García Ponce José Camilo

Matématicas para las Ciencias Aplicadas III
Tarea 6
Fecha de entrega: 2 de diciembre de 2022

1. Dibujar en el plano $x - y$ las siguientes funciones paramétricas.

a) $\vec{F}(t) = sin(2t)\hat{i} + cos(t)\hat{j},\ 0 \leq t \leq \pi.$

b) $\vec{F}(t) = cos^2(t)\hat{i} + sin^2(t)\hat{j},\ 0 \leq t \leq 2\pi.$

2. Calcular las integrales de línea de los campos vectoriales a lo largo de las trayectorias dadas.

a) $\vec{F}(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$. Trayectoria: $\vec{S}(t) = sin(t)\hat{i} + cos(t)\hat{j} + t\ \hat{k},\ 0 \leq t \leq 2\pi$

b) $\vec{F}(x, y, z) = \frac{1}{z^2+1}\hat{i} + x(1 + y^2)\hat{j} + e^y\hat{k}$. Trayectoria: $\vec{s}(t) = (1 + t^2)^2\ \hat{i} +\ \hat{j} + t\ \hat{k},\ 0 \leq t \leq 1$

c) $\vec{F} = x^3\hat{i} + 3zy^2\hat{j} - x^2y\hat{k}$ a lo largo de la recta que parte del punto (3,2,1) al punto (0,0,0).

3. Resolver las siguientes integrales de línea.

a) $\int_s (y^4 + x^3)dx + 2x^6dy$, donde s es la trayectoria cerrada definida por las rectas $x = 0, x = 1, y = 0$ y $y = 1$.

b) Sea $\oint = xy^2\ \hat{i} - yx^2\ \hat{j}$ y la trayectoria s es la elipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

c) $\int_s (x^5 - 2xy^3)dx - 3x^2y^2dy$, donde s está parametrizada por $(t^8, t^{10}), 0 \leq t \leq 1$.

4. Hallar la longitud de arco de cada una de las siguientes curvas.

a) $x = \sqrt{t},\ y = \frac{t^2}{8} + \frac{1}{4t}$, desde t = 1 hasta t=3.

b) $x = cos^4t,\ y = sin^4t,\ 0 \leq t \leq \pi/2$ Hint: $y = (1 - \sqrt{x})^2,\ 0 \leq x \leq 1$

c) $x = a(cost + t\ sint),\ y = a(sint - t\ cost),\ 0 \leq t \leq 2\pi$

García Ponce José Camilo

1) Dibuja en el plano Oxy las siguientes funciones paramétricas

a) $\vec{F}(t) = \sin(2t)\hat{\imath} + \cos(t)\hat{\jmath}$, $0 \leq t \leq \pi$

veamos algunos valores

si $t=0$, entonces $\vec{F}(0) = \sin(0)\hat{\imath} + \cos(0)\hat{\jmath} = (0, 1)$

$t = \pi/6$, $\vec{F}(\pi/6) = \sin(\pi/3)\hat{\imath} + \cos(\pi/6)\hat{\jmath} = \left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$

$t = \frac{2\pi}{6}$, $\vec{F}(\frac{2\pi}{6}) = \sin(\frac{2\pi}{3})\hat{\imath} + \cos(\frac{2\pi}{6})\hat{\jmath} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

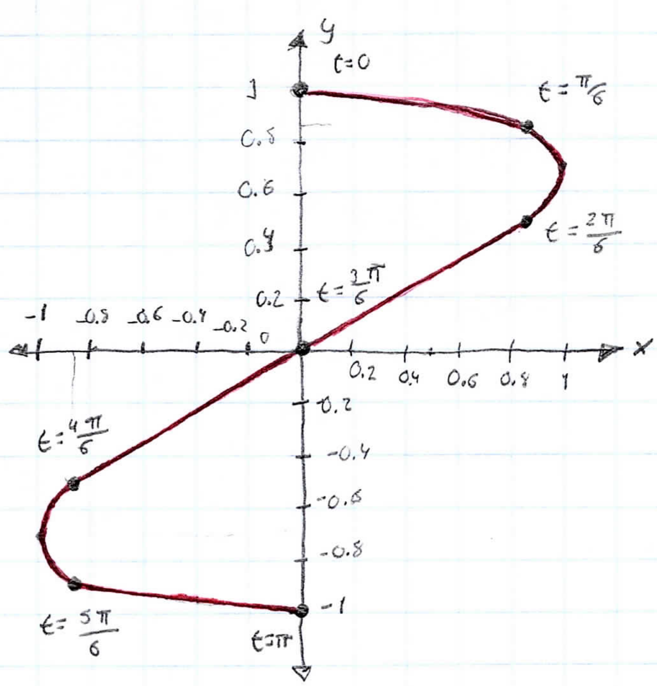
$t = \frac{3\pi}{6}$, $\vec{F}(\frac{3\pi}{6}) = \sin(\pi)\hat{\imath} + \cos(\frac{3\pi}{6})\hat{\jmath} = (0, 0)$

