Puzzling Correlations and When to Find Them: Sovereign Spreads and Inflation*

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January 2021

Abstract

The paper explores a novel pattern in the correlation between default risk and currency depreciation risk for emerging economies. It confirms there is a positive correlation between currency depreciation risk and default risk, but it shows that such correlation disappears in the short horizon, broadly defined as less than three years. The pattern in the correlation is fairly robust to the measures of default risk and currency depreciation risk; as well as for controlling for other factors. I build a small open monetary economy model to take into account a new feature of the trade-offs associated with inflation and future output, and inflation and ability to repay. The model is able to match the untargeted correlation between depreciation risk and default risk for different maturities for Brazil. The model also quantifies the changes in the capital investment decision for Brazil between 1995 and 2010 and finds that Brazil invested around 1.1 % of GDP less because of the inflation risk related to the majority of their debt being in Brazilian Real.

*Acknowledgements: I am extremely grateful to Manuel Amador, Tim Kehoe, and Mark Wright for all their help, advice, and patience. I would like to thank Marco Bassetto, Carlo Galli, Victor Almeida, Stelios Fourakis, Agustin Samano, Carlos Esquivel for all the helpful discussions. I would also like to thank all participants at the University of Minnesota International Workshop and Minnesota-Wisconsin International Workshop for all their comments. I acknowledge and thank the generous support from the Beverly and Richard Fink Fellowship. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The remaining errros are mine.

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1 Introduction

The shift in the currency composition of developing countries' sovereign debt portfolios has brought the issues associated with nominal debt into the consciousness of the sovereign debt literature. In recent years, developing countries have started to borrow more in their local currency, a change from the status quo of the 1980s and 1990s (Du, Schreger (2016),Arslanalp, Tsuda (2014)). Associated with this decision, there is the risk of sovereigns inflating away real obligations. Theoretically, we can think of a sovereign using inflation as a way to partially default on a portion of their bonds. However, empirically inflation and default seem to be associated with being complements; the long-term risk of inflation and default have positive co-movements, and default episodes are associated with high inflation. Understanding how governments reduce real liabilities by using inflation while balancing the associated cost is paramount to understanding the actual mechanism for which inflation impacts the incentives and the ability to repay.

My paper studies the relationship between inflation and default, first empirically and then through a structural model. In the data, I find that there is only a positive correlation in the long horizon. To explain the lack of correlation in the short run, I merge two strands of the literature and build a small open economy model that incorporates how inflation affects capital accumulation. I use the framework to measure the decrease in investment associated with issuing debt in local currency for Brazil from 1995 to 2010. I also show the changes in welfare associated as a result and decompose it between the insurance that local debt offers versus the lower output as a result of lower investment.

While a significant part of the literature has focused mostly on how inflation changes the incentives to default, this paper wrestles with the question of how inflation changes the country's future ability to repay its debt through inflation's effect on capital accumulation. The mechanism explored in this paper questions the conventional wisdom that overcoming the original sin of developing countries was a triumph without negative consequences. Furthermore, it focuses on how the consequences of inflation change between short and long horizons.

I show that for a group of developing countries, the correlation between implied yields in currency forwards and default risk is high in the long run and disappears in the short run. Similarly, the correlation between inflation risk, measured by the spreads in bonds linked to inflation, and default risk is also only positive in the long run. Recent literature has focused only on comparing the realized inflation with default risk or comparing long-term inflation risk and default risk. Galli (2020) documented that the correlation was positive, however, when the horizon shortens, the positive correlation disappears.

I use Bloomberg to get prices from secondary markets for bonds and currency futures. I rely on the fact that all countries in my sample issue part of their bonds in USD, so the spreads between local bonds issued in USD and US treasury bonds is the expected default. As a robustness check for default risk, using credit default swaps (CDS) gives similar results. Following Du, Schreger (2016) amongst others, I use the implied yield in futures to measure the expected depreciation of a currency versus the USD at different maturities. Finally, to get a better measure of expected inflation in the short term, I use the spreads of Local Currency (LCU) bonds indexed-to-inflation and LCU bonds not indexed to inflation. The underlying assumption is that the probability of default is the same for both, and the only difference is the expected inflation.

I find that the correlation between expected depreciation of a currency and expected default goes on average from 0.07 in the 1-year maturity structure to 0.48 in the 5-year maturity. The result is robust to all countries, and to using CDS instead of spread. Similarly, the correlation between expected inflation and expected default goes from 0.15 at 1 year to 0.76 at 5 years. The expected inflation is only done for Brazil, Mexico, and Chile due to data constraints. The 5-year correlations are inline with what has been found by Galli (2020) and in similar magnitudes to the correlation between realized inflation and default found in Arellano et al. (2020). This paper is the first one, to my knowledge, to document the short-term correlation and the differences in term structures.

Once again, the shape of the correlation curves for different maturities are at least mildly surprising. A simple way of thinking about the relationship between inflation and default would be that the government uses money creation to implicitly default on debt, therefore we should expect to see a negative correlation between expected inflation and expected default. On the other hand, if we expect both to be caused by reactions to negative shocks, then we should expect high positive correlations. Previously in the literature, the empirical relationship between these has been either ignored or assumed to be positive and constant for all maturities.

With these new correlations in mind, I build a model which incorporates classic elements of monetary models, a Cash-in-Advance constraint (CIA) à la Stockman (1981), into a small open economy framework with lack of commitment in both monetary policy and debt repayment. The model is able to capture the trade-off between inflation and future output by incorporating that future capital decreases with inflation. As a result, the government faces a trade-off between lowering their default probability today by increasing it tomorrow through lower capital and expected output tomorrow. My approach is new to the literature as it blends two different parts of the sovereign debt literature. It combines the work done in adding capital to sovereign debt models with the new emerging models of the interaction between monetary policy and default. Most of the others in the literature take output to be a stochastic process, and inflation is assumed to only matter in the price of new bonds and how much you have to repay today, which implies that the correlation between expected inflation and default does not vary with the maturity structure.

