García Ponce José Camilo

Matemáticas para Ciencias de la Tierra III
Matemáticas para las Ciencias Aplicadas III
Tarea 1

Fecha de entrega: **Viernes 2 de septiembre de 2022.**

Sumas de Riemann

1. Estimar el volumen de los siguientes sólidos por sumas de Riemann (5 puntos):

$$V \approx \sum_{i=1}^{m} \sum_{i=1}^{n} f(x_i, y_j) \Delta A$$

   a) El sólido que yace por debajo de $f(x, y) = 2\cos(xy)$ y por encima del rectángulo $D = [0, \pi/2] \times [0, \pi/2]$. Escoger la suma de Riemann tal que $m = n = 2$ y escoger el punto a evaluar como la esquina superior izquierda de cada dominio parcial. 3.875

   b) El sólido que yace por encima del rectángulo $R = [0, 2] \times [0, 4]$ y por debajo de $f(x, y) = 4 - x - y^2$. Escoger la suma de Riemann tal que $m = n = 2$ y escoger los puntos a evaluar como la esquina superior derecha de cada dominio parcial. -60

   c) Estima el volumen del sólido que se encuentra arriba del rectángulo $R = [0, 2] \times [-1, 1]$ y debajo del paraboloide elíptico $z = 16 - x^2 - 2y^2$. Para ello, divide el rectángulo R en 16 subrectángulos iguales y elige el punto a evaluar como la esquina superior derecha de cada dominio parcial. 53.5

   d) Estima el volumen del sólido que se encuentra arriba del rectángulo $R = [0, 2] \times [-1, 1]$ y debajo de la gráfica $f(x, y) = 1 + 6xy^2$. Para ello, divide el rectángulo R en 16 subrectángulos iguales y elige el punto a evaluar como la esquina superior izquierda de cada dominio parcial. 10.75

Integrales Dobles

2. Calcula las siguientes integrales dobles y dibuja la región de integración (5 puntos).

   a) $\int_0^2 \int_1^2 (x - 3y^2)\, dydx$ -12

   b) $\int_{-1}^1 \int_{2x^2}^{1+x^2} (x + 2y)\, dydx$ 2.1333

   c) $\int_1^2 \int_1^{7-3y} \frac{2}{12y-3y^2}\, dxdy$ 0.287

   d) $\int_1^2 \int_y^{y^3} e^{\frac{x}{y}}\, dxdy$ 21.852

Tarea 1 García Ponce José Camilo

1) Sumas de Riemann

a) $f(x,y) = 2\cos(xy)$, dominio $D = [0, \pi/2] \times [0, \pi/2]$

$m = n = 2$ (partir en 4) y el punto la esquina superior izq

$D = [0, \pi/4, \pi/2] \times [0, \pi/4, \pi/2]$

$= \{ (0,\frac{\pi}{2}), (\frac{\pi}{4},\frac{\pi}{2}), (\frac{\pi}{2},\frac{\pi}{2}),$
$(0,\frac{\pi}{4}), (\frac{\pi}{4},\frac{\pi}{4}), (\frac{\pi}{2},\frac{\pi}{4}),$
$(0,0), (\frac{\pi}{4},0), (\frac{\pi}{2},0) \}$

notemos que cada dominio parcial tiene area de

$\frac{\pi}{4} \cdot \frac{\pi}{4} = \frac{\pi^2}{16} \simeq 0.616$

$\Delta s_i \simeq 0.616$

ahora tomemos los puntos

$P_1 = (0, \frac{\pi}{4})$ $P_3 = (\frac{\pi}{4}, \frac{\pi}{4})$

$P_2 = (0, \frac{\pi}{2})$ $P_4 = (\frac{\pi}{4}, \frac{\pi}{2})$

ahora evaluemos los puntos

$f(P_1) = 2\cos(0 \cdot \frac{\pi}{4}) = 2\cos(0) = 2$

$f(P_2) = 2\cos(0 \cdot \frac{\pi}{2}) = 2\cos(0) = 2$

$f(P_3) = 2\cos(\frac{\pi}{4} \cdot \frac{\pi}{4}) = 1{,}631$

$f(P_4) = 2\cos(\frac{\pi}{4} \cdot \frac{\pi}{2}) = 0{,}661$

$$\sum_{i=1}^{4} f(P_i)\Delta s_i = f(P_1)\Delta s_1 + f(P_2)\Delta s_2 + f(P_3)\Delta s_3 + f(P_4)\Delta s_4$$