$t = \frac{4\pi}{6}$, $\vec{F}(\frac{4\pi}{6}) = \sin(\frac{4\pi}{3})\hat{\imath} + \cos(\frac{4\pi}{6})\hat{\jmath} = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$t = \frac{5\pi}{6}$, $\vec{F}(\frac{5\pi}{6}) = \sin(\frac{5}{3}\pi)\hat{\imath} + \cos(\frac{5\pi}{6})\hat{\jmath} = \left(-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\right)$

$t = \pi$, $\vec{F}(\pi) = \sin(2\pi)\hat{\imath} + \cos(\pi)\hat{\jmath} = (0, -1)$

entonces



b) $\vec{F}(t) = \cos^2(t)\,\hat{\imath} + \sin^2(t)\,\hat{\jmath}$, $0 \leq t \leq 2\pi$

veamos algunos valores

$t=0$, $\vec{F}(0) = \cos^2(0)\,\hat{\imath} + \sin^2(0)\,\hat{\jmath} = (0,1)$

extra

$t=\frac{\pi}{2}$ $\vec{F}(\frac{\pi}{2}) = (0,1)$

$t=\frac{\pi}{3}$, $\vec{F}(\frac{\pi}{3}) = \cos^2(\frac{\pi}{3})\,\hat{\imath} + \sin^2(\frac{\pi}{3})\,\hat{\jmath} = (\frac{1}{4}, \frac{3}{4})$

$t=\frac{2}{3}\pi$, $\vec{F}(\frac{2}{3}\pi) = \cos^2(\frac{2}{3}\pi)\,\hat{\imath} + \sin^2(\frac{2}{3}\pi) = (\frac{1}{4}, \frac{3}{4})$

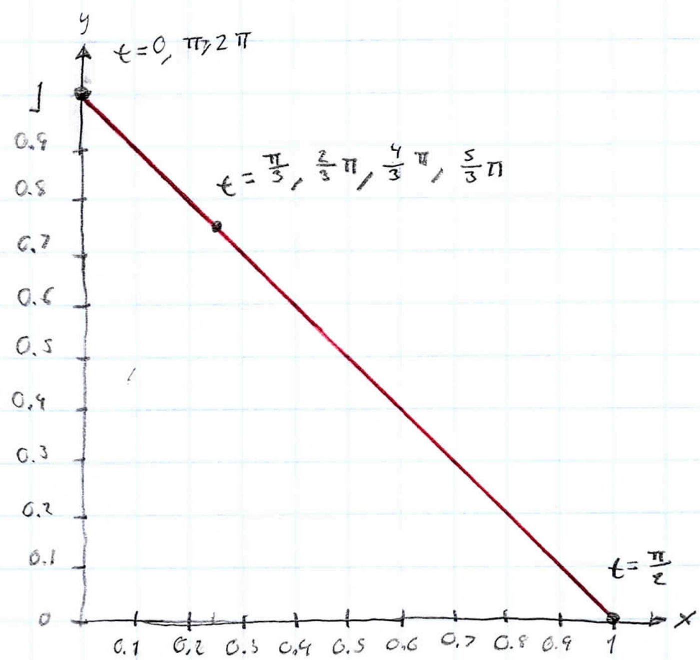
$t=\pi$, $\vec{F}(\pi) = \cos^2(\pi)\,\hat{\imath} + \sin^2(\pi) = (1, 0)$

$t=\frac{4}{3}\pi$, $\vec{F}(\frac{4}{3}\pi) = \cos^2(\frac{4}{3}\pi)\,\hat{\imath} + \sin^2(\frac{4}{3}\pi) = (\frac{1}{4}, \frac{3}{4})$

