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Matématicas para las Ciencias Aplicadas III
Tarea 4
Fecha de entrega: 26 de octubre de 2022

1.Usando los cambios de variable propuestos encontrar los valores de las integrales.

✓ a) El cambio de variable es $x = \frac{u}{v}$ y $y = uv$. La integral a resolver es $\int\int \sqrt{xy^3}dxdy$. Donde D es la región limitada por $xy = 1$, $xy = 9$, $y = x$ y $y = 4x$.

✓ b)El cambio de variable $x = lnu$ y $y = v$, $\int_0^1 \int_0^1 e^x cos(e^x)\, dx\, dy$.

c) El cambio de variable es $u = x + y$ y $y = uv$. Probar que $\int_0^1 \int_0^{1-x} (e^{\frac{y}{x+y}})\, dy\, dx = \frac{e-1}{2}$

2. Proponer cambios de variables para resolver las siguientes integrales:

✓ a)$\int\int_D (x-y)^2\, dx\, dy$ donde D es el paralelogramo con vértice en (0,0), (1,1), (2,0) y (1,-1).

✓ b) $\int_1^2 \int_{x+2}^{x+3} \frac{dydx}{\sqrt{xy-x^2}}$

✓ 3. Calcular el área de la superficie de la esfera $x^2 + y^2 + z^2 = 1$ que yace por encima de la elipse $x^2 + (\frac{y}{a})^2 \leq 1$ (a es una constante tal que $0 \leq a \leq 1$)

4. Demostrar que cada una de las siguientes funciones tiene la propiedad de que el volumen por debajo de su gráfica es igual al área de la superficie de la gráfica para cualquier dominio.

✓ a) $f(x,y) = 1$

✓ b) $f(x,y) = cosh(\sqrt{x^2 + y^2} + c)$, c es constante

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1) Usando los cambios de variables propuestos encontrar los valores de las integrales

a) El cambio de variable es $x = \frac{u}{v}$ y $y = uv$

La integral es $\iint \sqrt{xy^3}\, dx dy$ donde D es la region limitada

por $xy = 1$, $xy = 9$, $y = x$ y $y = 4x$ → notemos que son dos dominios, uno en el cuadrante 1 y el otro en el 3

veamos los limites

$x = \frac{u}{v}$ $\qquad$ $y = uv$

$xy = \frac{u}{v} \cdot uv = u^2$

$xy = 1 \Rightarrow u^2 = 1$ $\qquad$ $y = x \Rightarrow uv = \frac{u}{v} \Rightarrow v^2 = 1$

$xy = 9 \Rightarrow u^2 = 9$ $\qquad$ $y = 4x \Rightarrow uv = 4\frac{u}{v} \Rightarrow v^2 = 4$

entonces $\begin{cases} u = \pm\sqrt{1} = \pm 1 \\ u = \pm\sqrt{9} = \pm 3 \end{cases}$ $\quad$ $\begin{cases} v = \pm\sqrt{1} = \pm 1 \\ v = \pm\sqrt{4} = \pm 2 \end{cases}$

como en el dominio original tenemos dos areas, cuadrante 1 y 3, entonces u y v deben ser positivos o negativos al mismo tiempo, tomemos positivos y multipliquemos por 2 la integral $\begin{cases} 1 \leq u \leq 3 \\ 1 \leq v \leq 2 \end{cases}$

ahora veamos la funcion

$f(x,y) = \sqrt{xy^3} \Rightarrow \sqrt{\frac{u}{v}(uv)^3} = \sqrt{\frac{u}{v} u^3 v^3} = \sqrt{u^4 v^2} = \sqrt{u^4}\sqrt{v^2}$