$$= 2(0.616) + 2(0.616) + 1.631(0.616) + 0.661(0.616)$$

$$= 1.232 + 1.232 + 1.004 + 0.407 = 3.875\, u^3$$

b) $f(x,y) = 4 - x - y^2$, dominio $D = [0,2] \times [0,4]$

$m = n = 2$ (partir en 4) , y el punto la esquina superior der

$D = [0, 1, 2] \times [0, 2, 4]$

$$= \{(0,4), (1,4), (2,4), (0,2), (1,2), (2,2), (0,0), (1,0), (2,0)\}$$

notemos que cada dominio parcial tiene area de $1 \cdot 2 = 2$

$\Delta s_i = 2$

ahora tomamos los puntos

$P_1 = (1,4)$ $P_3 = (2,4)$

$P_2 = (1,2)$ $P_4 = (2,2)$

ahora evaluamos los puntos

$f(P_1) = 4 - 1 - 4^2 = -13$

$f(P_2) = 4 - 1 - 2^2 = -1$

$f(P_3) = 4 - 2 - 4^2 = -14$

$f(P_4) = 4 - 2 - 2^2 = -2$

$$\sum_{i=1}^{4} f(P_i)\Delta s_i = f(P_1)\Delta s_1 + f(P_2)\Delta s_2 + f(P_3)\Delta s_3 + f(P_4)\Delta s_4$$

$$= -13(2) + -1(2) + -14(2) + -2(2)$$

$$= -26 - 2 - 28 - 4 = -60\ u^3$$

o $60\ u^3$?

c) $f(x,y) = 16-x^2-2y^2$, dominio $D=[0,2]\times[-1,1]$

partir en 16 , el punto la esquina superior der

$$D=[0,\tfrac{1}{2},1,\tfrac{3}{2},2]\times[-1,-\tfrac{1}{2},0,\tfrac{1}{2},1]$$

$$= \{(0,1),(\tfrac{1}{2},1),(1,1),(\tfrac{3}{2},1),(2,1),$$
$$(0,\tfrac{1}{2}),(\tfrac{1}{2},\tfrac{1}{2}),(1,\tfrac{1}{2}),(\tfrac{3}{2},\tfrac{1}{2}),(2,\tfrac{1}{2}),$$
$$(0,0),(\tfrac{1}{2},0),(1,0),(\tfrac{3}{2},0),(2,0),$$
$$(0,-\tfrac{1}{2}),(\tfrac{1}{2},-\tfrac{1}{2}),(1,-\tfrac{1}{2}),(\tfrac{3}{2},-\tfrac{1}{2}),(2,-\tfrac{1}{2}),$$
$$(0,-1),(\tfrac{1}{2},-1),(1,-1),(\tfrac{3}{2},-1),(2,-1),\}$$

notemos que cada dominio parcial tiene área de $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$

$\Delta s_i = \frac{1}{4}$

ahora tomamos los puntos

$P_1 = (\frac{1}{2},1)$ $P_5=(1,1)$ $P_9=(\frac{3}{2},1)$ $P_{13}=(2,1)$

$P_2=(\frac{1}{2},\frac{1}{2})$ $P_6=(1,\frac{1}{2})$ $P_{10}=(\frac{3}{2},\frac{1}{2})$ $P_{14}=(2,\frac{1}{2})$

$P_3=(\frac{1}{2},0)$ $P_7=(1,0)$ $P_{11}=(\frac{3}{2}0)$ $P_{15}=(2,0)$

$P_4=(\frac{1}{2},-\frac{1}{2})$ $P_8=(1,-\frac{1}{2})$ $P_{12}=(\frac{3}{2},-\frac{1}{2})$ $P_{16}=(2,-\frac{1}{2})$

