Puzzling Correlations and When to Find Them: Sovereign Spreads and Inflation

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PRELIMINARY AND CLICK HERE FOR UPDATED VERSION *

The paper explores a new novel pattern in correlations between default risk and currency depreciation risk. It uses data from secondary bond markets to proxy default and inflation risk for six developing countries. The default risk is proxied by using the yield from USD denominated bonds, and the currency depreciation risk is proxied by the spreads between inflation-linked bonds and not-linked bonds. I look at the implied yield in exchange rate forwards between LCU and USD. When the results are compared to the ones found in previous studies, it shows that the positive co-movements between currency depreciation risk and default risk disappears in the short horizon, broadly defined as less than two years. These relations remain when doing robustness checks by using other measures for currency depreciation risk. I use the correlation curve for different maturities, to discipline a model that explores a new mechanism of the trade offs associated with inflation and investment. The model is used to evaluate welfare loss for Brazil between 1995 and 2010, and quantify underinvestment because inflation risks.

I. Introduction

The shift in the currency composition of developing countries sovereign debt portfolio has brought the issues associated nominal debt into the consciousness of the sovereign debt literature. In recent years, developing countries have started to borrow more in their local currency, a change from the status quo of the 1980s and 1990s. With this decision, the risk of sovereigns inflating away real obligations has started become more relevant. Theoretically, we can think of a sovereign using inflation as a way to partially default on a portion of their bonds. However, empirically inflation and default have been more associated with being

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complements; we have seen both long-term risk co-move, and default episodes are associated with high inflation. Understanding how governments would like to reduce real liabilities by using inflation while balancing the associated cost is paramount to understanding the actual mechanism for which inflation impacts the incentives and the ability to repay.

My paper answers the question how inflation affects future ability to repay and how to it might vary depending on the time frame we are looking at. While a lot of the literature has focused mostly on how inflation changes the incentives to default, this paper also tries wrestles with the question of how it changes the future ability to repay through capital accumulation or decumulation. This mechanism brings into question the conventional wisdom that overcoming the original sin of developing countries was a triumph with no negative consequences. Furthermore, it focuses on how it changes between short and long run. It empirically documents how these properties can be seen in the correlations between inflation risk and default risk at different maturities, by looking at the changes in prices in secondary markets for bonds and currency.

First, I empirically show that for a group of developing countries the correlation between implied yields in currency forwards and default risk disappears in the short run while it is high in the long run. Similarly, the correlation between inflation risk and default risk also seems to only be relevant in the long run. Recent literature has focused only on comparing the realized inflation with default risk, or comparing long-term inflation risk and default risk. The new facts are surprising since previous literature have treated it as flat, and is not easily explainable. It suggests a new mechanism is needed for models of nominal debt with lack of commitment.

With these new correlations in mind, I build a model that makes inflation not be memoryless. My framework incorporates classic elements of monetary models, like a Cash-in-Advance constraint (CIA) à la Stockman (1981), into a small open economy framework with lack of commitment. The model is able to speak to the trade-offs between inflation and future output by incorporating that future capital decreases with inflation. It also means that the government faces a trade off between lowering their default probability today by increasing it tomorrow. The approach is new to the literature since most take output to be a stochastic process where inflation only matters in the price of new bonds and how much you have to repay today, which implies that the correlation between expected inflation and default does not vary with the maturity structure.

Finally, I use my model to measure the effects that borrowing in nominal debt had on Brazil between 1995-2010. Brazil is an interesting example as they had faced hyperinflation before this time period and experienced rapid growth after 1995. They also implemented reforms to get their inflation down but their central bank is not fully independent and their debt is denominated in local currency. So it provides an environment in where inflation and default risk, nominal debt, and large capital investment are all present. The model is able to match the moments fairly well, and finds large yearly underinvestment attributed to the risk of future inflation. The result is surprising not only in magnitude but also given that Brazil is usually regarded as a success story of growth and how to get inflation under control.

I use Bloomberg to get prices from secondary markets for bonds and currency futures. I rely on the fact that all of the countries issue part of their bonds in USD, so the spreads between local bonds issued in USD and US treasury bonds is the expected inflation. As a robustness check, using credit default swaps (CDS) gives similar results. Following Du and Schreger (2016) amongst others I use the implied yield in futures to measure the expected depreciation of a currency against USD. Finally, to get a better measure of expected inflation in the short term I use the spreads of Local Currency (LCU) bonds indexed-to-inflation and LCU bonds not indexed to inflation. The underlying assumption is that the probability of default is the same for both, and the only difference is the expected inflation.

The model is a small open economy with households, government, and international investors. Government issues nominal debt, and chooses government consumption and money growth. Households choose private consumption, and choose money balances and capital for next period. Households have a CIA constraint on both consumption and investment, and face adjustment costs of their money balances and capital. It is important to the model that the government can't change taxes or chose capital directly. Taxes are taken to be exogenous and so have the only source of government revenue to be seigniorage. Through the CIA is that inflation distorts the capital investment decision; by lowering the real money balances, households decrease their capital holdings to smooth out consumption. Households don't internalize the effect of capital in next periods default probability. Then I use simulated method of moments to match the facts for Brazil 1995-2010.