$= u^2 v = F(u,v)$

x y y dependen de u y v , entonces usamos el jacobiano

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \left\|\begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{matrix}\right\| = \left\|\begin{matrix} \frac{\partial}{\partial u}\left(\frac{u}{v}\right) & \frac{\partial}{\partial v}\left(\frac{u}{v}\right) \\ \frac{\partial}{\partial u}(uv) & \frac{\partial}{\partial v}(uv) \end{matrix}\right\|$$

$$= \left\|\begin{matrix} \frac{1}{v} & -\frac{u}{v^2} \\ v & u \end{matrix}\right\| = \left|\left(\frac{1}{v}\,u\right) - \left(v\,\frac{-u}{v^2}\right)\right|$$

$$= \left|\frac{u}{v} - \left(-\frac{u}{v}\right)\right| = \left|\frac{u}{v} + \frac{u}{v}\right| = \left|2\frac{u}{v}\right| = 2\frac{u}{v}$$

entonces limites $\begin{cases} 1 \leq u \leq 3 \\ 1 \leq v \leq 2 \end{cases}$

jacobiano $\left\{ \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = 2\frac{u}{v} \right.$

funcion $\left\{ F(u,v) = u^2 v \right.$

$$2\int_1^2\int_1^3 u^2 v\, 2\frac{u}{v}\, du\, dv$$

$$\phi(v) = \int_1^3 u^2 v\, 2\frac{u}{v}\, du = \int_1^3 2u^3\, du = 2\int_1^3 u^3\, du = 2\left[\frac{u^4}{4}\Big|_1^3\right]$$

$$= 2\left[\frac{3^4}{4} - \frac{1^4}{4}\right] = 2\left[\frac{81}{4} - \frac{1}{4}\right] = 2\left[\frac{80}{4}\right] = 2[20] = 40$$

$$\Rightarrow 2\int_1^2 \phi(v)\,dv = 2\int_1^2 40\,dv = 80\int_1^2 dv = 80\left[v\Big|_1^2\right] =$$

$$80\,[2-1] = 80\,[1] = 80$$

b) El cambio de variable $x = \ln(u)$ y $y = v$

$$\int_0^1 \int_0^1 e^x \cos(e^x)\, dx\, dy$$

veamos los limite

$x=0 \Rightarrow \ln(u)=0 \Rightarrow u=1$ $\qquad y=0 \Rightarrow v=0$

$x=1 \Rightarrow \ln(u)=1 \Rightarrow u=e$ $\qquad y=1 \Rightarrow v=1$

entons $\begin{cases} u=1 \\ u=e \end{cases}$ $\begin{cases} y=0 \\ y=1 \end{cases}$

$$\begin{cases} 1 \le u \le e \\ 0 \le v \le 1 \end{cases}$$

ahora veamos la funcion

$$f(x,y) = e^x \cos(e^x) \Rightarrow e^{\ln(u)} \cos(e^{\ln(u)}) = u\cos(u) = F(u,v)$$

x y y dependen de u y v, entons usamos

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right|$$

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \left\| \begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{matrix} \right\| = \left\| \begin{matrix} \frac{\partial}{\partial u}(\ln(u)) & \frac{\partial}{\partial v}(\ln(u)) \\ \frac{\partial}{\partial u}(v) & \frac{\partial}{\partial v}(v) \end{matrix} \right\|$$

$$= \left\| \begin{matrix} \frac{1}{u} & 0 \\ 0 & 1 \end{matrix} \right\| = \left| \left(\frac{1}{u} \cdot 1\right) - (0 \cdot 0) \right| = \left| \frac{1}{u} - 0 \right| = \frac{|1|}{|u|} = \frac{1}{u}$$

entonces

limites $\begin{cases} 1 \leq u \leq e \\ 0 \leq v \leq 1 \end{cases}$

jacobiano $\left\{ \left| \dfrac{\partial(x,y)}{\partial(u,v)} \right| = \dfrac{1}{u} \right.$

funcion $\{ F(u,v) = u\cos(u)$

$$\int_0^1 \int_1^e u\cos(u)\frac{1}{u}\,du\,dv$$

$$\phi(v) = \int_1^e u\cos(u)\frac{1}{u}\,du = \int_1^e \cos(u)\,du = \operatorname{sen}(u)\Big|_1^e$$

$$= [\operatorname{sen}(e) - \operatorname{sen}(1)] = \operatorname{sen}(e) - \operatorname{sen}(1)$$

$$\Rightarrow \int_0^1 \phi(v)\,dv = \int_0^1 \operatorname{sen}(e) - \operatorname{sen}(1)\,dv = \operatorname{sen}(e) - \operatorname{sen}(1)\int_0^1 dv$$