ahora evaluamos los punto

$f(P_1)=16-(\frac{1}{2})^2-2(1)^2=\frac{55}{4}$

$f(P_2)=16-(\frac{1}{2})^2-2(\frac{1}{2})^2=\frac{61}{4}$

$f(P_3)=16-(\frac{1}{2})^2-2(0)^2=\frac{63}{4}$

$f(P_4)=16-(\frac{1}{2})^2-2(-\frac{1}{2})^2=\frac{61}{4}$

$f(P_5)=16-(1)^2-2(1)^2=13$

$f(P_6)=16-(1)^2-2(\frac{1}{2})^2=\frac{58}{4}$

$f(P_7)=16-(1)^2-2(0)^2=15$

$f(P_8)=16-(1)^2-2(-\frac{1}{2})^2=\frac{58}{4}$

$f(P_9)=16-(\frac{3}{2})^2-2(1)^2=\frac{47}{4}$

$f(P_{10})=16-(\frac{3}{2})^2-2(\frac{1}{2})^2=\frac{53}{4}$

$f(P_{11})=16-(\frac{3}{2})^2-2(0)^2=\frac{55}{4}$

$f(P_{12})=16-(\frac{3}{2})^2-2(-\frac{1}{2})^2=\frac{53}{4}$

$f(P_{13})=16-(2)^2-2(1)^2$
$=10$

$f(P_{14})=16-(2)^2-2(\frac{1}{2})^2$
$=\frac{46}{4}$

$f(P_{15})=16-(2)^2-2(0)^2$
$=12$

$f(P_{16})=16-(2)^2-2(-\frac{1}{2})^2$
$=\frac{46}{4}$

$$\sum_{i=1}^{16} f(P_i)\Delta s_i = f(P_1)\Delta s_1 + f(P_2)\Delta s_2 + f(P_3)\Delta s_3 + f(P_4)\Delta s_4 + f(P_5)\Delta s_5$$
$$+ f(P_6)\Delta s_6 + f(P_7)\Delta s_7 + f(P_8)\Delta s_8 + f(P_9)\Delta s_9 + f(P_{10})\Delta s_{10}$$
$$+ f(P_{11})\Delta s_{11} + f(P_{12})\Delta s_{12} + f(P_{13})\Delta s_{13} + f(P_{14})\Delta s_{14} + f(P_{15})\Delta s_{15}$$
$$+ f(P_{16})\Delta s_{16}$$

$$= \frac{55}{4}\left(\frac{1}{4}\right) + \frac{61}{4}\left(\frac{1}{4}\right) + \frac{63}{4}\left(\frac{1}{4}\right) + \frac{61}{4}\left(\frac{1}{4}\right)$$

$$+ 13\left(\frac{1}{4}\right) + \frac{58}{4}\left(\frac{1}{4}\right) + 15\left(\frac{1}{4}\right) + \frac{58}{4}\left(\frac{1}{4}\right)$$

$$+ \frac{47}{4}\left(\frac{1}{4}\right) + \frac{53}{4}\left(\frac{1}{4}\right) + \frac{55}{4}\left(\frac{1}{4}\right) + \frac{53}{4}\left(\frac{1}{4}\right)$$

$$+ 10\left(\frac{1}{4}\right) + \frac{46}{4}\left(\frac{1}{4}\right) + 12\left(\frac{1}{4}\right) + \frac{46}{4}\left(\frac{1}{4}\right)$$

$$= \frac{55}{16} + \frac{61}{16} + \frac{63}{16} + \frac{61}{16} + \frac{52}{16} + \frac{58}{16} + \frac{60}{16} + \frac{58}{16}$$

$$+ \frac{47}{16} + \frac{53}{16} + \frac{55}{16} + \frac{53}{16} + \frac{40}{16} + \frac{46}{16} + \frac{48}{16} + \frac{46}{16}$$

$$= \frac{856}{16} = \frac{107}{2} = 53.5\ u^3$$

d) $f(x,y) = 1+6xy^2$, dominio $D = [0,2] \times [-1,1]$

partir en 16, el punto la esquina superior izq

$D = [0, \frac{1}{2}, 1, \frac{3}{2}, 2] \times [-1, -\frac{1}{2}, 0, \frac{1}{2}, 1]$