I find that correlation between expected depreciation of a currency and expected default goes on average from 0.07 in the 1-year maturity structure to 0.48 in the 5-year maturity. The result is robust to all countries, and to using CDS instead of spread. Similarly, the correlation between expected inflation and expected default goes for 0.15 at 1 year to 0.76 at 5 years. The expected inflation is only done for Brazil, Turkey, and Peru due to data constraints. The 5-year correlations are inline with has been found by Galli (2020) and in similar magnitudes to the correlation between realized inflation and default found in Arellano, Bai, Mihalache (2020). This paper is the first one, to my knowledge, to document the short-term correlation, and the differences in term structures.

For Brazil between 1995 and 2010, I also find that investment was on average about 1.1 % of GDP lower every year compared to a baseline in which they had borrowed in dollars or linked-to-inflation debt. It is mostly due to the difference in the amount of capital that they are able to sustain. Borrowing in real debt leads to fewer defaults and less seigniorage, and so the return on capital is higher. It is a surprising find in the monetary literature as Gomme (1993), De Gregorio

(1993) and Jones and Manuelli (1995) all find modest effects of inflation on capital accumulation. The main difference is that in my framework there is lack of commitment with respect to debt and the government has extra incentives to use seigniorage to lower debt obligations and get revenue. Section 3 will discuss more in-depth the model and the validity of the assumptions.

My road-map for the paper is to first go into the empirical work, and focus on showing the correlations between inflation risk and default risk. The empirical work should be thought of as being at least a hint of what the mechanism the literature missing is. Then build a sovereign default model that has a capital and CIA on consumption and investment as to incorporate the trade offs associated with inflation and future ability to repay. The main idea being that inflation affects future capital and is very correlated with itself. To test the model, it will be use to match moments from Brazil 1995-2010. And given those match, I will look at the counterfactual of what would had happen if they had borrowed in real debt rather than nominal debt.

A. Related Literature

The topic of inflation and sovereign debt isn't a new one, and this paper owes a lot to its intellectual predecessors. The general framework is building on sovereign default models following the path of Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2006) and subsequent work since. More specifically, it contributes to three different strands of the literature.

In terms of money and sovereign default, the closest work to this is Galli (2020), Roettger (2019) and Sunder-Plassmann (2018). They consider the interaction of monetary policy in default decisions. I add to their work by adding capital which makes inflation have an effect on future output and repayment. It allows me to explore the dynamic trade-off that inflation poses, by reducing default today but increasing it tomorrow. Similarly, Gomez-Gonzalez (2019) explores the role of inflation-linked debt, while abstracting from inflationary and capital concerns. Aguiar Amador Farhi Gopinath (2013) explore the credibility of joining monetary unions and exposures to self-fulling crises, something this paper doesn't consider because of tractability. Finally, Arellano, Bai, Mihalache (2020) merge a New Keynesian open economy with a sovereign debt framework. Their work includes capital, but monetary policy follows a rule and government borrows in real debt. In contrast, I focus on an environment in where there is strategic inflation to lower real liabilities. Because of tractability, this paper abstracts from the portfolio choice between nominal and real debt, which is analyzed by Ottonello Perez (2019), Engel Park (2018) and Du et al. (2016).

The paper also adds to the literature that studies soverign defualt and capital. Gordon and Guerron-Quintana (2018), and Park (2017) both focus on the interactions between capital accumulation and sovereign spreads. They show that it is not trivial since capital changes both the value function to default and to repay, something that is present in the model. Esquivel (2020) studies the impact of giant oil discoveries on the sovereign debt, and shows they increase the probability of default. I build on their work by integrating their capital adjustment costs, while adding money and nominal debt. The model produces an inverse relationship between inflation and capital investment through the CIA.

Finally, there is a long tradition in monetary frameworks of including capital. Tobin (1965) shows that an increase in the monetary growth rate leads to capital formation in the long run. However, an anti-Tobin effect is also possible, as shown by Stockman (1981) and Abel (1985). Cooley and Hanson (1989, 1991) also find anti-Tobin effect when labor supply is endogenous. I follow in the tradition of Stockman (1981) and Abel (1985) and build a model with a CIA constraint on capital investment. Compared to their model, this paper deals with both a small open economy, and the government lacks commitment. Wright (2011), Janiak, Santos Monteiro (2011), and Arawatari (2018) all also focus on the negative relationship between money growth and inflation.