$$= \operatorname{sen}(e) - \operatorname{sen}(1)\left[v\Big|_0^1\right] = \operatorname{sen}(e) - \operatorname{sen}(1)[1-0]$$

$$= \operatorname{sen}(e) - \operatorname{sen}(1)[1] = \operatorname{sen}(e) - \operatorname{sen}(1) \approx -0.43$$

c) El cambio de variable es $u=x+y$ y $y=uv$

probar que $\displaystyle\int_0^1\int_0^{1-x} (e^{\frac{y}{x+y}})\,dy\,dx = \frac{e-1}{2}$

$u=x+y \Rightarrow u=x+uv$

$x=u-uv$

veamos los limites, veamos que $x=u(1-v)=u-uv$

tenemos que

$y=0 \Rightarrow y=0 \Rightarrow uv=0 \Rightarrow v=0$, $u=0$

$y=1-x \Rightarrow y+x=1 \Rightarrow u=1 \Rightarrow u=1$

$x=0 \Rightarrow u-uv=0 \Rightarrow$ usemos $u=1 \Rightarrow 1-v=0 \Rightarrow v=1$

$x=1 \Rightarrow u-uv=1 \Rightarrow$ usemos $u=1 \Rightarrow 1-v=1 \Rightarrow v=0$

(con $u=0$ se indefine)

entonces $\begin{cases} 0\le u\le 1 \\ 0\le v\le 1\end{cases}$

ahora veamos la funcion

$f(x,y)=e^{\frac{y}{x+y}} \Rightarrow e^{v}=F(u,v)$

u y v dependen de x y y (usando $\begin{cases} u=x+y \\ v=\frac{y}{x+y}\end{cases}$), entonces

usamos $\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|}$

$$\left|\frac{\partial(u,v)}{\partial(x,y)}\right| = \left\|\begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{matrix}\right\| = \left\|\begin{matrix} \frac{\partial}{\partial x}(x+y) & \frac{\partial}{\partial y}(x+y) \\ \frac{\partial}{\partial x}\left(\frac{y}{x+y}\right) & \frac{\partial}{\partial y}\left(\frac{y}{x+y}\right)\end{matrix}\right\|$$

$$= \left| \begin{vmatrix} 1 & 1 \\ \frac{x}{(x+y)^2} & \frac{-y}{(x+y)^2} \end{vmatrix} \right| = \left| \left(1 \cdot \frac{-y}{(x+y)^2}\right) - \left(1 \cdot \frac{x}{(x+y)^2}\right) \right| = \left| \frac{-y}{(x+y)^2} - \frac{x}{(x+y)^2} \right|$$

$$= \left| -\frac{y+x}{(x+y)^2} \right| = \left| -\frac{1}{x+y} \right| = \frac{1}{x+y} = \frac{1}{4} \quad \Rightarrow \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \frac{1}{\left| \frac{\partial(u,v)}{\partial(x,y)} \right|} =$$

$$\frac{1}{\frac{1}{4}} = 4$$

entonces

$$\text{limites} \begin{cases} 0 \le u \le 1 \\ 0 \le v \le 1 \end{cases}$$

$$\text{jacobiano} \left\{ \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = 4 \right.$$

$$\text{funcion} \left\{ F(u,v) = e^v \right.$$

$$\int_0^1 \int_0^1 e^v u \, dv \, du$$

$$\phi(u) = \int_0^1 e^v u \, dv = u \int_0^1 e^v dv = u \left[e^v \Big|_0^1 \right] =$$

$$u[e^1 - e^0] = ue - u1 = ue - u = u(e-1)$$

$$\Rightarrow \int_0^1 \phi(u) \, du = \int_0^1 u(e-1) \, du = (e-1) \int_0^1 u \, du$$