$= \{(0,1), (\frac{1}{2},1), (1,1), (\frac{3}{2},1), (2,1),$
$(0,\frac{1}{2}), (\frac{1}{2},\frac{1}{2}), (1,\frac{1}{2}), (\frac{3}{2},\frac{1}{2}), (2,\frac{1}{2}),$
$(0,0), (\frac{1}{2},0), (1,0), (\frac{3}{2},0), (2,0),$
$(0,-\frac{1}{2}), (\frac{1}{2},-\frac{1}{2}), (1,-\frac{1}{2}), (\frac{3}{2},-\frac{1}{2}), (2,-\frac{1}{2}),$
$(0,-1), (\frac{1}{2},-1), (1,-1), (\frac{3}{2},-1), (2,-1)\}$

notemos que cada dominio parcial tiene area de

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$\Delta s_i = \frac{1}{4}$

ahora tomamos los puntos

$P_1 = (0,1)$ $P_5 = (\frac{1}{2},1)$ $P_9 = (1,1)$ $P_{13} = (\frac{3}{2},1)$

$P_2 = (0,\frac{1}{2})$ $P_6 = (\frac{1}{2},1)$ $P_{10} = (1,\frac{1}{2})$ $P_{14} = (\frac{3}{2},\frac{1}{2})$

$P_3 = (0,0)$ $P_7 = (\frac{1}{2},0)$ $P_{11} = (1,0)$ $P_{15} = (\frac{3}{2},0)$

$P_4 = (0,-\frac{1}{2})$ $P_8 = (\frac{1}{2},-\frac{1}{2})$ $P_{12} = (1,-\frac{1}{2})$ $P_{16} = (\frac{3}{2},-\frac{1}{2})$

ahora evaluamos los puntos

$f(P_1) = 1+6(0)(1)^2 = 1$ $f(P_7) = 1+6(\frac{1}{2})(0)^2 = 1$ $f(P_{13}) = 1+6(\frac{3}{2})(1)^2 = 10$

$f(P_2) = 1+6(0)(\frac{1}{2})^2 = 1$ $f(P_8) = 1+6(\frac{1}{2})(-\frac{1}{2})^2 = \frac{7}{4}$ $f(P_{14}) = 1+6(\frac{3}{2})(\frac{1}{2})^2 = \frac{13}{4}$

$f(P_3) = 1+6(0)(0)^2 = 1$ $f(P_9) = 1+6(1)(1)^2 = 7$ $f(P_{15}) = 1+6(\frac{3}{2})(0)^2 = 1$

$f(P_4) = 1+6(0)(-\frac{1}{2})^2 = 1$ $f(P_{10}) = 1+6(1)(\frac{1}{2})^2 = \frac{10}{4}$ $f(P_{16}) = 1+6(\frac{3}{2})(-\frac{1}{2})^2 = \frac{13}{4}$

$f(P_5) = 1+6(\frac{1}{2})(1)^2 = 4$ $f(P_{11}) = 1+6(1)(0)^2 = 1$

$f(P_6) = 1+6(\frac{1}{2})(\frac{1}{2})^2 = \frac{7}{4}$ $f(P_{12}) = 1+6(1)(-\frac{1}{2})^2 = \frac{10}{4}$

$$\sum_{i=1}^{16} f(P_i)\Delta s_i = f(P_1)\Delta s_1 + f(P_2)\Delta s_2 + f(P_3)\Delta s_3 + f(P_4)\Delta s_4$$
$$+ f(P_5)\Delta s_5 + f(P_6)\Delta s_6 + f(P_7)\Delta s_7 + f(P_8)\Delta s_8$$
$$+ f(P_9)\Delta s_9 + f(P_{10})\Delta s_{10} + f(P_{11})\Delta s_{11} + f(P_{12})\Delta s_{12}$$
$$+ f(P_{13})\Delta s_{13} + f(P_{14})\Delta s_{14} + f(P_{15})\Delta s_{15} + f(P_{16})\Delta s_{16}$$

$$= 1\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right)$$
$$+ 4\left(\tfrac{1}{4}\right) + \tfrac{7}{4}\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right) + \tfrac{7}{4}\left(\tfrac{1}{4}\right)$$
$$+ 7\left(\tfrac{1}{4}\right) + \tfrac{10}{4}\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right) + \tfrac{10}{4}\left(\tfrac{1}{4}\right)$$
$$+ 10\left(\tfrac{1}{4}\right) + \tfrac{13}{4}\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right) + \tfrac{13}{4}\left(\tfrac{1}{4}\right)$$