$t=\frac{5}{3}\pi$, $\vec{F}(\frac{5}{3}\pi) = \cos^2(\frac{5}{3}\pi)\,\hat{\imath} + \sin^2(\frac{5}{3}\pi) = (\frac{1}{4}, \frac{3}{4})$

$t=2\pi$, $\vec{F}(2\pi) = \cos^2(2\pi)\,\hat{\imath} + \sin^2(2\pi) = (1, 0)$

entonces



2) Calcula las integrales de línea de los campos vectoriales a lo largo de las trayectorias

a) $\vec{F}(x,y,z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, trayectoria $\vec{s}(t) = \sin(t)\hat{\imath} + \cos(t)\hat{\jmath} + t\hat{k}$, $0 \le t \le 2\pi$

$$\vec{s}'(t) = (\cos(t), -\sin(t), 1)$$

$$\int_C \vec{F} = \int_a^b \vec{F}(\vec{s}(t)) \cdot \vec{s}'(t)\, dt$$

$$\vec{F}(\vec{s}(t)) = (\sin(t), \cos(t), t)$$

$$\vec{F}(\vec{s}(t)) \cdot \vec{s}'(t) = (\sin(t)\cos(t) - \sin(t)\cos(t) + t) = t$$

$$\Rightarrow \int_0^{2\pi} t\, dt = \frac{t^2}{2}\Big|_0^{2\pi} = \left[\frac{(2\pi)^2}{2} - \frac{0^2}{2}\right] = 2\pi^2$$

b) $\vec{F}(x,y,z) = \frac{1}{z^2+1}\hat{\imath} + x(1+y^2)\hat{\jmath} + e^y\hat{k}$, trayectoria $\vec{s}(t) = (1+t^2)^2\hat{\imath} + 1\hat{\jmath} + t\hat{k}$, $0 \le t \le 1$

$$\vec{s}'(t) = (2(1+t^2)\cdot 2t, 0, 1) = (4t + 4t^3, 0, 1)$$

$$\vec{F}(\vec{s}(t)) = \left(\frac{1}{t^2+1}, (1+t^2)^2(1+1^2), e^1\right)$$

$$\vec{F}(\vec{s}(t)) \cdot \vec{s}'(t) = \left(\frac{4t(1+t^2)}{t^2+1} + 0 + e\right) = 4t + e$$

$$\int_C \vec{F} = \int_a^b \vec{F}(\vec{s}(t)) \cdot \vec{s}'(t)\, dt = \int_0^1 4t + e\, dt = 4\int_0^1 t\, dt + e\int_0^1 dt$$

$$= 4\left[\frac{t^2}{2}\Big|_0^1\right] + e\left[t\Big|_0^1\right] = 4\left[\frac{1^2}{2} - \frac{0^2}{2}\right] + e[1-0] = 4\left(\tfrac{1}{2}\right) + e(1)$$

$$= 2 + e$$

c) $\bar{F}(x,y,z) = x^3\hat{\imath} + 3zy^2\hat{\jmath} - x^2y\hat{k}$, con trayectoria recta que parte de $(3,2,1)$ a $(0,0,0)$

veamos que sea $\bar{s}(t)$

$$u = P_i + t\,[P_t - P_i] = (3,2,1) + t\,[(0,0,0)-(3,2,1)]$$
$$= (3,2,1) + t\,(-3,-2,-1) = (3-3t,\ 2-2t,\ 1-t) \quad \text{con } t\in[0,1]$$

$$\bar{s}(t) = (3-3t,\ 2-2t,\ 1-t)$$

$$\bar{s}'(t) = (-3,-2,-1)$$

$$\bar{F}(\bar{s}(t)) = [(3-3t)^3,\ 3(2-2t)^2(1-t),\ -(3-3t)^2(2-2t)]$$
$$= (-27t^3+81t^2-81t+27,\ -12t^3+36t^2-36t+12,\ 18t^3-54t^2+54t-18)$$

$$\bar{F}(\bar{s}(t))\cdot\bar{s}'(t) = (81t^3-243t^2+243t-81+24t^3-72t^2+72t-24$$
$$-18t^3+54t^2-54t+18)$$
$$= 87t^3-261t^2+261t-87$$

$$\int_C \bar{F} = \int_a^b \bar{F}(\bar{s}(t))\cdot\bar{s}'(t)\,dt = \int_0^1 87t^3-261t^2+261t-87\,dt$$