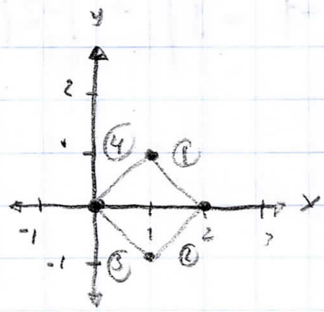
$$= (e-1)\left[\frac{u^2}{2} \Big|_0^1 \right] = (e-1)\left[\frac{1^2}{2} - \frac{0^2}{2} \right] = (e-1)\left[\frac{1}{2}\right]$$

$= \frac{e-1}{2}$ por lo tanto si esta bien

2) Proponer cambios de variables para resolver las siguiente integrales

a) $\iint_D (x-y)^2\, dx\, dy$ con D es el paralelogramo con vertices

$$(0,0),\ (1,1),\ (2,0)\ y\ (1,-1)$$



① $x = 2-y \Rightarrow x+y=2$

② $x = 2+y \Rightarrow x-y=2$

③ $x = -y \Rightarrow x+y=0$

④ $x = y \Rightarrow x-y=0$

propongamos

$$u = x+y \quad y \quad v = x-y$$

entonces los limites son

$$\begin{cases} x+y=2 = u \\ x+y=0 = u \end{cases} \qquad \begin{cases} x-y=2 \\ x-y=0 \end{cases}$$

$$\begin{cases} 0 \le u \le 2 \\ 0 \le v \le 2 \end{cases}$$

ahora veamos la funcion

$$f(x,y) = (x-y)^2 \Rightarrow (v)^2 = v^2 = F(u,v)$$

u y v dependen de x y Y entonces $\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|}$

$$\left|\frac{\partial(u,v)}{\partial(x,y)}\right| = \left\|\begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{matrix}\right\| = \left\|\begin{matrix} \frac{\partial}{\partial x}(x+y) & \frac{\partial}{\partial y}(x+y) \\ \frac{\partial}{\partial x}(x-y) & \frac{\partial}{\partial y}(x-y)\end{matrix}\right\|$$

$$= \left\|\begin{matrix} 1 & 1 \\ 1 & -1\end{matrix}\right\| = |(1\cdot -1)-(1\cdot 1)| = |-1-1| = |-2| = 2$$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|} = \frac{1}{2}$$

entonces limites $\begin{cases} 0 \le u \le 2 \\ 0 \le v \le 2 \end{cases}$

Jacobiano $\left\{ \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{2} \right.$

funcion $\left\{ F(u,v) = v^2 \right.$

$$\int_0^2\int_0^2 v^2 \frac{1}{2}\, dv\, du$$

$$\phi(u) = \int_0^2 v^2 \frac{1}{2}\, dv = \frac{1}{2}\int_0^2 v^2\, dv = \frac{1}{2}\left[\frac{v^3}{3}\Big|_0^2\right] = \frac{1}{2}\left[\frac{2^3}{3} - \frac{0^3}{3}\right]$$

$$= \frac{1}{2}\left[\frac{8}{3}\right] = \frac{8}{6} = \frac{4}{3}$$

$$\Rightarrow \int_0^2 \phi(u)\, du = \int_0^2 \frac{4}{3}\, du = \frac{4}{3}\int_0^2 du = \frac{4}{3}\left[u\Big|_0^2\right] = \frac{4}{3}[2-0]$$

$$= \frac{4}{3}[2] = \frac{8}{3} \approx 2.666$$

b) $$\int_1^2 \int_{x+2}^{x+3} \frac{1}{\sqrt{xy-x^2}} \, dy\,dx$$

veamos los cambios de variables

$$\begin{cases} y = x+2 \Rightarrow y-x = 2 \\ y = x+3 \Rightarrow y-x = 3 \end{cases} \qquad \begin{cases} x = 2 \\ x = 1 \end{cases}$$

proposongamos

$$u = x \qquad v = y-x$$

entonces los limites son

$$\begin{cases} 1 \leq u \leq 2 \\ 2 \leq v \leq 3 \end{cases}$$

ahora veamos la funcion

$$f(x,y) = \frac{1}{\sqrt{xy-x^2}} = \frac{1}{\sqrt{x(y-x)}} \Rightarrow \frac{1}{\sqrt{uv}} = F(u,v)$$

u y v dependen de x y y entonces $\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|}$