$$= \frac{4}{16} + \frac{4}{16} + \frac{4}{16} + \frac{4}{16} + \frac{16}{16} + \frac{7}{16} + \frac{4}{16} + \frac{7}{16}$$
$$+ \frac{28}{16} + \frac{10}{16} + \frac{4}{16} + \frac{10}{16} + \frac{40}{16} + \frac{13}{16} + \frac{4}{16} + \frac{13}{16}$$

$$= \frac{172}{16} = \frac{43}{4} = 10.75\ u^3$$

2) Integrales dobles

a) $$\int_0^2 \underbrace{\int_1^2 (x-3y^2)\, dy}_{\phi(x)}\, dx$$

$$\phi(x) = \int_1^2 (x-3y^2)\,dy = \int_1^2 x\, dy - \int_1^2 3y^2\, dy$$

$$= x\int_1^2 dy - 3\int_1^2 y^2\, dy = x\left[y\Big|_1^2\right] - 3\left[\frac{y^3}{3}\Big|_1^2\right]$$

$$= x[2-1] - 3\left[\frac{8}{3} - \frac{1}{3}\right] = x(1) - 3\left(\frac{7}{3}\right) = x-7$$

$$\Rightarrow \int_0^2 x-7\,dx = \int_0^2 x\,dx - \int_0^2 7\,dx = \int_0^2 x\,dx - 7\int_0^2 dx$$

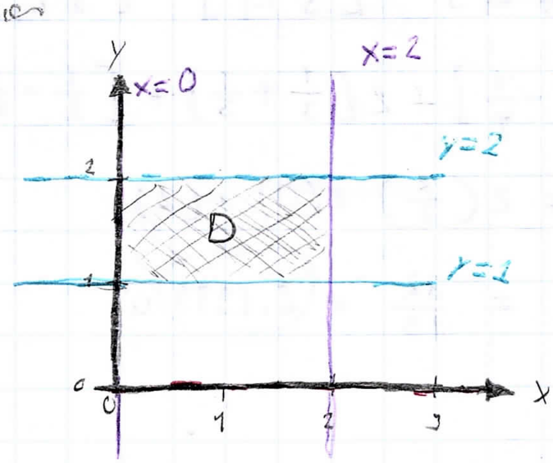
$$= \left[\frac{x^2}{2}\Big|_0^2\right] - 7\left[x\Big|_0^2\right] = \left[\frac{4}{2} - \frac{0}{2}\right] - 7[2-0]$$

$$= 2 - 7(2) = 2-14 = -12\ u^3$$

-Region de integracion

$x=0$ $y=1$

$x=2$ $y=2$



b) $$\int_{-1}^{1}\underbrace{\int_{2x^2}^{1+x^2}(x+2y)\,dy\,dx}_{\phi(x)}$$

$$\phi(x) = \int_{2x^2}^{1+x^2} x+2y\,dy = \int_{2x^2}^{1+x^2} x\,dy + \int_{2x^2}^{1+x^2} 2y\,dy = x\int_{2x^2}^{1+x^2} dy + 2\int_{2x^2}^{1+x^2} y\,dy$$

$$= x\left[y\Big|_{2x^2}^{1+x^2}\right] + 2\left[\frac{y^2}{2}\Big|_{2x^2}^{1+x^2}\right] = x\left[1+x^2-2x^2\right] + 2\left[\frac{(1+x^2)^2}{2} - \frac{(2x^2)^2}{2}\right]$$

$$= x\left[1-x^2\right] + 2\left[\frac{-3x^4+2x^2+1}{2}\right] = x - x^3 - 3x^4 + 2x^2 + 1$$

$$= -3x^4 - x^3 + 2x^2 + x + 1$$

$$\Rightarrow \int_{-1}^{1} -3x^4 - x^3 + 2x^2 + x + 1\,dx = \int_{-1}^{1} -3x^4\,dx - \int_{-1}^{1} x^3\,dx + \int_{-1}^{1} 2x^2\,dx + \int_{-1}^{1} x\,dx + \int_{-1}^{1} dx$$

