Práctica 3: Semántica

1. Reglas de semántica operacional del intérprete:

   - ***Variables y nil***

     A continuación se describen las variables y nil:

$$\frac{}{\text{Var x} \not\to}$$

$$\frac{}{\text{Num n} \not\to}$$

$$\frac{}{\text{BoolE b} \not\to}$$

$$\frac{}{\text{Nil} \not\to}$$

   - ***Concatenacion, Head y Tail***

     A continuación se describen las concatenacion, head y tail:

$$\frac{}{\text{Cons (Nil, } e1) \to e1}$$

$$\frac{e2 \to e4}{\text{Cons (Val } e1, e2) \to \text{Cons (Val } e1, e4)}$$

$$\frac{e1 \to e3}{\text{Cons } (e1, e2) \to \text{Cons } (e3, e2)}$$

$$\frac{}{\text{Head (Cons } (e1, e2)) \to e1}$$

$$\frac{}{\text{Head (Nil)} \to \text{Nil}}$$

$$\frac{e1 \to e2}{\text{Head } (e1) \to \text{Head } (e2)}$$

$$\frac{}{\text{Tail (Cons } (e1, e2)) \to e2}$$

$$\frac{}{\text{Tail (Nil)} \to \text{Nil}}$$

$$\frac{e1 \to e2}{\text{Tail } (e1) \to \text{Tail } (e2)}$$

- ***Suma***

  A continuación se describe la suma:

$$\frac{}{\text{Plus (Num n1, Num n2)} \to \text{Num [n1 + n2]}}$$

$$\frac{e2 \to e3}{\text{Plus (Num n1, } e2) \to \text{Plus (Num n1, } e3)}$$

$$\frac{e1 \to e3}{\text{Plus } (e1, e2) \to \text{Plus } (e3, e2)}$$

- ***Resta***

  A continuación se describe la resta:

$$\frac{}{\text{Minus (Num n1, Num n2)} \to \text{Num [n1 - n2]}}$$

$$\frac{e2 \to e3}{\text{Minus (Num n1, } e2) \to \text{Minus (Num n1, } e3)}$$

$$\frac{e1 \to e3}{\text{Minus } (e1, e2) \to \text{Minus } (e3, e2)}$$

- ***Multiplicación***

  A continuación se describe la multiplicación:

$$\frac{}{\text{Times (Num n1, Num n2)} \to \text{Num [n1 * n2]}}$$

$$\frac{e2 \to e3}{\text{Times (Num n1, } e2) \to \text{Times (Num n1, } e3)}$$

$$\frac{e1 \to e3}{\text{Times } (e1, e2) \to \text{Times } (e3, e2)}$$

- ***División***

  A continuación se describe la división:

$$\frac{}{\text{Div (Num n1, Num n2)} \to \text{Num (Div(n1, n2))}}$$

$$\frac{e2 \to e3}{\text{Div (Num n1, } e2\text{)} \to \text{Div (Num n1, } e3\text{)}}$$

$$\frac{e1 \to e3}{\text{Div (}e1\text{, } e2\text{)} \to \text{Div (}e3\text{, } e2\text{)}}$$

- ***Módulo***

  A continuación se describe el módulo:

$$\frac{}{\text{Mod (Num n1, Num n2)} \to \text{Num (Mod(n1, n2))}}$$

$$\frac{e2 \to e3}{\text{Mod (Num n1, } e2\text{)} \to \text{Mod (Num n1, } e3\text{)}}$$

$$\frac{e1 \to e3}{\text{Mod (}e1\text{, } e2\text{)} \to \text{Mod (}e3\text{, } e2\text{)}}$$