The model is a small open economy with households, government, and international investors. Government issues nominal debt and chooses government consumption and money growth. Households choose private consumption and choose money balances and capital for the next period. Households face a CIA constraint on both consumption and investment, and face adjustment costs of their money balances and capital. In the model, the government does not have access to complete tax instruments, and as such does not choose capital directly.

Households do not internalize that their capital decision changes the default probability tomorrow and therefore underinvest relative to what the government would like. The taxes being exogenous to the model captures some political economy forces that do not allow for quick responses in tax rates; they also match the empirical evidence that emerging market countries use seigniorage instead of taxes to smooth out business cycle fluctuations. Therefore, taxes are taken to be exogenous and so the only source of time-varying government revenue is seigniorage. It is through the CIA that inflation distorts the capital investment decision; by lowering the real money balances, households decrease their capital holdings to smooth out consumption. In other words, the government uses seigniorage to raise revenue and lower the value of their liabilities, but to the households, it is a tax on current wealth and they react by lowering their investment to smooth out changes in consumption. I use Simulated Method of Moments to match Brazil 1995-2010.

Brazil is an interesting case as it faced hyperinflation before 1995 and experienced rapid growth afterward. The government implemented reforms to lower inflation, but their central bank is not independent. During this period, Brazil was able to reduce their inflation, experienced an increase in capital investment, and they borrowed mostly in Brazilian Reals. And thanks to a commodity boom, they had a high TFP growth rate. By 2010, Brazil was considered one of the giants of the emerging markets. Finally, Bank of Brazil could make withdrawal unlimited funds from the central bank to fund the government spending. Brazil provides an environment in where inflation risk, default risk, nominal debt, and large capital investment are all present. The model is able to measure the effects that borrowing on their own currency had on Brazil.

I use Simulated Method of Moments to match Brazil 1995-2010. The model is able to match business cycle moments well and reproduces the untargeted correlations between inflation risk and default risk for different horizons. The model measures the effect that borrowing on their own currency had on Brazil and finds large yearly underinvestment attributed to the risk of future inflation. The result is surprising not only in magnitude but also given that Brazil is usually regarded as a success story of growth and how to get inflation under control.

For Brazil between 1995 and 2010, I find that investment was on average about 1.1 % of GDP lower every year compared to a baseline in which they had borrowed in dollars or linked-to-inflation debt. It is mostly due to the difference in the amount of capital that they are able to sustain. Borrowing in real debt leads to less seigniorage, and so the return on capital is higher. I also find welfare gains of about 3%, coming mostly from differences in capital accumulation. It is a contrasting result to the traditional monetary literature as Gomme (1993), De Gregorio (1993) and Jones, Manuelli (1995) all find modest effects of inflation on capital accumulation. The main difference is that in my framework there is lack of commitment with respect to debt and the government has extra incentives to use seigniorage to lower debt obligations and get revenue. Section 3 will discuss more in-depth the model and the validity of the assumptions.

My road-map for the paper is to first go into the empirical work and focus on showing the correlations between inflation risk and default risk. The empirical work should be thought of as being at least a hint of what the mechanism the literature missing is. Then build a sovereign default model that has a capital and CIA on consumption and investment so as to incorporate the trade-offs associated with inflation and future ability to repay. The main idea is that inflation affects future capital and is very correlated with itself. To test the model, it will be used to match moments from Brazil 1995-2010. And given that match, I will look at the counterfactual of what would happen if they had borrowed in real debt rather than nominal debt. Finally, I would look at the welfare effects and how they decompose between capital accumulation and ex-post insurance.

1.1 Related Literature

The topic of inflation and sovereign debt is not a new one, and this paper owes a lot to its intellectual predecessors. The general framework builds on sovereign default models following the path of Eaton, Gersovitz (1981), Arellano (2008), Aguiar, Gopinath (2006) and subsequent work since. More specifically, it contributes to three different strands of the literature.

In terms of money and sovereign default, the closest work to this is Galli (2020), Roettger (2019) and Sunder-Plassmann (2020). They consider the interaction of monetary policy in default decisions. I add to their work by adding capital which makes inflation affect future output and repayment. It allows me to explore the dynamic trade-off that inflation poses, by reducing default today but increasing it tomorrow. Similarly, Gomez-Gonzalez (2019) explores the role of inflation-linked debt, while abstracting from inflationary and capital concerns. Aguiar et al. (2013) explore the credibility of joining monetary unions and exposure to self-fulling crises, something this paper does not consider because of tractability. Finally, Arellano et al. (2020) merges a New Keynesian open economy with a sovereign debt framework. Their work includes capital, but monetary policy follows a rule and government borrows in real debt. In contrast, I focus on an environment in where there is strategic inflation to lower real liabilities. Because of tractability, this paper abstracts from the portfolio choice between nominal and real debt, which is analyzed by Ottonello, Perez (2019), Engel, Park (2018) and Du, Schreger (2016).

The paper also adds to the literature that studies sovereign default and capital. Gordon, Guerron-Quintana (2018), and Park (2014) both focus on the interactions between capital accumulation and sovereign spreads. They show that capital does not trivially lower default since capital changes both the value function to default and to repay, something that is present in the model. Esquivel (2020) studies the impact of giant oil discoveries on the sovereign debt, and shows that they increase the probability of default. I build on these works by integrating capital adjustment costs, while adding money and nominal debt.