$$= 87\int_0^1 t^3\,dt - 261\int_0^1 t^2\,dt + 261\int_0^1 t\,dt - 87\int_0^1 dt$$

$$= 87\left.\frac{t^4}{4}\right|_0^1 - 261\left.\frac{t^3}{3}\right|_0^1 + 261\left.\frac{t^2}{2}\right|_0^1 - 87\left.t\right|_0^1$$

$$= 87\left(\tfrac{1}{4}\right) - 261\left(\tfrac{1}{3}\right) + 261\left(\tfrac{1}{2}\right) - 87(1) = \frac{87}{4} - 87 + \frac{261}{2} - 87$$

$$= -\frac{87}{4}$$

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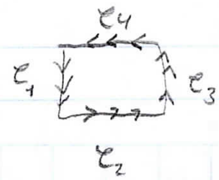

3) Resuelve las siguientes integrales de línea

a) $$\int_C (y^4+x^3)\,dx + (2x^6)\,dy = \int_C [\underbrace{(y^4+x^3),(2x^6)}_{F(x,y)}]\cdot(dx,dy) = \int_C \bar{F}(\bar{s}(t))\cdot\bar{s}'(t)$$

con trayectoria cerrada definida por las rectas $x=0$, $x=1$, $y=0$ y $y=1$

C_1 = recta $x=0$ $\begin{cases} x=0 \\ y=t \end{cases}$ reparametrizamos $h(a+b-t)$ $\bar{h}(t)=(0,1-t)$

$t\in[0,1]$ $\bar{h}'(t)=(0,-1)$

C_2 = recta $y=0$ $\begin{cases} x=t \\ y=0 \end{cases}$ $t\in[0,1]$ $\bar{h}(t)=(t,0)$ $\bar{h}'(t)=(1,0)$

C_3 = recta $x=1$ $\begin{cases} x=1 \\ y=t \end{cases}$ $t\in[0,1]$ $\bar{h}(t)=(1,t)$ $\bar{h}'(t)=(0,1)$

C_4 = recta $y=1$ $\begin{cases} x=t \\ y=1 \end{cases}$ reparametrizamos $h(a+b-t)$ $\bar{h}(t)=(1-t,1)$

$t\in[0,1]$ $\bar{h}'(t)=(-1,0)$

entonces $$\int_C \bar{F} = \int_{C_1}\bar{F} + \int_{C_2}\bar{F} + \int_{C_3}\bar{F} + \int_{C_4}\bar{F}$$

$$\int_{C_1}\bar{F} = \int_a^b [((1-t)^4+0^3,\ 2(0)^6)\cdot(0,-1)]\,dt = \int_0^1 0\,dt = 0$$

$$\int_{C_2}\bar{F} = \int_a^b [((0)^4+t^3,\ 2(t)^6)\cdot(1,0)]\,dt = \int_0^1 t^3\,dt = \frac{t^4}{4}\Big|_0^1 = \frac{1}{4}$$

$$\int_{C_3}\bar{F} = \int_a^b [((t)^4+(1)^3,\ 2(1)^6)\cdot(0,1)]\,dt = \int_0^1 2\,dt = 2\left[t\Big|_0^1\right] = 2$$

$$\int_{C_4}\bar{F} = \int_a^b [((1)^4+(1-t)^3,\ 2(1-t)^6)\cdot(-1,0)]\,dt = \int_0^1 -1-(1-t)^3\,dt$$

sustitución $u = 1-t$

$du = -1\,dt$

$\Rightarrow \begin{cases} 1 \Rightarrow 0 \\ 0 \Rightarrow 1 \end{cases}$

$$= -1\int_0^1 dt - \int_0^1 (1-t)^3\,dt = -1\left[t\Big|_0^1\right] - \int_1^0 -u^3\,du$$

$$= -1(1) + \frac{u^4}{4}\Big|_1^0 = -1 + \left[\frac{0^4}{4} - \frac{1^4}{4}\right] = -1 - \frac{1}{4} = -\frac{5}{4}$$

entonces $\displaystyle\int_C \bar{F} = 0 + \frac{1}{4} + 2 - \frac{5}{4} = 1$ usando sentido antihorario

b) Sea $f = xy^2\,\hat{\imath} - yx^2\,\hat{\jmath}$ y la trayectoria s es la elipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

