} + C_1x + C_2$$

pero $y'(0)=3$

entonces $y'(0) = 3 = C_1(0^2+1) = C_1$ entonces $C_1 = 3$

y ademas $y(0) = 0 = C_1\frac{0^3}{3} + C_1 0 + C_2 = C_2$ entonces $C_2 = 0$

por lo tanto $y = x^3 + 3x$

FIRST CLASS

2) Resuelve las siguientes ecuaciones diferenciales usando metodos de inspeccion

(a) $2t\,ds + s(2+s^2t)dt = 0$

$$2t\,ds + 2s\,dt + s^3t\,dt = 0$$

$$\underbrace{t\,ds + s\,dt}_{\text{inspeccion}} + \frac{s^3t}{2}dt = 0$$

multiplicamos por $\frac{1}{(xy)^3}$ (caso 5)

$$\frac{1}{(xy)^3}\left[t\,ds + s\,dt + \frac{s^3t}{2}dt\right] = 0$$

$$\frac{t\,ds + s\,dt}{(xy)^3} + \frac{1}{2t^2}dt = 0$$

usamos tablitas

$$d\left(\frac{-1}{2(xy)^2}\right) + \frac{1}{2t^2}dt = 0$$

integramos

$$\int d\left(\frac{-1}{2(xy)^2}\right) + \int \frac{1}{2t^2}dt + c = 0$$

$$\frac{-1}{2(xy)^2} - \frac{1}{2t} + c = 0$$

(b) $x\,dx + y\,dy + 4y^3(x^2+y^2)dy = 0$

multiplicar por $\cdot \frac{1}{x^2+y^2}$ (caso 6)

$$\frac{1}{x^2+y^2}\left[x\,dx + y\,dy + 4y^3(x^2+y^2)dy\right] = 0$$

$$\frac{x\,dy + y\,d\cancel{x}}{x^2+y^2} + 4y^3\,dy = 0$$

usamos tabla

$$4y^3dy + d\left(\tfrac{1}{2}\ln(x^2+y^2)\right) = 0$$

integremos

$$\int 4y^3dy + \int d\left(\tfrac{1}{2}\ln(x^2+y^2)\right) + c = 0$$

$$y^4 + \tfrac{1}{2}\ln(x^2+y^2) + c = 0$$

(c) $(x + x^4 + 2x^2y^2 + y^4)dx + ydy = 0$

$$x\,dx + y\,dy + (x^4 + 2x^2y^2 + y^4)dx = 0$$

multiplicamos por $\frac{1}{(x^2+y^2)^2}$ (caso 6)

$$\frac{1}{(x^2+y^2)^2}\left[x\,dx + y\,dy + (x^4+2x^2y^2+y^4)dx\right] = 0$$

$$\frac{x\,dx + y\,dy}{(x^2+y^2)^2} + dx = 0$$

usamos tablitas

$$d\left(\frac{-1}{2(x^2+y^2)}\right) + dx = 0$$

integremos

$$\int d\left(\frac{-1}{2(x^2+y^2)}\right) + \int dx + c = 0$$

$$-\frac{1}{2(x^2+y^2)} + x + c = 0$$