II. Empirical

I use Bloomberg to get monthly prices from secondary bond and currency markets, and focus on the time period between 1/1/2004 to 2/1/2020. I use the earliest time frame possible and end before the COVID epidemic since the paper won't have anything to say about the epidemiology factors of the current crisis (A better discussion can be found in Arellano Bai Mihalache (2020) since they focus on COVID implications on sovereign debt and default). I look at Brazil, Colombia, Mexico, Argentina, Peru, Chile and Turkey. All of these are countries that borrow in local currency and have nonzero default probabilities. In future work, I would like to expand the group of countries but there are data requirements that make the list hard to add to.

$$Spreads_T = r_{USD\ Foreign\ Bonds,T} - r_{US\ Bonds,T} \tag{1}$$

I measure the risk of default by the spread for bonds that are denominated in USD compared to US Bonds. I get the updated data of the US nominal yield curve from Gurkaynak, Sack, Wright (2006) and use it as the benchmark of the risk free rate for different maturities. For the local bonds denominated in USD, I use the yield-to-maturity for the average price traded for that month. All countries have enough of their debt denominated in USD and the markets are active enough. Since the bonds are denominated in foreign currency they are not exposed to currency depreciation risk, so the remaining of the spread should be the default risk.

$$i_T - i_{T^*} = \frac{Fwd_T}{Spot} \tag{2}$$

The currency depreciation risk is measured by using the implied yield in exchange rate forwards between LCU and USD. It measures the expected currency depreciation over the US dollar, following Following Du and Schreger (2016). While it serves a good proxy for the expected inflation in the long term, in the short term it has the problem of Fama (1988) regression that shows that in the short-term forwards are bad predictors of future inflation. It is still a worthwhile object to consider for two reasons. One it is the price that a foreign investor would have to pay in order to hedge currency risk, regardless of the maturity. Secondly, the data is more widely available than other measures of expected inflation.

$$Spreads_{i\,T} = r_{Inflation-Linked\,Bonds,T} - r_{Not-Linked\,Bonds,T} \tag{3}$$

As a way to measure expected inflation, I use the spreads between bonds issued in LCU that are indexed to inflation and bonds issued in LCU that are not. In the price of both bonds, default risk is present. However if we assume that both bonds have the same probability of being defaulted on, then the difference should be the expected inflation. It is a new way of measuring expected inflation. The downside is that in order to get the measure of expected inflation, a lot of data is needed, specifically sovereigns need to issue bonds issued that are index to inflation and not. The requirements limit the countries to just be Brazil, Turkey, and Peru.

An important point to make is that the correlations are between inflation risk and default risk in the same maturity. So the puzzle comes from that in a year if there is inflation, it does not correlate with default, but if it is in 5 years then it correlates highly with default. The theory of this paper is that inflation in 5 years means inflation for the next 5, which implies the decimation capital stock, while inflation tomorrow means only on period, which means it is already priced in and other factors dominate.

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	3M	**6M**	**1Y**	**2Y**	**3Y**	**4Y**	**5Y**	**10Y**
Mexico	-0.007	0.063	0.132	0.259	0.374	0.445	0.377	
Brazil	0.012	0.128	-0.003	0.128	0.331	0.33	0.485	0.506
Peru	-0.016	-0.009	-0.029	0.394	0.241		0.42	
Colombia	-0.049	-0.79	0.039		0.129			
Turkey			0.21	0.662	0.472		0.467	
Argentina			0.558		0.66			
Chile			0.151	0.405	0.412		0.769	
Average	-0.015	-0.152	0.151	0.370	0.374	0.388	0.504	0.506

Table 1—Correlations of Spreads on USD denominated debt and Implied Yield of Currency Futures

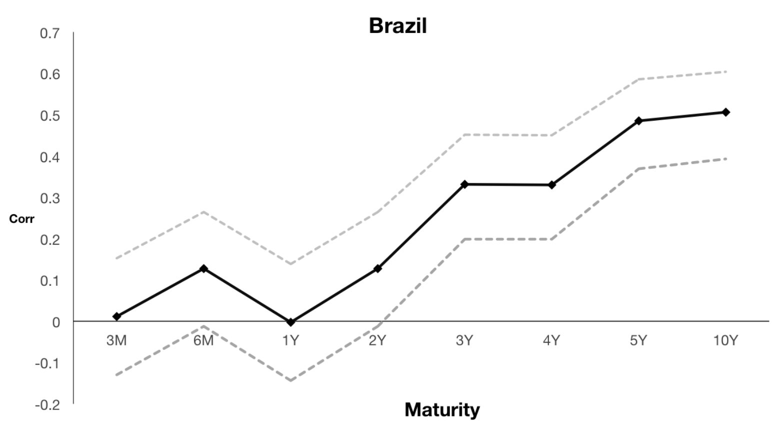
Table 1 shows the correlation between correlation between default risk (Equation 1) and currency depreciation risk (Equation 2). The remarkable quality is the way in which almost all of the countries exhibit statistically zero correlation if the maturity is less than 1 year, with Argentina being the exception. The general pattern is that the correlations become stronger as the maturity becomes farther into the future. It is somewhat instructive to look at the averages and see that the 1-year has a correlation of 0.151 while the 5 year has .504, almost 5 times higher.