Finally, there is a long tradition in monetary frameworks of including capital. Tobin (1965) shows that an increase in the monetary growth rate leads to capital formation in the long run. However, an anti-Tobin effect is also possible, as shown by Stockman (1981) and

Abel (1985). Cooley, Hansen (1989, 1991) also find anti-Tobin effect when labor supply is endogenous. I follow in the tradition of Stockman (1981) and Abel (1985) and build a model with a CIA constraint on capital investment. Compared to their model, this paper deals with both a small open economy, and the government lacks commitment. Aruoba et al. (2011), Janiak, Monteiro (2011), and Arawatari et al. (2018) all also focus on the negative relationship between money growth and inflation.

2 Facts

I use Bloomberg to get monthly prices from secondary bond and currency markets, and focus on the period between 1/1/2004 to 2/1/2020. I use the earliest time frame possible and end before the COVID epidemic. The sample contains Brazil, Colombia, Mexico, Argentina, Peru, Chile and Turkey. All of these are countries that borrow both in local currency and in USD.

First, I measure the default risk. Following the standard way it is done in the literature. I define spreads as:

$$Spreads_T = r_{USD\ Foreign\ Bonds,T} - r_{US\ Bonds,T} \tag{1}$$

Spreads therefore are the difference in yields between bonds denominated in USD compared to US Bonds [1]. All countries have enough of their debt denominated in USD and the markets are active enough. For the local bonds denominated in USD, I take the average price traded for that month and calculate the yield-to-maturity and use that as a the $_{USD\ Foreign\ Bonds,T}$. Since the bonds are denominated in foreign currency, they are not exposed to currency depreciation risk, so I interpret the spreads to measure the risk of default

Measuring the expected inflation is a lot harder and less clean. First, I define the currency depreciation risk as:

[1] I get the data of the US nominal yield curve from Gürkaynak et al. (2007) and use it as the benchmark of the risk-free rate for different maturities.

$$i_T - i_T* = \frac{Fwd_T}{Spot} \tag{2}$$

The currency depreciation risk is measured by using the implied yield in exchange rate forwards between LCU and USD. It measures the currency depreciation risk over the US dollar for a fixed maturity, following Du, Schreger (2016). While it serves a good proxy for the expected inflation in the long term, in the short term the interpretation of it as expected inflation has the problem of Fama, French (1988) regression that shows that in the short-term forwards are bad predictors of future inflation. It is still a worthwhile object to consider for two reasons. One it is the price that a foreign investor would have to pay in order to hedge currency risk, regardless of the maturity. Secondly, the data is more widely available than other measures of expected inflation.

$$Spreads_{i\,T} = r_{Inflation-Linked\,Bonds,T} - r_{Not-Linked\,Bonds,T} \tag{3}$$

As a way to better capture the expected inflation, I use the spread between bonds issued in LCU that are indexed to inflation and bonds issued in LCU that are not. In the price of both bonds, default risk is present. However if we assume that both bonds have the same probability of being defaulted on, then the difference should be the expected inflation. It is a new way of measuring expected inflation. The downside is that in order to get the measure of expected inflation, sovereigns need to issue bonds issued that are index to inflation and not. The requirements limit the countries to just Brazil, Chile, and Mexico.

Table 1 shows the correlation between default risk (Equation 1) and currency depreciation risk (Equation 2). Notice that most countries exhibit statistically zero correlation if the maturity is less than 1 year, with Argentina being the exception. The general pattern is that the correlations become stronger as the maturity becomes farther into the future. It is instructive to look at the averages and see that the 1-year has a correlation of 0.151 while the 5 year has .504, almost 5 times higher.

	3M	**6M**	**1Y**	**2Y**	**3Y**	**4Y**	**5Y**	**10Y**
Mexico	-0.007	0.063	0.132	0.259	0.374	0.445	0.377	
Brazil	0.012	0.128	-0.003	0.128	0.331	0.33	0.485	0.506
Peru	-0.016	-0.009	-0.029	0.394	0.241		0.42	
Colombia	-0.049	-0.79	0.039		0.129			
Turkey			0.21	0.662	0.472		0.467	
Argentina			0.558		0.66			
Chile			0.151	0.405	0.412		0.769	
Average	-0.015	-0.152	0.151	0.370	0.374	0.388	0.504	0.506

Table 1: Correlations of Spreads on USD denominated debt and Implied Yield of Currency Futures

	1Y	**2Y**	**3Y**	**4Y**	**5Y**	**8Y**
Brazil	0.263	0.439	0.771	0.813	0.813	
Mexico		0.252	0.22	0.43	0.505	0.502
Chile	0.169	0.329		0.36	0.485	
Average	0.216	0.340	0.495	0.534	0.601	0.502

Table 2: Correlations of Spreads on LCU Linked-to-Inflation Bonds and LCU Not Linked Bonds

Table 2 shows the correlation between correlation between default risk (Equation 1) and inflation risk (Equation 3). Due to the data requirements, most countries are dropped from the sample when switching over to Table 2. A similar pattern emerges as Table 1, where in the short run (less than 3 years) the correlation is low, yet in the longer horizon it rises. An interesting feature is that the correlations in the long term, both in Table 1 and Table 2, have similar magnitudes to the correlations of realized inflation and default risk in found previous literature (Galli (2020), Arellano et al. (2020)) .

An important point to make is that the correlations are between inflation risk and default risk in the same maturity. Therefore, the result comes from that in one year if there is inflation, it does not correlate with default, but if it is in 5 years then it correlates highly with default. The theory of this paper is that inflation in 5 years means inflation for the next 5, which implies the decimation capital stock, while inflation tomorrow means only on the period, which means it is already priced in and other factors dominate.