$$\left|\frac{\partial(u,v)}{\partial(x,y)}\right| = \left\|\begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{matrix}\right\| = \left\|\begin{matrix} \frac{\partial}{\partial x}(x) & \frac{\partial}{\partial y}(x) \\ \frac{\partial}{\partial x}(y-x) & \frac{\partial}{\partial y}(y-x) \end{matrix}\right\|$$

$$= \left\|\begin{matrix} 1 & 0 \\ -1 & 1 \end{matrix}\right\| = |(1 \cdot 1) - (-1 \cdot 0)| = |1-0| = |1| = 1$$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|} = \frac{1}{1} = 1$$

entonces límites $\begin{cases} 1 \leq u \leq 2 \\ 2 \leq v \leq 3 \end{cases}$

Jacobiano $\left\{ \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = 1 \right.$

función $\left\{ F(u,v) = \frac{1}{\sqrt{uv}} \right.$

$$\int_1^2 \int_2^3 \frac{1}{\sqrt{uv}} \, 1 \, dv \, du$$

$$\phi(u) = \int_2^3 \frac{1}{\sqrt{uv}} dv = \int_2^3 \frac{1}{\sqrt{u}\sqrt{v}} dv = \frac{1}{\sqrt{u}} \int_2^3 \frac{1}{\sqrt{v}} dv = \frac{1}{\sqrt{u}} \int_2^3 \frac{1}{v^{\frac{1}{2}}} dv$$

$$= \frac{1}{\sqrt{u}} \left[2\sqrt{v} \Big|_2^3 \right] = \frac{1}{\sqrt{u}} \left[2\sqrt{3} - 2\sqrt{2} \right] = \frac{2\sqrt{3} - 2\sqrt{2}}{\sqrt{u}} = \frac{2(\sqrt{3} - \sqrt{2})}{\sqrt{u}}$$

$$\Rightarrow \int_1^2 \phi(u) \, du = \int_1^2 \frac{2(\sqrt{3} - \sqrt{2})}{\sqrt{u}} du = 2(\sqrt{3} - \sqrt{2}) \int_1^2 \frac{1}{\sqrt{u}} du$$

$$= 2(\sqrt{3} - \sqrt{2}) \left[2\sqrt{u} \Big|_1^2 \right] = 2(\sqrt{3} - \sqrt{2}) \left[2\sqrt{2} - 2\sqrt{1} \right]$$

$$= 2(\sqrt{3} - \sqrt{2}) \left[2\sqrt{2} - 2 \right] = (\sqrt{3} - \sqrt{2})(4\sqrt{2} - 4)$$

$$= 4\sqrt{6} - 4\sqrt{3} - 4\sqrt{4} + 4\sqrt{2} = 4\sqrt{6} - 4\sqrt{3} - 8 + 4\sqrt{2} \approx 0.526$$

3) Calcula el area de la superficie de la esfera $x^2+y^2+z^2=1$

que yace por encima de la elipse $x^2+\left(\frac{y}{a}\right)^2 \leq 1$ (a una constante tal que $0 \leq a \leq 1$)

tenemos funcion implicita

$$x^2+y^2+z^2-1=0$$

$$\sigma = \iint_D \frac{\sqrt{\left(\frac{\partial F}{\partial x}\right)^2+\left(\frac{\partial F}{\partial y}\right)^2+\left(\frac{\partial F}{\partial z}\right)^2}}{\frac{\partial F}{\partial z}}\, dx\, dy$$

$$\left.\begin{aligned} \frac{\partial F}{\partial x} &= \frac{\partial}{\partial x}(x^2+y^2+z^2-1) = 2x \\ \frac{\partial F}{\partial y} &= \frac{\partial}{\partial y}(x^2+y^2+z^2-1) = 2y \\ \frac{\partial F}{\partial z} &= \frac{\partial}{\partial z}(x^2+y^2+z^2-1) = 2z \end{aligned}\right\} \Rightarrow \sqrt{(2x)^2+(2y)^2+(2z)^2} = 2\sqrt{x^2+y^2+z^2}$$