veamos la parametrización $\bar{h}(t)$

usemos la siguiente parametrización

$$\begin{cases} x = a\cos\theta \\ y = b\,\text{sen}\,\theta \end{cases} \quad \text{con} \quad \theta \in [0, 2\pi]$$

entonces $\bar{h}(\theta) = (a\cos\theta,\ b\,\text{sen}\,\theta)$

$$\bar{h}'(\theta) = (a\,\text{sen}\,\theta,\ -b\cos\theta)$$

$$\bar{F}(\bar{h}(\theta)) = (a\cos\theta\,(b\,\text{sen}\,\theta)^2,\ -(a\cos\theta)^2\,b\,\text{sen}\,\theta) = (a\cos\theta\, b^2\text{sen}^2\theta,\ -a^2\cos^2\theta\, b\,\text{sen}\,\theta)$$

$$\bar{F}(\bar{h}(\theta))\cdot\bar{h}'(\theta) = (a^2b^2\cos\theta\,\text{sen}^3\theta + a^2b^2\cos^3\theta\,\text{sen}\,\theta)$$

$$= (a^2b^2)(\cos\theta\,\text{sen}^3\theta + \cos^3\theta\,\text{sen}\,\theta)$$

$$= (a^2b^2)(\cos\theta\,\text{sen}\,\theta)(\text{sen}^2\theta + \cos^2\theta)$$

$$= a^2b^2\cos\theta\,\text{sen}\,\theta$$

$$\int_{\mathcal{C}} \bar{F} = \int_a^b \bar{F}(\bar{h}(\theta))\cdot\bar{h}'(\theta)\,d\theta = \int_0^{2\pi} a^2b^2\cos\theta\,\text{sen}\,\theta\,d\theta = a^2b^2\int_0^{2\pi}\cos\theta\,\text{sen}\,\theta\,d\theta$$

sustitución $u = \text{sen}\,\theta$

$du = \cos\theta\,d\theta$

$$= a^2b^2\int_0^0 u\,du = 0$$

$$\begin{cases} 2\pi \Rightarrow 0 \\ 0 \Rightarrow 0 \end{cases}$$

-1

-1

c) $\int_C (x^5-2xy^3)\,dx - (3x^2y^2)\,dy = \int_C [\underbrace{(x^5-2xy^3), (-3x^2y^2)}_{F(x,y)}] \cdot (dx, dy)$, con

C parametrizada por (t^8, t^{10}), $0 \leq t \leq 1$

$\bar{s}(t) = (t^8, t^{10})$

$\bar{s}'(t) = (8t^7, 10t^9)$

$\bar{F}(\bar{s}(t)) = (t^{40} - 2(t^8)(t^{30}), -3(t^{16})(t^{20})) = (t^{40} - 2t^{38}, -3t^{36})$

$\bar{F}(\bar{s}(t)) \cdot \bar{s}'(t) = (8t^{47} - 16t^{45} - 30t^{45}) = 8t^{47} - 46t^{45}$

$$\int_C \bar{F} = \int_a^b \bar{F}(\bar{s}(t)) \cdot \bar{s}'(t)\,dt = \int_0^1 8t^{47} - 46t^{45}\,dt$$

$$= 8\int_0^1 t^{47}\,dt - 46\int_0^1 t^{45}\,dt = 8\left[\frac{t^{48}}{48}\Big|_0^1\right] - 46\left[\frac{t^{46}}{46}\Big|_0^1\right]$$

$$= 8\left[\frac{1^{48}}{48} - \frac{0^{48}}{48}\right] - 46\left[\frac{1^{46}}{46} - \frac{0^{46}}{46}\right] = 8\,\frac{1}{48} - 46\,\frac{1}{46} = \frac{1}{6} - 1$$

$$= -\frac{5}{6}$$

4) Hallar la longitud de arco de cada una de las siguientes curvas

a) $x=\sqrt{t}$, $y=\frac{t^2}{8}+\frac{1}{4t}$, desde $t=1$ hasta $t=3$

$$S_p = \int_a^b \|\gamma'(t)\|\, dt$$

$$\gamma(t) = \left(\sqrt{t}, \frac{t^2}{8}+\frac{1}{4t}\right) \qquad \gamma'(t) = \left(\frac{1}{2\sqrt{t}}, \frac{t}{4}-\frac{1}{4t^2}\right) = \left(\frac{1}{2\sqrt{t}}, \frac{t^3-1}{4t^2}\right)$$