	1Y	**2Y**	**3Y**	**4Y**	**5Y**	**8Y**
Brazil	0.263	0.439	0.771	0.813	0.813	
Mexico		0.252	0.22	0.43	0.505	0.502
Chile	0.169	0.329		0.36	0.485	
Average	0.216	0.340	0.495	0.534	0.601	0.502

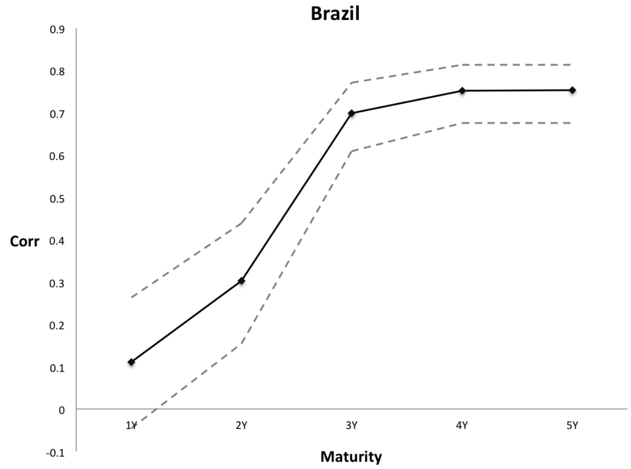
Table 2—Correlations of Spreads on LCU Linked-to-Inflation Bonds and LCU Not Linked Bonds

Due to the data requirements, most countries are lost when switching over to Table 2. It shows the correlation between correlation between default risk (Equation 1) and inflation risk (Equation 3). A similar pattern emerges where in the short run (less than 3 years) the correlation is low, yet in the longer horizon it rises. An interesting feature is that the correlations at long term, both in Table 1 and Table 2, have similar magnitudes to the correlations of realized inflation and default risk in previous literature.

Correlation between Default Risk and Currency Depreciation Risk



Correlation between Inflation Risk and Default Risk



The general shape of the correlations is fairly robust to using quarterly, daily, or even yearly data. I also check that using CDS for default risk does not change the results. While one might worry about the effects coming from the Great Recession, the results hold whether or not the time period includes it, but does mechanically increase the standard errors because of the loss of data. Running a panel regression on the spreads vs. US bonds against the implied yields while using country and time fixed effects also does not change the significance of the short and long maturities.

III. Model

The model will be an infinite-horizon small open economy model, where time is discrete and indexed by $t \in \{0, 1, 2, \dots\}$. There are 3 types of agents: Households, government, and international lenders. Households are atomistic, with measure one, and will choose private consumption, and money balances and capital for next period. Money is used for transactions this period which gives rise to a cash-in-advance constraint on household consumption and investment. Households also face a quadratic adjustment cost of changing both their money balances and capital. The benevolent government will have endogenous government spending, issue nominal debt, and have strategic default and inflation. Taxes are fixed and so the only source of revenue that the government can change is seigniorage. Finally, foreign and risk neutral lenders buy bonds from the government, and have a zero profit condition.

A. Timing

The timing in the model is as follows: At the start of each period, the government can be either in good standing or in financial autarky, depending on its default history. The state of the world is summarized by nominal bonds from the government B_t, money balances M_t, a stochastic productivity shock θ_t, and capital inherited from a previous period K_t. The government then announces its choices of default, bonds (0 if they are in default), money balances, and government expenditure. While the government doesn't have commitment from period to period, it is natural to assume they have commitment within a period and their announcements are binding. The households, taking the government policy functions, prices, and state of the world as a given, decide their consumption, money balances and investment for next year. Finally production and consumption happens and if the country isn't in default, investors are paid in final goods.

B. Households

Preferences

The representative household has preferences given at time T by:

$$\mathbb{E}_T \left[\sum_{t=T}^{\infty} \beta^t (U(C_t) + (1 - L_t)) + U_G(G_t) \right]$$

Where $\mathbb{E}_T$ is the expectation operator conditional on date T, $0 < \beta < 1$ is the discount factor, $0 \leq L_t \leq 1$ is the labor given that period, and $U(\cdot)$ is the utility function on private consumption assuming the standard conditions. $U_G(\cdot)$ is the utility of consuming the government expenditure, but its separable from the private consumption and since households take government as given, they take it as a fixed transfer.

Budget Constraint

$$C_t + \frac{M_{t+1}^{hh}}{P_t} + I_t = \frac{M_t^{hh}}{P_t} + (1 - \tau)(\theta_t F\left(K_t, L_t\right) - \frac{\psi}{2} \left(\frac{M_{t+1}^{hh}}{M_t^{hh}} - 1 \right)^2)$$

The left side of the budget constraint for the household is simple; they either consume, invest, or buy money balances for next period. The right side is the income, they receive real money balances and they produce. Production is taxed exogenously at a constant rate, and they also face a cost of changing their money balances. While not being standard, the cost is meant to capture the cost of inflation on output in the present period, and so its made to be in the tradition of Rotemberg (1982). The constraint will help keep inflation down in the calibration as otherwise the government faces only the misallocation of government consumption with respect to private consumption as the only cost of money growth.

Investment is defined by:

$$K_{t+1} - (1 - \delta)K_t = I_t - \phi(K_t, K_{t+1})$$

δ is the constant depreciation rate, and $\phi(\cdot,\cdot)$ is the capital adjustment cost. It is standard, as seen for example in Gordon and Guerron-Quintana (2018) and Esquivel (2020). It will be used to match the volatility of investment.