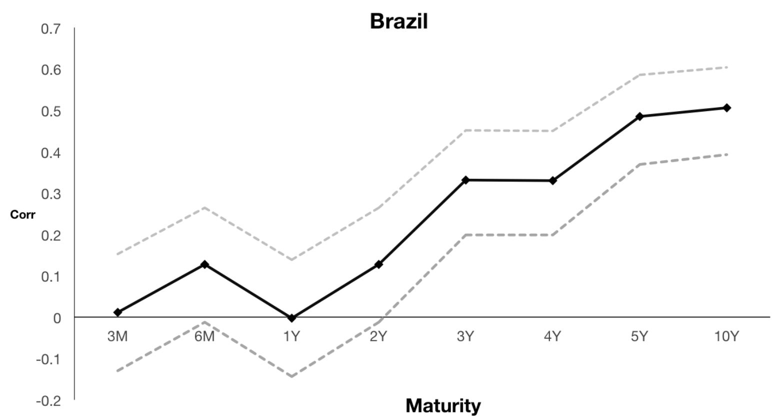

Figure 1: Correlation between Default Risk and Currency Depreciation Risk

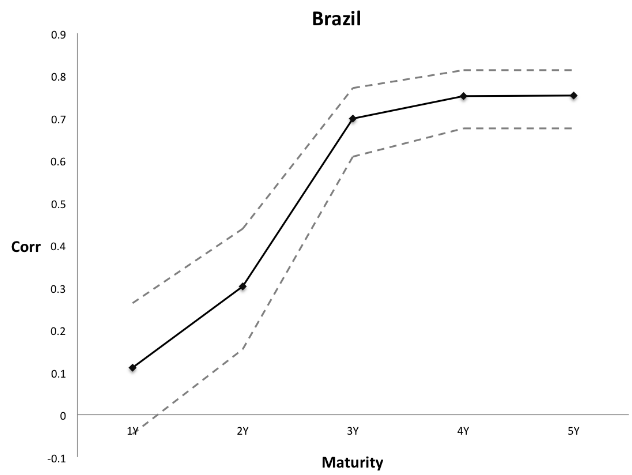

Figure 2: Correlation between Inflation Risk and Default Risk

Figures 1 and 2 show the correlations for different horizons for Brazil. The general shape of the correlations is fairly robust to using quarterly, daily, or even yearly data. I also check that using Credit Default Swaps (CDS) for default risk does not change the results. While one might worry about the effects coming from the Great Recession, the results hold whether or not the time period includes it, but does mechanically increase the standard errors because of the loss of data. Running a panel regression on the spreads vs. US bonds on the implied yields while using country and time fixed effects also does not change the significance of the short and long maturities.

3 Model

The model is an infinite-horizon small open economy model, where time is discrete and indexed by $t \in \{0, 1, 2, \ldots\}$. There are 3 types of agents: Domestic Households, a government, and international lenders. Households are atomistic with measure one and will choose private consumption, investment, and money balances for the next period. Money is used for transactions during this period, which gives rise to a cash-in-advance constraint on household consumption and investment. Households also face a quadratic adjustment cost of changing both their money balances and capital. The government has chosen endogenous government spending, issued nominal debt and money supply, and has a default decision and inflation. Tax rate is fixed and so the only source of revenue that the government can change is seigniorage. Finally, foreign and risk neutral lenders buy bonds from the government.

3.1 Timing

The timing in the model is as follows: At the start of each period, the government can be either in good standing or in financial autarky, depending on its default history. The state of the world is summarized by one-period nominal bonds from the government B_t, money balances M_t, a stochastic productivity shock θ_t, and capital inherited from a previous period K_t. The government then announces its choices of default, new bond issuance (0 if they are in default), money supply, and government spending. While the government doesn't have commitment from period to period, it has commitment within a period and these announcements are binding. The households, taking the government policy functions, prices, and state of the world as a given, decide their consumption, holdings of money balances, and investment. Finally, production and consumption take place and if the country is not in default, the holders of maturing bonds are paid.

3.2 Domestic Households

3.2.1 Preferences

Households have preferences given at time T by:

$$\mathbb{E}_T \left[\sum_{t=T}^{\infty} \beta^t (U(C_t) + (1 - L_t)) + U_G(G_t) \right]$$

Where $\mathbb{E}_T$ is the expectation operator conditional on date T, $0 < \beta < 1$ is the discount factor, $0 \leq L_t \leq 1$ is the labor given that period, and $U(\cdot)$ is the utility function on private consumption assuming the standard conditions. The function $U_G(\cdot)$ is the utility of consuming the government expenditure, but its separable from the private consumption and since households take government as given, they take it as a fixed transfer.

3.2.2 Budget Constraint

At every time t, households face the budget constraint of:

$$C_t + \frac{M_{t+1}^{hh}}{P_t} + I_t = \frac{M_t^{hh}}{P_t} + (1 - \tau)(\theta_t F\left(K_t, L_t\right) - \frac{\psi}{2} \left(\frac{M_{t+1}^{hh}}{M_t^{hh}} - 1\right)^2)$$

The left side of the budget constraint for the household is simple; they either consume, C_t, invest in physical capital, I , or hold money balances for the next period ,M. The right side is the income: they receive real money balances and they produce net of a tax. This production is taxed exogenously at a constant rate, and they also face the cost of changing their money balances. While not being standard, the cost is meant to capture the cost of inflation on output in the present period. I made in the tradition of Rotemberg (1982).

Investment is :

$$K_{t+1} - (1 - \delta)K_t = I_t - \phi(K_t, K_{t+1})$$

where δ is the constant depreciation rate, and $\phi(\cdot, \cdot)$ is the capital adjustment cost. This adjustment cost is standard in small open economy models, as in Gordon and Gordon, Guerron-Quintana (2018) and Esquivel (2020). It will be used to match the volatility of investment.

Households do not have access to international markets. They can only smooth consumption by saving in either money balances or capital. Capital has a stochastic return because L_t and θ_t will be stochastic. On the other hand, money also has a stochastic return. The government uses seigniorage, it will work as an inflation tax on the household's money holdings [2]. The tax will look as a wealth tax on households, so they will have to decrease their investment and consumption. However, they will choose to decrease capital to smooth out consumption in bad times.