$z = \pm\sqrt{1-x^2-y^2}$

como es el casquete de arriba tomamos +

$$\sigma = \iint_D \frac{2\sqrt{x^2+y^2+z^2}}{2z}\, dx\, dy = \iint_D \frac{\sqrt{x^2+y^2+z^2}}{z}\, dx\, dy$$

$$\sigma = \iint_D \frac{\sqrt{x^2+y^2+\left(\sqrt{1-x^2-y^2}\right)^2}}{\sqrt{1-x^2-y^2}}\, dx\, dy = \iint_D \frac{\sqrt{x^2+y^2+1-x^2-y^2}}{\sqrt{1-x^2-y^2}}\, dx\, dy$$

$$= \iint_D \frac{\sqrt{1}}{\sqrt{1-x^2-y^2}}\, dx\, dy = \iint_D \frac{1}{\sqrt{1-x^2-y^2}}\, dx\, dy$$

ahora veamos el dominio D, no usaremos polares, para no morir

limites

imagen al final

$$x^2+\left(\frac{y}{a}\right)^2=1$$

$$y=\pm a\sqrt{1-x^2}$$

$$\begin{cases} -1\le x\le 1 \\ -a\sqrt{1-x^2}\le y\le a\sqrt{1-x^2} \end{cases}$$

entonces

$$\sigma=\int_{-1}^{1}\int_{-a\sqrt{1-x^2}}^{a\sqrt{1-x^2}}\frac{1}{\sqrt{1-x^2-y^2}}\,dy\,dx$$

$$\phi(x)=\int_{-a\sqrt{1-x^2}}^{a\sqrt{1-x^2}}\frac{1}{\sqrt{1-x^2-y^2}}\,dy=\int_{-a\sqrt{1-x^2}}^{a\sqrt{1-x^2}}\frac{1}{\sqrt{\sqrt{1-x^2}^2-y^2}}\,dy$$

ahora usando una tabla de integrales $\displaystyle\int\frac{1}{\sqrt{a^2-u^2}}du=\operatorname{sen}^{-1}\left(\frac{u}{a}\right)+C$

$$=\operatorname{sen}^{-1}\left(\frac{y}{\sqrt{1-x^2}}\right)\Bigg|_{-a\sqrt{1-x^2}}^{a\sqrt{1-x^2}}=\left[\operatorname{sen}^{-1}\left(\frac{a\cancel{\sqrt{1-x^2}}}{\cancel{\sqrt{1-x^2}}}\right)-\operatorname{sen}^{-1}\left(\frac{-a\cancel{\sqrt{1-x^2}}}{\cancel{\sqrt{1-x^2}}}\right)\right]$$

$$=\left[\operatorname{sen}^{-1}(a)-\operatorname{sen}^{-1}(-a)\right]=\left[\operatorname{sen}^{-1}(a)-(-\operatorname{sen}^{-1}(a))\right]$$

$$=\left[\operatorname{sen}^{-1}(a)+\operatorname{sen}^{-1}(a)\right]=2\operatorname{sen}^{-1}(a)$$

$$\Rightarrow\int_{-1}^{1}\phi(x)\,dx=\int_{-1}^{1}2\operatorname{sen}^{-1}(a)=2\operatorname{sen}^{-1}(a)\int_{-1}^{1}dx$$

$= 2\,sen^{-1}(a)\left[x\Big|_{-1}^{1}\right] = 2\,sen^{-1}(a)[1-(-1)] = 2\,sen^{-1}(a)[2]$

$= 4\,sen^{-1}(a)$

4) Demuestra que cada una de las siguientes funciones tienen la propiedad de que el volumen por debajo de su grafica es igual al área de la superficie de la grafica para cualquier dominio

a) $f(x,y)=1$ veamos cual es el area de superficie

tenemos funcion explicita

$$\sigma = \iint_D \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1}\; dx\,dy$$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}(1) = 0 \\ \frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}(1) = 0 \end{aligned}\right\} \Rightarrow \sqrt{0^2+0^2+1} = \sqrt{1} = 1$$

$$\sigma = \iint_D 1\, dx\,dy$$ area de superficie

ahora veamos que el volumen por debajo es

$$V = \iint_D f(x,y)\, dx\,dy = \iint_D 1\, dx\,dy$$

entonces $\sigma = \iint_D 1\, dx\,dy = V$ por lo tanto si se cumple

b) $f(x,y) = \cosh(\sqrt{x^2+y^2}+c)$, c es una constante

tenemos función explícita

$$\sigma = \iint_D \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1}\, dx\,dy$$