Households do not have access to save in international markets. They can only smooth consumption by saving in either money balances or capital. Capital has a stochastic return given by the expected derivative of the production function. In the other hand, money in this economy can be thought of a type of government debt, which is also needed to buy the final good (The next section will talk in more detail about it). Inflation, in this economy, is a kin to partial default of the government debt held by the households, since as the government increases the money supply, the real money balances go down. The lack of a safe asset plays an important role: household want to use their stock of capital to smooth out consumption in bad times, exactly when the return of capital goes down; however the adjustment costs of capital prevents big swings in the capital stock.

Cash-in-Advance (CIA)

Households will face a cash-in-advance constraint:

$$C_t + I_t \leq \frac{M_t^{hh}}{P_t}$$

The usual story is that there are 2 people in a household. One is a worker and the other is a shopper. At the beginning of the period the shopper take the money balances and goes to buy investment and consumption for next period in a decentralized markets. Simultaneously and independently, the worker sells his labor in a decentralized market for labor and gets paid at the end of the period and choose money balances for next period. The set up is a natural extension which creates the need for money and gives rise to the CIA.

The constraint, while new to the sovereign debt literature, has a long history in monetary frameworks. Stockman (1981) and Abel (1985) provide the original CIA constraint on capital investment in the Neoclassical growth model to show that inflation can lead to lower capital accumulation/growth since inflation acts as a tax on agents' holdings of real money balances, ultimately reducing the capital stock. Cooley and Hanson (1989, 1991) find similar results when they use endogenous labor supply. Recently, Wright (2011) has shown that this CIA can be thought of as a lower bound for the effect of money on capital accumulation and welfare since it doesn't capture moments related to velocity, and misses market structures like bargaining, which could have large welfare effects.

CIA will be a main component of my results. The interaction, while simple, is elegant. As government increases the money supply, it will drive the price of money in term of good to also go up. In turn, the CIA will tighten, and households will adjust by reducing investment to smooth out the decrease in consumption.

C. Government

The Government is benevolent and so has the same utility as the households. So the flow utility at period T is just:

$$U(C_t) + (1 - L_t) + U_G(G_t)$$

where the government does choose G_t and takes into account how their decision will change C_t and L_t.

Similarly, the budget constraint of the government is

$$G_t + \mathbb{D}_t \frac{B_t}{P_t} = \mathbb{D}_t q(\cdot) B_{t+1} + (\frac{M_{t+1}}{P_t} - \frac{M_t}{P_t}) + \tau \theta_t F(K_t, L_t) - \frac{\psi}{2}\left(\frac{M_{t+1}}{M_t} - 1\right)^2$$

where G_t is government consumption, $\mathbb{D}_t$ is if the government is in default or not, B is nominal government bonds, q is the price at which the bonds are traded, and M is the money balances

The left-hand side is expenditures, which are only government consumption, and repaying debt, if there is any. The right-hand side is the income, which breakdown into new debt, seigniorage, and tax income. It should be clear to see that seigniorage is just the difference between the money balances issued today and the ones owed to them today.

D. International Lenders

International lenders are assumed to be infinitely, and so make zero profits in expectations. Since bonds can be both diluted through inflation and can be defaulted, the expected inflation will appear in the pricing of the risk neutral along side the default probability.

It gives rise to the following pricing equation:

$$q(B_{t+1}, M_{t+1}, \theta_t, K_{t+1}) = \frac{1}{(1+r)} \mathbb{E}_t \left[\frac{1 - D_t(B', M', \theta', K')}{(1 + \pi_{t+1})}\right]$$

where D_t is the default probability and $(1 + \pi_{t+1})$ is the inflation tomorrow.

Given that we have defined the price, we can now look at the model equivalent of the derived default and inflation risk in the empirical part.

So default risk at time t+i while on period t is :

$$\mathbb{D}_{t+i} = \mathbb{E}_t[D_{t+i}]$$

and since in the model the inflation risk and the currency depreciation is the same we just have inflation risk for the next period being:

$$\mathbb{E}_t[1 + \pi_{t+1}] = \mathbb{E}_t[\frac{P_{t+1}}{P_t}]$$

$$= \mathbb{E}_t[\frac{P_{t+1}}{P_t}\frac{M_{t+1}}{M_{t+1}}\frac{M_t}{M_t}]$$
$$= \mathbb{E}_t[\frac{M_t}{P_t}\frac{P_{t+1}}{M_{t+1}}\frac{M_{t+1}}{M_t}]$$
$$= \frac{M_t}{P_t}\frac{M_{t+1}}{M_t}\mathbb{E}_t[\frac{P_{t+1}}{M_{t+1}}]$$

This will be an easier object to compute, and we can similarly, define inflation risk for time as t+j instead.

IV. Equilibrium

I focus on a Markov perfect equilibrium where the government understands the effect of its choices on the household allocations, and prices while takes futures policies as given. Households are atomistic and take both government current and future policy choices as given as well as prices. I drop the time subscript and adopt " ' " to mean the next period value.