- ***Igualdad***

  A continuación se describe la igualdad:

$$\frac{}{\text{Eq (Num n1, Num n2)} \to \text{Bool (n1 == n2)}}$$

$$\frac{e2 \to e3}{\text{Eq (Num n1, } e2\text{)} \to \text{Eq (Num n1, } e3\text{)}}$$

$$\frac{}{\text{Eq (Nil, Nil)} \to \text{BoolE True}}$$

$$\frac{}{\text{Eq ((Cons (}e1\text{, } e2\text{)), Nil)} \to \text{BoolE False}}$$

$$\frac{}{\text{Eq (Nil, (Cons (}e1\text{, } e2\text{))) } \to \text{BoolE False}}$$

$$\frac{}{\text{Eq ((Cons } (e1,\ e2)),\ (\text{Cons } (e3,\ e4))) \rightarrow \text{And ((Eq } (e1,\ e3)),\ (\text{Eq } (e2,\ e4)))}$$

$$\frac{e1 \rightarrow e3}{\text{Eq } (e1,\ \text{Nil}) \rightarrow \text{Eq } (e3,\ \text{Nil})}$$

$$\frac{e2 \rightarrow e3}{\text{Eq (Nil, } e2) \rightarrow \text{Eq (Nil, } e3)}$$

$$\frac{e3 \rightarrow e4}{\text{Eq ((Cons } (e1,\ e2)),\ e3) \rightarrow \text{Eq ((Cons } (e1,\ e2)),\ e4)}$$

$$\frac{e1 \rightarrow e3}{\text{Eq } (e1,\ e2) \rightarrow \text{Eq } (e3,\ e2)}$$

- ***Menor que***

  A continuación se describe el menor que:

$$\frac{}{\text{Lt (Num n1, Num n2)} \rightarrow \text{Bool } (n1 < n2)}$$

$$\frac{e2 \rightarrow e3}{\text{Lt (Num n1, } e2) \rightarrow \text{Lt (Num n1, } e3)}$$

$$\frac{e1 \rightarrow e3}{\text{Lt } (e1,\ e2) \rightarrow \text{Lt } (e3,\ e2)}$$

- ***Mayor que***

  A continuación se describe el mayor que:

$$\frac{}{\text{Gt (Num n1, Num n2)} \rightarrow \text{Bool } (n1 > n2)}$$

$$\frac{e2 \rightarrow e3}{\text{Gt (Num n1, } e2) \rightarrow \text{Gt (Num n1, } e3)}$$

$$\frac{e1 \rightarrow e3}{\text{Gt } (e1,\ e2) \rightarrow \text{Gt } (e3,\ e2)}$$

- ***Menor o igual que***

  A continuación se describe el menor o igual que:

$$\frac{}{\text{Le (Num n1, Num n2)} \to \text{Bool } (n1 <= n2)}$$

$$\frac{e2 \to e3}{\text{Le (Num n1, } e2) \to \text{Le (Num n1, } e3)}$$

$$\frac{e1 \to e3}{\text{Le } (e1, e2) \to \text{Le } (e3, e2)}$$

- ***Mayor o igual que***

  A continuación se describe el mayor o igual que:

$$\frac{}{\text{Ge (Num n1, Num n2)} \to \text{BoolE } (n1 >= n2)}$$

$$\frac{e2 \to e3}{\text{Ge (Num n1, } e2) \to \text{Ge (Num n1, } e3)}$$

$$\frac{e1 \to e3}{\text{Ge } (e1, e2) \to \text{Ge } (e3, e2)}$$

- ***And***

  A continuación se describe el And:

$$\frac{}{\text{And (BoolE b1, BoolE b2)} \to \text{BoolE } (b1 \&\& b2)}$$

$$\frac{e2 \to e3}{\text{And (BoolE b1, } e2) \to \text{And (BoolE b1, } e3)}$$

$$\frac{e1 \to e3}{\text{And } (e1, e2) \to \text{And } (e3, e2)}$$

- ***Or***

  A continuación se describe el Or:

$$\frac{}{\text{Or (BoolE b1, BoolE b2)} \to \text{BoolE } (b1 \| \| b2)}$$

$$\frac{e2 \to e3}{\text{Or (BoolE b1, } e2) \to \text{Or (BoolE b1, } e3)}$$

$$\frac{e1 \to e3}{\text{Or } (e1, e2) \to \text{Or } (e3, e2)}$$

- ***Not***

  A continuación se describe el Not:

$$\frac{}{\text{Not (BoolE b1)} \to \text{BoolE (not b1)}}$$

$$\frac{e1 \to e2}{\text{Not } (e1) \to \text{Not } (e2)}$$

- ***Append***

  A continuación se describe el Append:

$$\frac{}{\text{Append (Nil, Nil)} \to \text{Nil}}$$

$$\frac{}{\text{Append (Nil, } e2) \to e2}$$

$$\frac{}{\text{Append } (e1, \text{Nil}) \to e1}$$

$$\frac{}{\text{Append ((Cons } e1\ e2), e3) \to \text{Cons } (e1, (\text{Append } (e2, e3)))}$$

$$\frac{e1 \to e3}{\text{Append } (e1, e2) \to \text{Append } (e3, e2)}$$

- ***App***

  A continuación se describe el App:

$$\frac{}{\text{App ((Lam x T } e1), e2) \to e1[x := e2]}$$

$$\frac{e1 \to e3}{\text{App } (e1, e2) \to \text{App } (e3, e2)}$$

- ***If***

  A continuación se describe el If:

$$\frac{}{\text{If ((BoolE b), } e2\text{, } e3) \rightarrow \text{if b then } e2 \text{ else } e3}$$

$$\frac{e1 \rightarrow e4}{\text{If } (e1\text{, } e2\text{, } e3) \rightarrow \text{If } (e4\text{, } e2\text{, } e3)}$$

- ***Let***

  A continuación se describe el Let:

$$\frac{}{\text{Let (x, } e1\text{, } e2) \rightarrow e2[x := \text{Fix(x, } e1)]}$$

- ***Lam***

  A continuación se describe el Lam:

$$\frac{}{\text{Lam(x, t, e)} \not\rightarrow}$$

- ***Fix***

  A continuación se describe el Fix:

$$\frac{}{\text{Fix(x, } e) \rightarrow e[x := \text{Fix(x, } e)]}$$

- ***Otro Caso***

  A continuación se describe cualquier otro caso:

$$\frac{}{\text{S} \not\rightarrow}$$

2. Reglas del algoritmo de generación de restricciones:

   - ***Variables y nil***

     A continuación se describen las variables y nil:

$$\frac{}{\text{rest(Var } x_i) \mapsto [\![\text{ } x_i \text{ }]\!] = X_i}$$

$$\frac{}{\text{rest(Num n)} \mapsto [\![\text{ Num n }]\!] = \text{Nat}}$$

$$\frac{}{\text{rest(BoolE b)} \mapsto [\![\text{ BoolE b }]\!] = \text{BooleaN}}$$

$$\frac{Y_i \text{ una variable nueva}}{\text{rest}(\text{Nil}_i) \mapsto [\![\text{Nil}_i]\!] = \text{ListOf}(\text{Y}_i)}$$

- ***Concatenacion, Head y Tail***

  A continuación se describen las concatenacion, head y tail:

$$\frac{\text{rest}(\text{e}_1) \mapsto \text{R}_1, rest(e_2) \mapsto \text{R}_2}{\text{rest}(\text{Cons } (\text{e}_1, e_2)) \mapsto \text{R}_1, R_2, [\![\text{Cons } (\text{e}_1, e_2)]\!] = \text{ListOf}([\![\text{e}_1]\!]),\ [\![\text{e}_2]\!] = \text{ListOf}([\![\text{e}_1]\!])}$$

$$\frac{\text{rest}(\text{e}_1) \mapsto \text{R}_1}{\text{rest}(\text{Head } (\text{e}_1)) \mapsto \text{R}_1, [\![\text{e}_1]\!] = \text{ListOf}([\![\text{Head } (\text{e}_1)]\!])}$$

$$\frac{\text{rest}(\text{e}_1) \mapsto \text{R}_1}{\text{rest}(\text{Tail } (\text{e}_1)) \mapsto \text{R}_1, [\![\text{Tail } (\text{e}_1)]\!] = [\![\text{e}_1]\!]}$$