$$\|\gamma'(t)\| = \sqrt{\left(\frac{1}{2\sqrt{t}}\right)^2+\left(\frac{t^3-1}{4t^2}\right)^2} = \sqrt{\frac{1}{4t}+\frac{t^6-2t^3+1}{16t^4}} = \sqrt{\frac{4t^3+t^6-2t^3+1}{16t^4}}$$

$$= \sqrt{\frac{t^6+2t^3+1}{16t^4}} = \frac{\sqrt{t^6+2t^3+1}}{\sqrt{16t^4}} = \frac{\sqrt{(t^3+1)^2}}{4t^2} = \frac{t^3+1}{4t^2}$$

$$S_p = \int_1^3 \frac{t^3+1}{4t^2}\, dt = \frac{1}{4}\int_1^3 \frac{t^3+1}{t^2}\, dt = \frac{1}{4}\left[\int_1^3 \frac{t^3}{t^2}\, dt + \int_1^3 \frac{1}{t^2}\, dt\right]$$

$$= \frac{1}{4}\left[\int_1^3 t\, dt + \int_1^3 t^{-2}\, dt\right] = \frac{1}{4}\left(\left[\frac{t^2}{2}\Big|_1^3\right] + \left[\frac{t^{-1}}{-1}\Big|_1^3\right]\right)$$

$$= \frac{1}{4}\left(\left[\frac{3^2}{2}-\frac{1^2}{2}\right] + \left(-\frac{1}{3}+\frac{1}{1}\right)\right] = \frac{1}{4}\left(4+\frac{2}{3}\right) = \frac{1}{4}\left(\frac{14}{3}\right)$$

$$= \frac{7}{6}$$

b) $x = \cos^4(t)$, $y = \sin^4(t)$, $0 \leq t \leq \pi/2$

$$Sp = \int_a^b \|\gamma'(t)\| \, dt$$

$\gamma(t) = (\cos^4(t), \sin^4(t))$, $\gamma'(t) = (-4\cos^3(t)\sin(t), \ 4\sin^3(t)\cos(t))$

$$\|\gamma'(t)\| = \sqrt{(-4\cos^3(t)\sin(t))^2 + (4\sin^3(t)\cos(t))^2}$$

$$= \sqrt{16\cos^6(t)\sin^2(t) + 16\sin^6(t)\cos^2(t)}$$

$$= \sqrt{16\cos^2(t)\sin^2(t)\,(\cos^4(t) + \sin^4(t))}$$

$$= 4\cos(t)\sin(t)\sqrt{\left(\frac{1+\cos(2t)}{2}\right)^2 + \left(\frac{1-\cos(2t)}{2}\right)^2}$$

ya que $\sin^2(t) = \dfrac{1-\cos(2t)}{2}$ y $\cos^2(t) = \dfrac{1+\cos(2t)}{2}$

$$= 2\sin(2t)\sqrt{\tfrac{1}{4}(1+\cos(2t))^2 + \tfrac{1}{4}(1-\cos(2t))^2}$$

ya que $\sin(2t) = 2\sin(t)\cos(t)$

$$= \sin(2t)\sqrt{4\left(\tfrac{1}{4}\right)\left[(1+\cos(2t))^2 + (1-\cos(2t))^2\right]}$$

$$= \sin(2t)\sqrt{(\cos^2(2t) + 2\cos(2t) + 1) + (\cos^2(2t) - 2\cos(2t) + 1)}$$

$$= \sin(2t)\sqrt{2\cos^2(2t) + 2}$$

entonces

$$Sp = \int_0^{\pi/2} \sin(2t)\sqrt{2\cos^2(2t)+2}\, dt$$

sustitución $u = \cos(2t)$
$du = -2\sin(2t)\,dt$

$\Rightarrow \begin{cases} \frac{\pi}{2} \Rightarrow -1 \\ 0 \Rightarrow 1 \end{cases}$

$$= \int_1^{-1} -\frac{1}{2}\sqrt{2u^2+2}\, du = \frac{1}{2}\int_{-1}^{1}\sqrt{2(u^2+1)}\, du = \frac{\sqrt{2}}{2}\int_{-1}^{1}\sqrt{u^2+1}\, du$$