3.2.3 Cash-in-Advance (CIA)

Households face a cash-in-advance constraint every period [3]:

$$C_t + I_t \leq \frac{M_t^{hh}}{P_t}$$

The constraint, while new to the sovereign debt literature, has a long history in monetary frameworks. Stockman (1981) and Abel (1985) provide the original CIA constraint on capital investment in the neoclassical growth model to show that inflation can lead to lower capital accumulation/growth since inflation acts as a tax on agents' holdings of real money balances, ultimately reducing the capital stock. Cooley, Hansen (1989, 1991) find similar results when they use endogenous labor supply. Recently, Aruoba et al. (2011) has shown that this CIA can be thought of as a lower bound for the effect of money on capital accumulation and welfare since it does not capture moments related to velocity, and misses market structures

[2]The timing on the cash-in-advance constraint is Svensson (1985) and not Lucas timing, which is to say HHs choose their money holdings one period ahead. This creates the situation in which the households have a fixed money holdings for their C and I this period, and so are exposed to the governments decision. Seigniorage acts as an unexpected shock to the real value of their money holdings

[3]Consider 2 people in a household. One is a worker and the other is a shopper. At the beginning of the period the shopper take the money balances and goes to buy investment and consumption for next period in a decentralized markets. Simultaneously and independently, the worker sells his labor in a decentralized market for labor and gets paid at the end of the period and choose money balances for next period. The set up is a natural extension which creates the need for money and gives rise to the CIA.

like bargaining, which could create large effects of the money supply on aggregate investment . The CIA is the a main component of my results. As government increases the money supply, it increases the price of money in terms of goods. In turn, the right hand side of the CIA will decrease, and households will have to adjust by reducing investment and consumption. Households will choose to reduce investment more to smooth out the decrease in consumption.

3.3 Government

The government is benevolent and values consumption stream as the households. The government's flow utility at period t is just:

$$U(C_t) + (1 - L_t) + U_G(G_t)$$

where the government chooses G_t.

The budget constraint of the government is

$$G_t + D_t \frac{B_t}{P_t} = D_t q(\cdot) B_{t+1} + \left(\frac{M_{t+1}}{P_t} - \frac{M_t}{P_t}\right) + \tau[\theta_t F\left(K_t, L_t\right) - \frac{\psi}{2}\left(\frac{M_{t+1}}{M_t} - 1\right)^2]$$

where G_t is government consumption, D_t is if the government is in default or not, B is nominal government bonds, q is the price at which the bonds are traded which depends on $B_{t+1}, M_{t+1}, \theta_t, K_{t+1}$, and M is the money balances.

The left-hand side is government expenditures and payments of maturing debt, if any. The right-hand side is the income, which breakdowns into new debt, seigniorage, and tax income. Tax revenue is net of costs. It should be clear to see that seigniorage is just the difference between the money balances issued today and the ones owed to them today.

3.4 International Lenders

International lenders are assumed to be infinitely lived and make zero profits in expectation. Recall that bonds are nominal. Therefore, the break-even condition includes both the inflation and default risk:

$$q(B_{t+1}, M_{t+1}, \theta_t, K_{t+1}) = \frac{1}{(1+r)}\mathbb{E}_t\left[\frac{1-D_t(B', M', \theta', K')}{(1+\pi_{t+1}(B', M', \theta', K'))}\right]$$

where D_t is the default probability and $(1 + \pi_{t+1})$ is the inflation tomorrow.

Given the price, we can now look at the model equivalent of the derived default and inflation risk in the empirical part.

The default risk at time t+i while on period t is :

$$\mathbb{D}_{t+i} = \mathbb{E}_t[D_{t+i}]$$

And since in the model the inflation risk and the currency depreciation are the same, we just have the inflation risk for the next period :

$$\mathbb{E}_t[1 + \pi_{t+1}] = \mathbb{E}_t[\frac{P_{t+1}}{P_t}]$$

$$\begin{aligned} &= \mathbb{E}_t[\frac{P_{t+1}}{P_t}\frac{M_{t+1}}{M_{t+1}}\frac{M_t}{M_t}] \\ &= \mathbb{E}_t[\frac{M_t}{P_t}\frac{P_{t+1}}{M_{t+1}}\frac{M_{t+1}}{M_t}] \\ &= \frac{M_t}{P_t}\frac{M_{t+1}}{M_t}\mathbb{E}_t[\frac{P_{t+1}}{M_{t+1}}] \end{aligned}$$

We can similarly define inflation risk for time as t+j:

$$\mathbb{E}_t[1 + \pi_{t+j}] = \frac{M_t}{P_t}\mathbb{E}_t[\frac{M_{t+j}}{M_t}]\mathbb{E}_t[\frac{P_{t+j}}{M_{t+j}}]$$

4 Equilibrium

I focus on a Markov perfect equilibrium, actions are only a function of payoff relevant state. In this case is where the government understands the effect of its choices on the household allocations and prices while takes futures policies as given. Households are atomistic and take both government current and future policy choices as given as well as prices. I drop the time subscript and adopt $'$ to mean the next period value.