A. Normalization

The state variables are θ, B,K,M, where θ is the exogenous productivity and the rest are endogenous choice variables. B,M are given by the government problem, and K by the households; with market clearing making the money demand of the household be equal to the money decision of the government. In this class of models, the ratio of B/M is a sufficient statistic for the government endogenous state. So I normalize all nominal variables by dividing by M and denote them with $\hat{}$. Therefore, the state becomes $(\theta, \hat{B}, K)$

Define the set of current government choices to be $s := (\mathbb{D}, \hat{B}', G, \mu)$ where $\mathbb{D}$ is the default decision, $\hat{B}'$ is the normalized future bonds, G is government consumption, and $\mu = \frac{M'}{M}$ is money growth. Government policy functions is then defined as a mapping from aggregate states to policy choices $\mathcal{H} : (\theta, \hat{B}, K) \rightarrow s$.

Since HHs take the current and future actions of the government as given, define S to be $S = (\theta, \hat{B}, K, s)$ and the inverse of the normalized price level is $m(S) = \frac{M}{P}$

B. Private Equilibrium

Definition 1 Private Sector Equilibrium (PSE):: Define a PSE as: Given aggregate state and current government policies S, future government polices $\mathcal{H}$, PSE is a $(C(S), L(S), \hat{M}^{hh}, K'(S), m(S))$ such that:

1) They solve the HHs maximization problem subject to budget constraint, and CIA

2) Market clears for money balances, $\int \hat{M}^{hh} = 1$

C. Recursive Problem

The recursive problem of the government can be described as:

$$V(\hat{B}, \theta, K) = max_{\mathbb{D}\in\{0,1\}} \left\{ (1-\mathbb{D})V^r(\hat{B}, \theta, K) + (\mathbb{D})V^d(\theta_{def}, K) \right\}$$

where: $\mathbb{D}$ is the default decision, B is the bonds held by international investors, θ is stochastic productivity, and K is capital inherited from the previous period.

The value function for repayment then is:

$$V^r(B, M, \theta, K) = max_{\hat{B}',\mu,G} \quad U(C(S), L(S), G) + \beta\mathbb{E}\left[V(\hat{B}', \theta', K'(S))\right]$$

subject to the feasibility:

$$C(S) + G + \hat{B}m(S) + I = q(\hat{B}', \theta, K'(S))\hat{B}'\mu m(S) + \theta F\left(K, L(S)\right) - \frac{\psi}{2}\left(\mu - 1\right)^2$$

and CIA:

$$C(S) + K'(S) - (1-\delta)K + \phi(K, K'(S)) \leq m(S)$$

and where $(C(S), L(S), K'(S), m(S))$ are in a PSE.

Similarly, the value function for default is:

$$\begin{array}{c} V^d(M, \theta, K) = max_{\mu,G} \quad U(C(S), L(S), G) + \Theta\beta\mathbb{E}\left[V(0, \theta', K'(S))\right] \\ +(1-\Theta)\beta\mathbb{E}\left[V^d(\theta', K'(S))\right] \end{array}$$

$$\begin{array}{c} ST :: C(S) + G + I(S) = \theta_{def}F\left(K, L(S)\right) - \frac{\psi}{2}\left(\mu - 1\right)^2 \\ C(s) + K'(s) - (1-\delta)K - \phi(K_t, K_{t+1}) \leq m(S) \end{array}$$

where $(C(S), L(S), K'(S), m(S))$ are in a PSE.

Definition 2 Markov Perfect Equilibrium (PSE):: A Markov perfect recursive equilibrium is

- Value functions $(V(\hat{B}, \theta, K), V^r(\hat{B}, \theta, K), V^d(\theta_{def}, K))$
- Policy functions $(\mathbb{D}, \hat{B}', G, \mu)$
- a PSE, $\mathcal{P}$

such that given the aggregate state and debt price functions q :

1) Policy functions maximize the government problem, given $\mathcal{P}$

2) $\mathcal{P}$ corresponds the value and policy functions

3) current policy functions $\in \mathcal{H}$

V. Mechanisms

The model exhibits a static trade off between defaulting and suffering a productivity penalty, and inflating away your obligations and suffering the cost of adjusting money balances. The sovereign tries to balance the costs as it smooths out consumption but cant decrease capital too quickly. Since default is binary, money growth can serve as a way of partially defaulting on both international lenders and households. The result is that the government prefers to inflate away some of its obligations in order to decrease the probability of default today.

Inflation also serves as an ex-post state contingency. You print money and reduce repayment in bad times. However, Problem is ex ante would like commitment, since it implies cheaper debt. Capital is also chosen sub-optimally, since it is chosen by hhs, who don't internalized effect of capital on default probability or price of bonds.

The dynamic trade off at pay is that in general, money growth tightens the CIA constraints which less investment. Similarly, all being equal, less capital implies a higher probability of default. The sovereign then faces a choice between lowing their default probability today but raising it tomorrow.