resolvamos $\int \sqrt{u^2+1}\, du$

sustitución trigonométrica $u = \tan\theta$
$du = \sec^2\theta \, d\theta$

$$\int \sqrt{1+\tan^2\theta}\, \sec^2\theta \, d\theta = \int \sqrt{\sec^2\theta}\, \sec^2\theta \, d\theta$$

propiedad trigonométrica

$$= \int \sec\theta \sec^2\theta \, d\theta = \int \sec^3\theta \, d\theta$$

usando una tabla de integrales tenemos

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln(\sec\theta + \tan\theta)$$

recordemos que $u = \tan\theta$ y

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$= \frac{1}{2}\sqrt{1+u^2}\,u + \frac{1}{2}\ln\left(\sqrt{1+u^2}+u\right)$$

por lo tanto ahora evaluemos

$$\frac{\sqrt{2}}{2}\left(\left[\frac{1}{2}\sqrt{1+u^2}\,u + \frac{1}{2}\ln\left(\sqrt{1+u^2}+u\right)\right|_{-1}^{1}\right)$$

$$= \frac{\sqrt{2}}{2}\left(\left[\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln(\sqrt{2}+1)\right] - \left[-\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln(\sqrt{2}-1)\right]\right)$$

$$= \frac{\sqrt{2}}{2}\left(\sqrt{2} + \ln\left[(\sqrt{2}+1)^{\frac{1}{2}}\right] + \ln\left[(\sqrt{2}-1)^{-\frac{1}{2}}\right]\right)$$

$y \ln(x) = \ln(x^y)$

$$= \frac{\sqrt{2}}{2}\left(\sqrt{2} + \ln\left((\sqrt{2}+1)^{\frac{1}{2}}(\sqrt{2}-1)^{-\frac{1}{2}}\right)\right)$$

$\ln(x)+\ln(y) = \ln(xy)$

$$= \frac{\sqrt{2}}{2}\left(\sqrt{2} + \ln\left(\frac{(\sqrt{2}+1)^{\frac{1}{2}}}{(\sqrt{2}-1)^{\frac{1}{2}}}\right)\right) = \frac{\sqrt{2}}{2}\left(\sqrt{2} + \ln\left(\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^{\frac{1}{2}}\right)\right)$$

$$= \frac{\sqrt{2}}{2}\left(\sqrt{2} + \frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\right) = \frac{\sqrt{2}}{2}\left(\sqrt{2} + \frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)\right)$$

$$= \frac{\sqrt{2}}{2}\left(\sqrt{2} + \frac{1}{2}\ln\left(\frac{(\sqrt{2}+1)(\sqrt{2}+1)}{2-1}\right)\right) = \frac{\sqrt{2}}{2}\left(\sqrt{2} + \frac{1}{2}\ln\left(2+2\sqrt{2}+1\right)\right) =$$

$$= \frac{\sqrt{2}}{2}\left(\sqrt{2} + \frac{1}{2}\ln(2\sqrt{2}+3)\right) = 1 + \frac{\sqrt{2}}{4}\ln(2\sqrt{2}+3) \approx 1.623$$

c) $x = a(\cos t + t\sin(t))$, $y = a(\sin t - t\cos(t))$, $0 \leq t \leq 2\pi$

$$Sp = \int_a^b \|\gamma'(t)\|\, dt$$

$$\gamma(t) = (a(\cos t + t\sin(t))) , (a(\sin t - t\cos(t)))$$

$$\gamma'(t) = (a(-\sin t + \sin(t) + t\cos(t)) , (a(\cos t - \cos t + t\sin(t)))$$

$$= (a\, t\cos(t) , a\, t\sin(t))$$

$$\|\gamma'(t)\| = \sqrt{(at\cos(t))^2 + (at\sin(t))^2} = \sqrt{a^2t^2\cos^2(t) + a^2t^2\sin^2(t)}$$

$$= \sqrt{a^2t^2(\cos^2(t) + \sin^2(t))} = \sqrt{a^2t^2} = at$$

$$Sp = \int_0^{2\pi} at\, dt = a\int_0^{2\pi} t\, dt = a\left[\frac{t^2}{2}\Big|_0^{2\pi}\right] = a\left[\frac{2\pi^2}{2} - \frac{0^2}{2}\right]$$

$$= a(2\pi^2) = 2a\pi^2$$