4.1 Normalization

The state variables are θ, B, K, M, where θ is the exogenous productivity. The values of B,M come from the a mapping between the policy function given by the solution to the governments problem and the state space. In the same vein, K is given by the mapping between the solution to the PSE and the state space. Finally, market clearing is the money demand of the household is equal to the money supply of the government. In this class of monetary models, the ratio of B/M is a sufficient statistic for the government endogenous state. I normalize all nominal variables by dividing by M and denote them with ˆ (hat). Therefore, the state becomes $(\theta, \hat{B}, K)$

Define the set of current government choices to be $s := (D, \hat{B}', G, \mu)$ where D is the default decision, $\hat{B}'$ is the normalized future bonds, G is government consumption, and $\mu = \frac{M'}{M}$ is money growth. Government policy functions is then defined as a mapping from aggregate states to policy choices $S = H(\theta, \hat{B}, K)$

Since HHs take the current and future actions of the government as given, define S to be $S = (\theta, \hat{B}, K, s)$ and the inverse of the normalized price level is $m(S) = \frac{M}{P}$

4.2 Private Equilibrium

Definition Private Sector Equilibrium (PSE):: Given aggregate state and current government policies S, the future government policy function $\mathcal{H}$, PSE is a set of functions $(C(S), L(S), \hat{M}^{hh}, K'(S), m(S))$ such that:

1. They are a solution to the the HHs maximization problem subject to budget constraint, and CIA

2. Market clears for money balances, $\int \hat{M}^{hh} = 1$

4.3 Recursive Problem

The recursive problem of the government can be described as:

$$V(\hat{B},\theta,K) = max_{D\in\{0,1\}} \left\{(1-D)V^r(\hat{B},\theta,K) + (D)V^d(\theta_{def},K)\right\}$$

where: D is the default decision, B is the bonds held by international investors, θ is stochastic productivity, and K is capital inherited from the previous period. I will denote the policy functions of the household as being functions of the state, e.g. $C(S)$ represents the consumption policy function of households.

The value function for repayment is:

$$V^r(B,M,\theta,K) = max_{\hat{B}',\mu,G} \quad U(C(S),L(S),G) + \beta\mathbb{E}\left[V(\hat{B}',\theta',K'(S))\right]$$

Subject to the feasibility:

$$C(S) + G + \hat{B}m(S) + I = q(\hat{B}',\theta,K'(S))\hat{B}'\mu m(S) + \theta F\left(K,L(S)\right) - \tfrac{\psi}{2}\left(\mu-1\right)^2$$

and CIA:

$$C(S) + K'(S) - (1-\delta)K + \phi(K,K'(S)) \leq m(S)$$

and where $(C(S), L(S), K'(S), m(S))$ are solutions to the Private Sector Equilibrium associated with the governments choices.

Similarly, the value function for default is:

$$\begin{gathered} V^d(M,\theta,K) = max_{\mu,G} \quad U(C(S),L(S),G) + \Theta\beta\mathbb{E}\left[V(0,\theta',K'(S))\right] \\ +(1-\Theta)\beta\mathbb{E}\left[V^d(\theta',K'(S))\right] \end{gathered}$$

$$\begin{gathered} ST :: C(S) + G + I(S) = \theta_{def}F\left(K,L(S)\right) - \tfrac{\psi}{2}\left(\mu-1\right)^2 \\ C(s) + K'(s) - (1-\delta)K - \phi(K_t,K_{t+1}) \leq m(S) \end{gathered}$$

where $(C(S), L(S), K'(S), m(S))$ are in a PSE.

Definition Markov Perfect Equilibrium:: A Markov perfect recursive equilibrium is

- Value functions $(V(\hat{B},\theta,K), V^r(\hat{B},\theta,K), V^d(\theta_{def},K))$
- Policy functions $(D, \hat{B}', G, \mu)$
- a PSE, $\mathcal{P}$

such that given the aggregate state and debt price functions q :

1. Policy functions maximize the government problem, given $\mathcal{P}$
2. $\mathcal{P}$ corresponds the value and policy functions
3. current policy functions $\in \mathcal{H}$

5 Quantitative Exercise

5.1 Brazil

The model is calibrated to Brazil. Brazil is an emerging market with a history of hyperinflation. After a series of reforms in the early 90s, Brazil managed to stop hyperinflation. However, the country continued to borrow in Brazilian Real, experienced inflation at higher rates than some of its neighbors, while also going through a period of relatively good GDP growth. While no default occurred during the 1995-2010 period, CDS rates remained constantly around 5%.

I use the World Development Indicators, and IMF data to get data on output, private and government consumption, capital formation, investment, debt, and inflation. I use Bloomberg to get information on bond prices and CDS spreads.

5.2 Parameterization

I calibrate the model to a quaterly frequency, as the data in the empirical section was also quarterly. I use standard functional forms in the literature. Household and government preferences are given by:

$$U = \frac{C^{1-\rho}}{1-\rho} + \frac{G^{1-\eta}}{1-\eta}$$

The production function is taken to be

$$F\left(K, L\right) = K^{1-\alpha} L^{\alpha}$$

while the productivity penalty is as in Chatterjee, Eyigungor (2012)

$$\theta_{def} = \theta - max\{0, d_0\theta - d_1\theta^2\}.$$

The capital adjustment cost is quadratic as in Gordon, Guerron-Quintana (2018)

$$\phi(K_t, K_{t+1}) = \phi_0 \times (K'(s) - (1-\delta)K)^2.$$

I take the exogenous productivity to follow an AR(1) process.

The parameters chosen exogenously are as follows:

Variable		Value	Source
U C curvature	ρ	2	Standard
U G curvature	η	4	Galli (2020)
Risk free rate	r	0.002	US treasury
Auto correlation	κ	0.967	Estimated
Output st dev	σ_ϵ	0.023	Estimated
Re-entry	Θ	0.04	Aguiar and Gopinath (2006)
Depreciation	δ	0.006	7% Yearly
Capital share	$1-\alpha$	0.37	Standard

Table 3: Exogenous Parameters

Table 1 shows the values of the exogenously chosen parameters. The curvature of the utility function of private consumption is standard in business cycle analysis. The curvature of the utility on the government expenditure is taken from Galli (2020), while the exact value isn't critical to the results, it is important that $\rho \leq \eta$. The depreciation rate is chosen to be consistent with IMF estimates and World Bank estimates, which have ranged from 6% to 11%. However, more work is needed to really get a good estimate of capital depreciation.