All put together imply that that the governments use inflation to reduce their money balances, household then lower capital too much, and so leave the sovereign in a worse state for next period. The impatience compared to the foreigners makes this trade worth it. Debt rollover is also preferred since in bad times, the marginal utility is high, the return on capital is low, but lowering capital too much becomes costly because of the adjustment costs.

VI. Quantitative Exercise

After a series of reforms in the early 90s, Brazil managed to stop hyperinflation. However, the country continued to borrow in Brazilian Real, experienced inflation at higher rates than some of its neighbors, while also going through a period of relatively good GDP growth. While no default occurred during the 1995-2010 period, CDS rates remained constantly around 5%. For a quantitative exercise, I put the model to the data and try to match key moments for this time period in Brazil.

I use World Development Indicators, and IMF data to get data on output, private and government consumption, capital formation, investment, debt, and inflation. I use Bloomberg to get information on bond prices and CDS spreads.

A. Parameterization

I use a quarterly model as my data in the empirical section was quarterly. I use standard functional forms in the literature.

Household and government preferences are

$$U = \frac{C^{1-\rho}}{1-\rho} + \frac{G^{1-\eta}}{1-\eta}$$

The production function is taken to be

$$F(K,L) = K^{1-\alpha}L^{\alpha}$$

while the productivity penalty is

$$\theta_{def} = \theta - max\{0, d_0\theta - d_1\theta^2\}.$$

And the capital adjustment cost is

$$\phi(K_t, K_{t+1}) = \phi_0 \times (K'(s) - (1-\delta)K)^2.$$

I take productivity to follow an AR(1) process.
The parameters chosen exogenously are as follows:

Variable		Value	Source
U C curvature	ρ	2	Literature
U G curvature	η	4	Galli (2020)
Risk free rate	r	0.006	US treasury
Auto correlation	κ	0.912	Estimated
Output st dev	σ_ϵ	0.023	Estimated
Re-entry	Θ	0.1	Aguiar and Gopinath (2006)
Depreciation	δ	0.020	7.7% Yearly
Capital share	$1-\alpha$	0.37	Literature

Table 3—Exogenous Parameters

Table 1 shows the values of the exogenously chosen parameters. The curvature of the utility function of private consumption is standard. However, the curvature of the utility on government expenditure is taken from Gali (2020), while the exact value isn't critical to the results, it is important that $\rho < \eta$. The depreciation rate is chosen to be consistent with IMF estimates and World Bank estimates. However, more work is needed to really get a good estimate of capital depreciation. Gordon and Guerron-Quintana (2018) have a discussion about the role of capital depreciation in sovereign default models and conclude that one period models will miss their target of investment-output ratio in the steady state without foreign lending.

B. Solution method

The solution method follows Gordon (2019), Dvorkin et al. (2018), Arellano et al. (2019), and Galli(2020). I add taste shocks to smooth out the probabilities of the policy functions of the government. Otherwise, the model struggles to converge due to having money and capital. I choose the smallest values such that the model converges.

C. Moments

Table 2 shows the moments used to match moments. The discount factor, while low, is still within the previously used range, and serves to drive the model towards impatience. The taxes are chosen to be exogenously, this can be seen as capturing some political economy forces that do not allow for quick responses in taxes. Here they are chosen to be lower than the "optimal" so the government has an extra incentive to use seigniorage to raise revenue. The costs do not have an natural way of interpreting the magnitude of the numbers. The default cost seem around what the literature uses, and so do the capital adjustment costs. Given that it is new, the Money Adjustment Cost has no literature to compare to. Arellano et al (2020) use Rotemberg (1982) adjustment costs and find the value to be even higher.

Variable		Value
Discount Factor	β	0.95
Taxes	τ	0.18
Money Adj. Cost	ϕ	53
Output Cost	d_0, d_1	-0.47, 0.55
Capital Adj. Cost	Φ	2.8

Table 4—Parameters used to match

While all numbers are co-chosen to match the targets, some seem more tied to certain targets. The discount factor pins down the debt service, the capital adjustment costs tracks the standard deviation of investment, and the output cost is responsible for the default probability, which in the data is taken from CDS spreads. The Money Adjustment Cost is necessary for the CPI inflation and the (I +C)/GDP.

Target	Data	Model
Debt Service/GDP	0.114	0.129
(I +C)/GDP	0.801	0.749
CPI Inflation	0.107	0.113
C/G	4.228	4.021
Default Prob.	0.05	0.05
σ_I/σ_Y	3.49	4.18