$t = \sqrt{x^2+y^2}+c$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\cosh(\sqrt{x^2+y^2}+c)) = \frac{\partial}{\partial t}\cosh(t) \cdot \frac{\partial}{\partial x} t$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c) \cdot \frac{\partial}{\partial x} \sqrt{x^2+y^2}+c$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c) \cdot \left[\frac{\partial}{\partial x}\sqrt{x^2+y^2} + \frac{\partial}{\partial x} c\right]$$

$t = x^2+y^2$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c)\left[\left[\frac{\partial}{\partial t}\sqrt{t} \cdot \frac{\partial}{\partial x} t\right] + 0\right]$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c)\left[\frac{1}{2\sqrt{x^2+y^2}} \cdot 2x\right]$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c)\left(\frac{x}{\sqrt{x^2+y^2}}\right) = \frac{x\operatorname{senh}(\sqrt{x^2+y^2}+c)}{\sqrt{x^2+y^2}}$$

$t = \sqrt{x^2+y^2}+c$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\cosh\sqrt{x^2+y^2}+c)) = \frac{\partial}{\partial t}\cosh(t) \cdot \frac{\partial}{\partial y} t$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c) \cdot \frac{\partial}{\partial y}\sqrt{x^2+y^2}+c$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c)\left[\frac{\partial}{\partial y}\sqrt{x^2+y^2} + \frac{\partial}{\partial y} c\right]$$

$t = x^2+y^2$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c)\left[\left(\frac{\partial}{\partial t}\sqrt{t} \cdot \frac{\partial}{\partial y} t\right) + 0\right]$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c)\left[\frac{1}{2\sqrt{x^2+y^2}} \cdot 2y\right]$$

$$= \operatorname{senh}(\sqrt{x^2+y^2}+c)\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{y\operatorname{senh}(\sqrt{x^2+y^2}+c)}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \sqrt{\left(\frac{x\,\mathrm{senh}(\sqrt{x^2+y^2}+c)}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y\,\mathrm{senh}(\sqrt{x^2+y^2}+c)}{\sqrt{x^2+y^2}}\right)^2 + 1}$$

$$= \sqrt{\frac{x^2\,\mathrm{senh}^2(\sqrt{x^2+y^2}+c)}{x^2+y^2} + \frac{y^2\,\mathrm{senh}^2(\sqrt{x^2+y^2}+c)}{x^2+y^2} + 1}$$

$$= \sqrt{\frac{x^2\,\mathrm{senh}^2(\sqrt{x^2+y^2}+c) + y^2\,\mathrm{senh}^2(\sqrt{x^2+y^2}+c)}{x^2+y^2} + 1}$$

$$= \sqrt{\frac{\cancel{(x^2+y^2)}(\mathrm{senh}^2(\sqrt{x^2+y^2}+c))}{\cancel{x^2+y^2}} + 1}$$

$$= \sqrt{\mathrm{senh}^2(\sqrt{x^2+y^2}+c) + 1}$$

identidad trigonométrica

$$\left(\cosh^2(x) - \mathrm{senh}^2(x) = 1\right)$$

$$= \sqrt{\cosh^2(\sqrt{x^2+y^2}+c)}$$

$$= \cosh(\sqrt{x^2+y^2}+c)$$

$$\sigma = \iint_D \cosh(\sqrt{x^2+y^2}+c)\,dx\,dy$$ área de superficie

ahora veamos el volumen por debajo es

$$V = \iint_D f(x,y)\, dx\,dy = \iint_D \cosh\left(\sqrt{x^2+y^2} + C\right) dx\,dy$$

entonces $\sigma = \iint_D \cosh\left(\sqrt{x^2+y^2} + C\right) dx\,dy = V$ por lo tanto sí se cumple