Gordon and Guerron-Quintana (2018) have a discussion about the role of capital depreciation in sovereign default models and conclude that one-period models will miss their target of investment-output ratio in the steady state without foreign lending.

5.3 Solution method

The solution method follows Gordon (2019), Dvorkin et al. (2018),and Galli (2020). I add taste shocks to smooth out the probabilities of the policy functions of the government. Otherwise, the model struggles to converge due to having money and capital. I choose the smallest values such that the model converges.

5.4 Targeted Moments

Table 2 shows the variables used to match moments from the data. For example, the discount factor, while low, is still within the previously used range and serves to drive the model towards impatience being the dominant force when borrowing. The taxes are chosen to be exogenously, this can be seen as capturing some political economy forces that do not allow for quick responses in tax rates. Here they are chosen to be lower than the "optimal" so the government has an extra incentive to use seigniorage to raise revenue. The costs do not have an natural way of interpreting the magnitude of the numbers. The default costs seem around what the literature uses, and so do the capital adjustment costs. Given that it is new, the money adjustment cost has no literature to compare with. Arellano et al. (2020) use Rotemberg adjustment costs and find the value to be a higher value.

Variable		Value
Discount Factor	β	0.98
Taxes	τ	0.18
Money Adj. Cost	ϕ	53
Output Cost	d_0, d_1	-0.47, 0.55
Capital Adj. Cost	Φ	2.8

Table 4: Parameters used to match

While all numbers are co-chosen to match the targets, some seem more tied to certain targets. The discount factor pins down the debt service, the capital adjustment costs tracks the standard deviation of investment, and the output cost upon default is responsible for the default probability, which in the data is taken from CDS spreads. The money adjustment cost is necessary for the CPI inflation and (I +C)/GDP.

Target	Data	Model
Debt Service/GDP	0.114	0.129
(I +C)/GDP	0.801	0.749
CPI Inflation	0.107	0.113
C/G	4.228	4.021
Default Prob.	0.05	0.05
σ_I/σ_Y	3.49	4.18

Table 5: Model vs. Data Comparison

5.5 Untargeted Moments

The model is able to deliver similar correlations to those in the data. The main mechanism is the trade-off between inflation and capital. With that, the realized inflation is correlated with low capital , which itself is highly correlated with high default. The reason is that high past realized inflation implies that in the past the CIA was tight and therefore it created low inflation. Similarly, future high inflation is correlated with low levels of investment, since it means that in the future it is expected to have a tight CIA and therefore low investment. In the short-run, the capital level is more or less fixed because of the capital adjustment cost , while in the long run it can fluctuate more as the capital adjustment costs are spread out. Finally, lower levels of capital are associated with higher levels of default as the repayment capacity becomes lesser, and governments can not keep rolling their over their debts or inflating them away.

Untargeted	Data	Model
$\rho(B, \pi_0)$	0.59	0.48
$\rho(\mathbb{D}_1, \pi_1)$	0.11	0.09
$\rho(\mathbb{D}_5, \pi_5)$	0.75	0.62
$\rho(Y, \mathbb{D}_0)$	-0.62	-0.43

Table 6: Untargeted moments

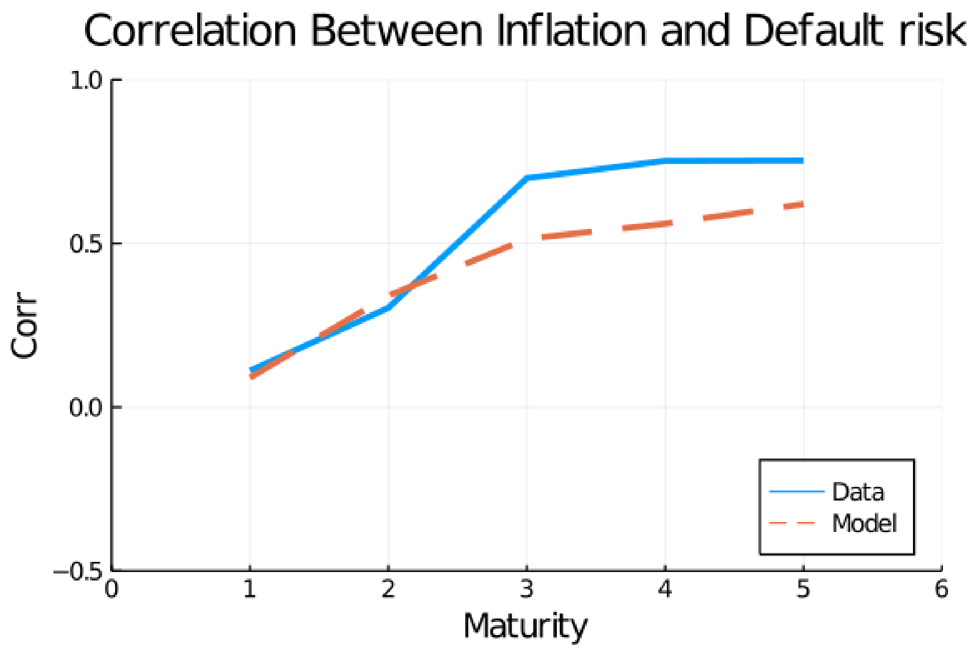

Figure 3: Comparing the correlations of expected inflation and expected default

Similarly, the model delivers the correlations found in the empirical section. Since inflation is correlated with its past self, inflation in 5 years informs us about what the inflation in 4 years probably was. Although the model matches the untargeted correlations closely, it does underestimate it in the long run.