$$= -3\int_{-1}^{1} x^4\,dx - \int_{-1}^{1} x^3\,dx + 2\int_{-1}^{1} x^2\,dx + \int_{-1}^{1} x\,dx + \int_{-1}^{1} dx$$

$$= -3\left[\frac{x^5}{5}\Big|_{-1}^{1}\right] - \left[\frac{x^4}{4}\Big|_{-1}^{1}\right] + 2\left[\frac{x^3}{3}\Big|_{-1}^{1}\right] + \left[\frac{x^2}{2}\Big|_{-1}^{1}\right] + \left[x\Big|_{-1}^{1}\right]$$

$$= -3\left[\frac{1}{5}+\frac{1}{5}\right] - \left[\frac{1}{4}-\frac{1}{4}\right] + 2\left[\frac{1}{3}+\frac{1}{3}\right] + \left[\frac{1}{2}-\frac{1}{2}\right] + \left[1+1\right]$$

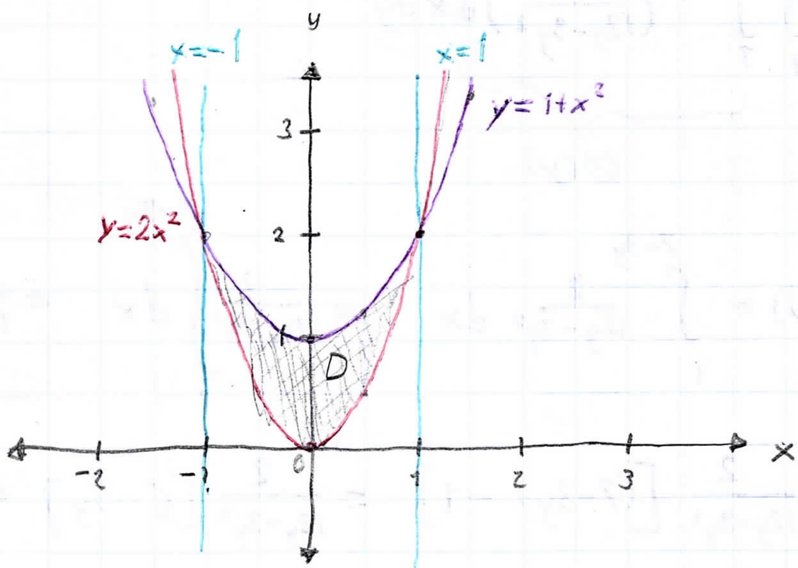
$$= -3\left(\frac{2}{5}\right) - (0) + 2\left(\frac{2}{3}\right) + (0) + (2)$$

$$= -\frac{6}{5} + \frac{4}{3} + 2 = \frac{32}{15} = 2.1333\,u^3$$

- Region de integración

$x = 1 \qquad y = 1 + x^2$

$x = -1 \qquad y = 2x^2$



c) $$\underbrace{\int_1^2 \int_1^{7-3y}}_{\varnothing(y)} \left(\frac{2}{12y-3y^2}\right) dx\, dy$$

$$\varnothing(y) = \int_1^{7-3y} \frac{2}{12y-3y^2}\, dx = \frac{2}{12y-3y^2} \int_1^{7-3y} dx = \frac{2}{12y-3y^2} \left[x \Big|_1^{7-3y} \right]$$

$$= \frac{2}{12y-3y^2} \left[(7-3y) - 1 \right] = \frac{2}{12y-3y^2} \left[6-3y \right] = \frac{12-6y}{12y-3y^2}$$

$$= \frac{(3)\ 4-2y}{(3)\ 4y-y^2} = \frac{4-2y}{4y-y^2}$$

$$\Rightarrow \int_1^2 \frac{4-2y}{4y-y^2}\, dy$$ sustitucion $u = 4y-y^2$, $du = 4-2y\ dy$

$$= \int_1^2 \frac{1}{u}\, du$$ por tablas de integrales? $= \left[\ln(|u|) \Big|_1^2 \right]$