- ***Suma***

  A continuación se describe la suma:

$$\frac{\text{rest}(\text{e}_1) \mapsto \text{R}_1 \quad rest(e_2) \mapsto \text{R}_2}{\text{rest}(\text{Plus}(\text{e}_1, e_2)) \mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{Nat},\ [\![e_2]\!] = \text{Nat},\ [\![\text{Plus}(\text{e}_1, e_2)]\!] = \text{Nat}}$$

- ***Resta***

  A continuación se describe la resta:

$$\frac{\text{rest}(\text{e}_1) \mapsto \text{R}_1 \quad rest(e_2) \mapsto \text{R}_2}{\text{rest}(\text{Minus}(\text{e}_1, e_2)) \mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{Nat},\ [\![e_2]\!] = \text{Nat},\ [\![\text{Minus}(\text{e}_1, e_2)]\!] = \text{Nat}}$$

- ***Multiplicación***

  A continuación se describe la multiplicación:

$$\frac{\text{rest}(\text{e}_1) \mapsto \text{R}_1 \quad rest(e_2) \mapsto \text{R}_2}{\text{rest}(\text{Times}(\text{e}_1, e_2)) \mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{Nat},\ [\![e_2]\!] = \text{Nat},\ [\![\text{Times}(\text{e}_1, e_2)]\!] = \text{Nat}}$$

- ***División***

  A continuación se describe la división:

$$\frac{\text{rest}(\text{e}_1) \mapsto \text{R}_1 \quad rest(e_2) \mapsto \text{R}_2}{\text{rest}(\text{Div}(\text{e}_1, e_2)) \mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{Nat},\ [\![e_2]\!] = \text{Nat},\ [\![\text{Div}(\text{e}_1, e_2)]\!] = \text{Nat}}$$

- ***Módulo***

  A continuación se describe el módulo:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2}{\text{rest}(\text{Mod}(\text{e}_1, e_2))\mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{Nat}, [\![e_2]\!] = \text{Nat}, [\![\text{Mod}(\text{e}_1, e_2)]\!] = \text{Nat}}$$

- ***Igualdad***

  A continuación se describe la igualdad:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2}{\text{rest}(\text{Eq}(\text{e}_1, e_2))\mapsto \text{R}_1, R_2, [\![e_1]\!] = [\![e_2]\!], [\![\text{Eq}(\text{e}_1, e_2)]\!] = \text{BooleaN}}$$

- ***Menor que***

  A continuación se describe el menor que:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2}{\text{rest}(\text{Lt}(\text{e}_1, e_2))\mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{Nat}, [\![e_2]\!] = \text{Nat}, [\![\text{Lt}(\text{e}_1, e_2)]\!] = \text{Nat}}$$

- ***Mayor que***

  A continuación se describe el mayor que:

$$\frac{\text{rest}(e_1) \mapsto R_1 \quad \text{rest}(e_2) \mapsto R_2}{\text{rest}(\text{Gt}(\text{e}_1, e_2))\mapsto R_1, R_2, [\![e_1]\!] = \text{Nat}, [\![e_2]\!] = \text{Nat}, [\![\text{Gt}(e_1, e_2)\ ]\!] = \text{Nat}}$$

- ***Menor o igual que***

  A continuación se describe el menor o igual que:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2}{\text{rest}(\text{Le}(\text{e}_1, e_2))\mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{Nat}, [\![e_2]\!] = \text{Nat}, [\![\text{Le}(\text{e}_1, e_2)]\!] = \text{Nat}}$$