5.6 Investment Counterfactual

Finally, the model can be used to estimate if Brazil under invested in this time period. I will solve two different models, one will be the one presented before, and the other one will be with everything the same except the government will have to borrow in real debt instead of nominal. Using the same parameters, I feed the series of productivity shocks to the two different models. I choose the initial level of capital of both models so that the nominal model can match the average investment during this period. Then I compare the investment decision of the households in both cases. This allows me to see what the households would have done if the debt was real.

The model finds that investment would have been on average about 1.1% of GDP higher every year if they would have borrowed in real terms. This result can is driven by the difference in the average level of capital at each steady state. The model with real debt is able to sustain on average more capital as there is more investment as there is less expected seigniorage and inflation. The difference between the average steady states is about 15% higher in the model with real debt. It should be noted that the exercise makes the change from all debt being nominal to all debt being real. However, in the data, borrowing is not done only in nominal terms, so this can be thought of as an upper bound. I also run a secondary exercise to determine the change in investment. Instead of choosing the initial capital to match the average investment period, I match the implied price of capital for 1995. I then compare the average investment and find it would have been .8% lower when the debt is nominal. The difference between the two results is mainly driven by the initial capital, and in the first exercise the initial capital is lower and so households choose to invest more as a percentage of output since the return on capital is higher.

5.7 Welfare Analysis

There exist trade-offs between nominal debt and real debt. Nominal debt offers the government debt that is more state-contingent, as the price level adjusts to negative shocks, and so they pay more in good times and less in bad times. However, since the government lacks commitment, it also creates the temptation to have higher inflation and lower value of liability ex-post. Additionally, real debt can accumulate more capital since there is less temptation to engage in seigniorage, and so have better output in general. The trade-off includes different levels of capital, debt, and inflation, which makes the welfare analysis not

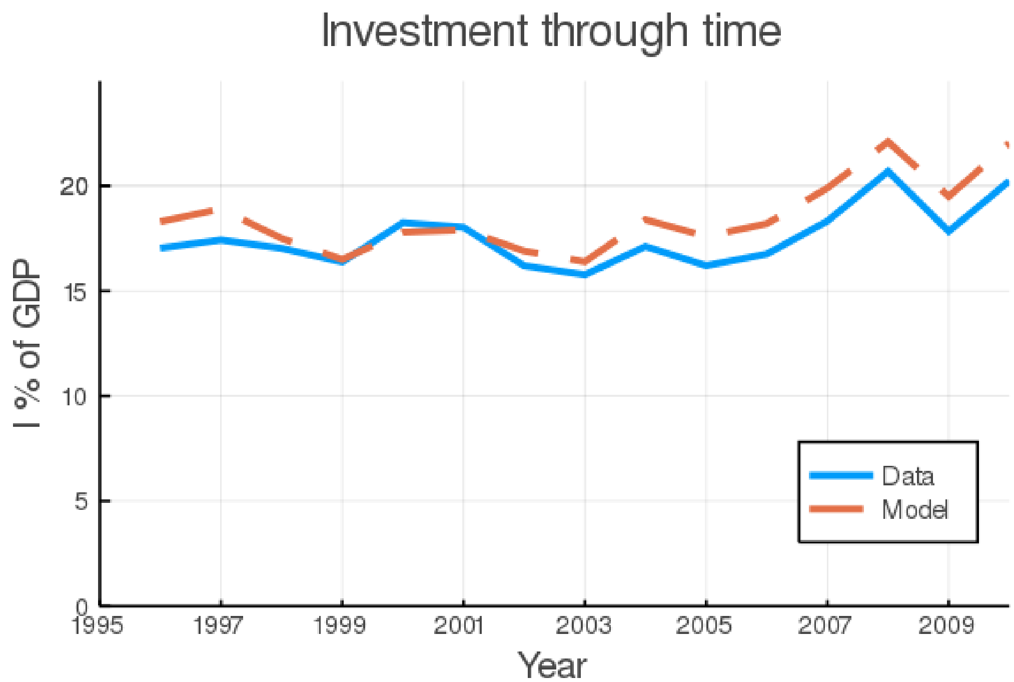

Figure 4: Aggregate investment in the data and the nominal model

be trivial, or has one debt strictly dominate the other one in terms of welfare.

Using the two models, one for real debt and the other one for nominal. I construct a series of consumption paths using the same parameters as the calibration. With both paths of consumption, I can find the welfare consumption equivalent (CE). I find that CE to be 3.3% higher in the real debt model, meaning the households would have to get 2.7% more of consumption to switch from a world with real debt to the one with nominal debt.

With that in mind, I decompose the change in welfare and see how much of the change is because of the higher capital and how much is due to the extra state contingency.

First, I do a consumption equivalent of the real model and one where the nominal model has the capital decision and money growth of the model with real debt. By keeping the capital and money decision fixed, it allows the second model to enjoy the state contingencies of nominal debt without the temptations. The difference should be attributed to the extra state contingency. I find that the CE would be .92%, meaning the households would have to get .92% more of consumption to give up the state contingency.

Afterward, I do a welfare comparison of a model with real debt, and one with real debt but keeping the money growth and debt decision to be the one of the nominal debt. Neither of the two has state-contingent debt, and the difference would be how much more capital is accumulated in the real debt model. The model finds that the CE would be 3.77%, meaning the households would have to get 3.77% more of consumption to give up the extra capital.

6 Conclusion

The paper is able to shed some light into new empirical facts over the correlations between default and inflation. Then I propose a theory about the trade-off associated with inflation and investment, and build on the new mechanism. The model is able to capture the trade-off between inflation and new investment. The model relies on seigniorage being a tool governments use to raise revenue and the incompleteness of the tax instrument, both of which are supported by the data. Finally, after showing that the model can match untargeted moments, I use it to quantify the underinvestment in Brazil between 1995 to 2010.

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