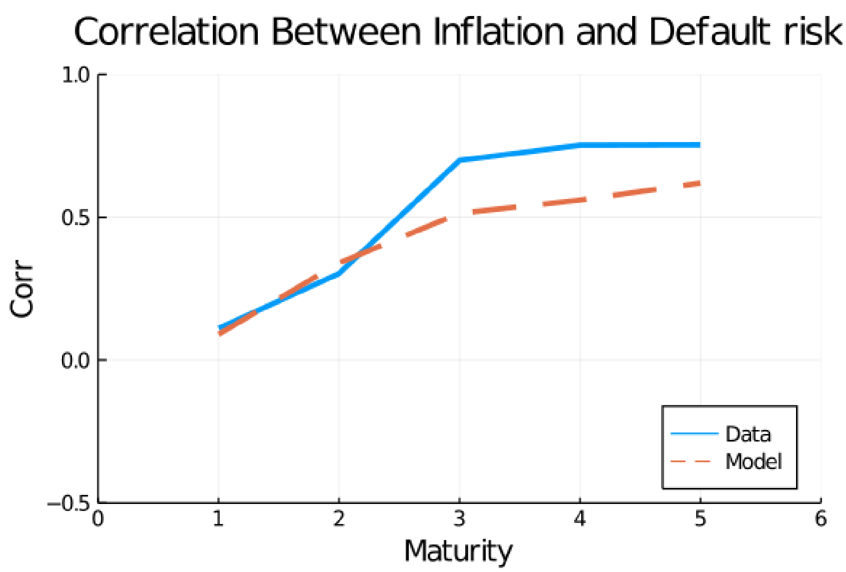
Table 5—Model vs. Data Comparison

The model is able to deliver similar correlations to those in the data. The main mechanism is the trade-off between inflation and capital. With that, the realized inflation is correlated with low capital which is highly correlated with high default. Similarly, future high inflation is correlated with low levels of investment. In the short-run, the capital level is more or less fixed, while in the long run it can fluctuate more as the capital adjustment costs are spread out.

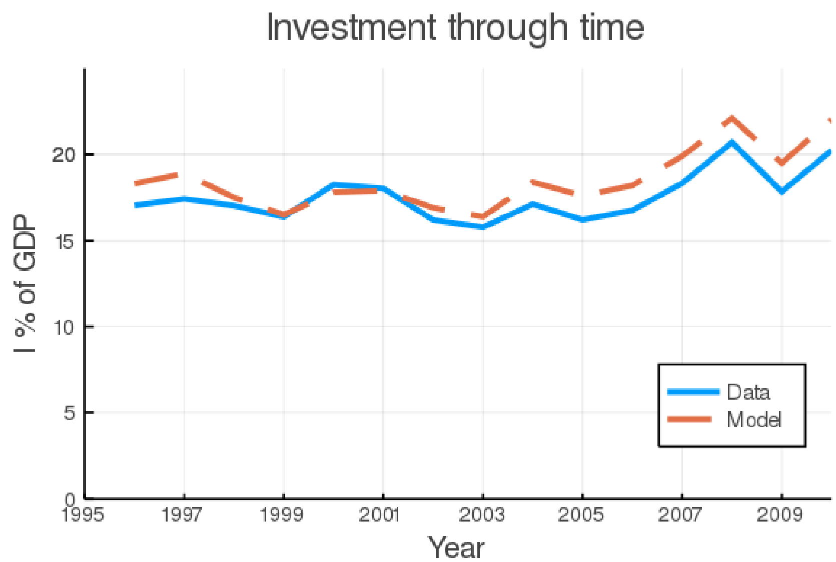
Untargeted	Data	Model
$\rho(B, \pi_0)$	0.59	0.48
$\rho(\mathbb{D}_1, \pi_1)$	0.11	0.09
$\rho(\mathbb{D}_5, \pi_5)$	0.75	0.62
$\rho(Y, \mathbb{D}_0)$	-0.62	-0.43

Table 6—Untargeted moments

Similarly, the model delivers the initial correlations found in the empirical section. Since inflation is correlated with itself, the more in the future we look, the more it informs about the path. The model matches the untargeted correlations closely, but it does underestimate it in the long run.



Finally, the model can be used to estimate if Brazil under invested in this time period. Using the same parameters, I feed productivity shocks to two different models, one nominal bonds and another one with real bonds. I start the capital of both models to so that the nominal model can match the investment pattern. Then I can compare investment decision in both cases and the difference can be attributed to borrowing in Brazilian Reals. In the data, the borrowing is not done only in nominal terms, so this can be thought of as an upper bound. At the end, investment would have been on average about 1.1% of GDP higher every year if they would have borrowed in real debt. This result can is driven by the difference in the average level of capital at each steady state. The model with real debt is able to sustain on average more capital as there is more investment as there is less expected seigniorage.



D. To Do

Welfare analysis is yet to be done. The conceptual problem is that nominal debt offers the sovereign debt that is more state contingent so there is less risk. However, real debt is able to accumulate more capital since there is less temptation to engage in seigniorage, and so have better output in general. The trade off include different level of capital, debt, and inflation, which makes the welfare analysis not be trivial.

VII. Conclusion

The paper is able to shed some light into new empirical facts over the correlations between default and inflation. Then I propose a theory about the trade off associated with inflation and investment, and build with the new mechanism. The model is able to capture the trade off between inflation and new investment. The model relies on seigniorage being a tool governments use to raise revenue and the incompleteness of the tax instrument, both of which are suppurted by the data. Finally, after showing that the model can match untargeted moments, I use it to quantify the underinvestment in Brazil between 1995 to 2010.