- ***Mayor o igual que***

  A continuación se describe el mayor o igual que:

$$\frac{\text{rest}(e_1) \mapsto R_1 \quad \text{rest}(e_2) \mapsto R_2}{\text{rest}(\text{Ge}(e_1,\ e_2)) \mapsto R_1, R_2, [\![e_1]\!] = \text{Nat}, [\![e_2]\!] = \text{Nat}, [\![\text{Ge}(e_1, e_2)\ ]\!] = \text{Nat}}$$

- ***And***

  A continuación se describe el And:

$$\frac{\text{rest}(e_1) \mapsto R_1 \quad \text{rest}(e_2) \mapsto R_2}{\text{rest}(\text{And}(e_1,\ e_2)) \mapsto R_1, R_2, [\![e_1]\!] = \text{BooleaN}, [\![e_2]\!] = \text{BooleaN}, [\![\text{And}(e_1, e_2)\ ]\!] = \text{BooleaN}}$$

- ***Or***

  A continuación se describe el Or:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2}{\text{rest}(\text{Or}(\text{e}_1, e_2))\mapsto \text{R}_1, R_2, [\![e_1]\!] = \text{BooleaN}, [\![e_2]\!] = \text{BooleaN}, [\![\text{Or}(\text{e}_1, e_2)]\!] = \text{BooleaN}}$$

- ***Not***

  A continuación se describe el Not:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1}{\text{rest}(\text{Not}(\text{e}_1))\mapsto \text{R}_1,[\![e_1]\!] = \text{BooleaN},\ [\![\text{Not}(\text{e}_1)]\!] = \text{BooleaN}}$$

- ***Append***

  A continuación se describe el Append:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2}{\text{rest}(\text{Append}(\text{e}_1, e_2))\mapsto \text{R}_1, R_2,[\![e_1]\!] = [\![e_2]\!],\ [\![\text{Append}(\text{e}_1, e_2)]\!] = [\![e_1]\!],\ [\![\text{Append}(\text{e}_1, e_2)]\!] = [\![e_2]\!]}$$

- ***App***

  A continuación se describe el App:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2}{\text{rest}(\text{App}(\text{e}_1, e_2))\mapsto \text{R}_1, R_2,[\![e_1]\!] = [\![e_2]\!] \to [\![\ \text{App}(\text{e}_1, e_2)]\!]}$$

- ***If***

  A continuación se describe el If:

$$\frac{\text{rest}(\text{e}_1)\mapsto \text{R}_1 \quad rest(e_2)\mapsto \text{R}_2 \quad rest(e_3)\mapsto \text{R}_3}{\text{rest}(\text{If}(\text{e}_1, e_2, e_3))\mapsto \text{R}_1, R_2, R_3,[\![e_1]\!] = \text{BooleaN},\ [\![e_2]\!] = [\![e_3]\!],\ [\![\text{If}(\text{e}_1, e_2, e_3)]\!] = [\![e_2]\!],\ [\![\text{If}(\text{e}_1, e_2, e_3)]\!] = [\![e_3]\!]}$$

- ***Let***

  A continuación se describe el Let:

$$\frac{\text{rest}(e_1)\ \mapsto R_1 \quad \text{rest}(\text{e}_2)\mapsto R_2}{\text{rest}(\text{Let}(\text{x}_i, e_1, e_2))\mapsto \text{R}_1, R_2, X_i = [\![e_1]\!],\ [\![\text{Let}(\text{x}_i, e_1, e_2)]\!] = [\![e_2]\!]}$$

- ***Lam***

  A continuación se describe el Lam:

$$\frac{\text{rest}(e_1)\ \mapsto R_1}{\text{rest}(\text{Lam}(\text{x}_i, t, e_1))\mapsto \text{R}_1,[\![(\text{Lam}(\text{x}_i, t, e_1))]\!] = \text{X}_i \to [\![e_1]\!]}$$

- ***Otro Caso***

  A continuación se describe cualquier otro caso:

$$\frac{}{\text{S}\mapsto \text{vacio}}$$