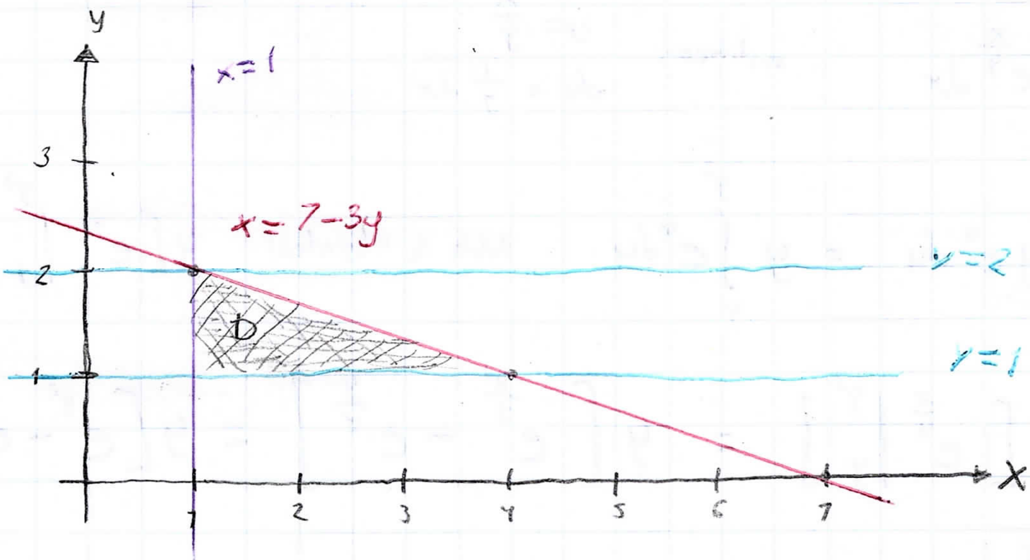
$$= \left[\ln(|4y-y^2|) \Big|_1^2 \right] = \left[\ln(|8-4|) - \ln(|4-1|) \right]$$

$= \left[\ln(4) - \ln(3) \right]$ propiedades de los $= \ln\left(\frac{4}{3}\right) \approx 0.287\, u^3$

- Region de integracion

$y = 2$ $\qquad$ $x = 7-3y$

$y = 1$ $\qquad$ $x = 1$



d) $\underbrace{\int_1^2 \int_y^{y^3} e^{\frac{x}{y}}\,dx\,dy}_{\varnothing(y)}$

$$\varnothing(y) = \int_y^{y^3} e^{\frac{x}{y}}\,dx \qquad \text{sustitucion} \quad u = \frac{x}{y} \quad du = \frac{1}{y}\,dx$$

$$= \int_y^{y^3} y e^u\,du = y \int_y^{y^3} e^u\,du \qquad \text{tabla de integrales!} \quad y\left[e^u \Big|_y^{y^3}\right]$$

$$= y\left[e^{\frac{x}{y}} \Big|_y^{y^3}\right] = y\left[e^{\frac{y^3}{y}} - e^{\frac{y}{y}}\right] = y\left[e^{y^2} - e\right]$$

$$= ye^{y^2} - ey$$

$$\Rightarrow \int_1^2 ye^{y^2} - ey\,dy = \int_1^2 ye^{y^2}\,dy - \int_1^2 ey\,dy$$

$$\int_1^2 ye^{y^2}\,dy \qquad \text{sustitucion} \quad u = y^2 \quad du = 2y\,dy$$

$$\Rightarrow \frac{1}{2}\int_1^2 e^u\,du = \frac{1}{2}\left[e^u \Big|_1^2\right]$$

$$= \frac{1}{2}\left[e^{y^2} \Big|_1^2\right]$$

$$= \frac{1}{2}\left[e^4 - e^1\right]$$

$$= \frac{e^4 - e}{2}$$

$$\int_1^2 ey\,dy = e\int_1^2 y\,dy$$

$$= e\left[\frac{y^2}{2}\Big|_1^2\right] = e\left[\frac{4}{2} - \frac{1}{2}\right]$$

$$= e\left(\frac{3}{2}\right)$$

juntando todo

$$\int_1^2 y e^{y^2} dy - \int_1^2 e y \, dy = \left[\frac{e^4 - e}{2} \right] - \left[\frac{3e}{2} \right]$$

$$= \frac{e^4 - e - 3e}{2} = \frac{e^4 - 4e}{2} \approx 21.862 \ u^3$$

-Region de integracion

$y = 2$ $\qquad x = y^3$

$y = 1$ $\qquad x = y$