Duplantier and Saleur Reply: Our model[1] describes a two-dimensional self-avoiding walk (SAW) on the hexagonal lattice, in the presence of hexagons forbidden with probability p, with annealed randomness. The centers of forbidden hexagons have a *site* percolation threshold at $p_c = \frac{1}{2}$. For $p < p_c$, the polymer is freely extended, while for $p > p_c$, it is dense. At p_c, a collapse transition occurs where the SAW takes the geometrical critical properties of a percolation hull, or of domain walls in the low-temperature phase of an Ising $O(n=1)$ model. This identification allows the exact calculation of all the exponents. As pointed out in Ref. 1 and in the preceding Comment[2] by Poole *et al.*, this SAW-forbidden-hexagon model is equivalent for any p to a SAW with nearest-neighbor (nn) attractions and a particular subclass of next-nearest-neighbor[3] (nnn) attractions, while the Θ point of polymers is usually modeled by simple nearest-neighbor attractive interactions. Usually, universality suggests that the introduction of nnn attractions does change the nonuniversal location of the Θ point in the space of microscopic parameters, but not the long-distance infrared tricritical properties. However, this is not rigorous, and it could be that this model of SAW with annealed obstacles leads to a different "Θ' point," as commented in Ref. 2. It is possible[4] that some local anisotropy (curvature) effects are relevant at this Θ' point. (Note that on the hexagonal lattice a walk always turns at each step.) In such a case, the order of the transitions could be higher. But, remarkably enough, the identity of a polymer model with annealed obstacles[5] to the standard model of the tricritical Θ point can actually be established[6] in higher space dimension. Consider the standard Edwards continuum model

$$P(\mathbf{r}) = \exp\left[-\frac{1}{2}\int_0^S \left(\frac{d\mathbf{r}}{ds}\right)^2 ds - \tfrac{1}{2} b \int_0^S ds \int_0^S ds' \,\delta^d(\mathbf{r}(s) - \mathbf{r}(s'))\right], \tag{1}$$

where $\mathbf{r}(s)$ is the configuration in d space of the chain of length S, and where b is the excluded-volume term. Following Ref. 5, we introduce an annealed-disorder set of impurities by multiplying $P(\mathbf{r})$ by

$$P_{\text{dis}}\{\mathbf{r}, \mathbf{R}_i\} = \exp\left[-\beta \sum_{i=1}^{N} \int_0^S ds\, \delta^d(\mathbf{r}(s) - \mathbf{R}_i)\right], \tag{2}$$

where the $\mathbf{R}_i$'s are the positions of the N impurities. The annealed average $\langle \cdots \rangle$ of P_{dis} over the $\mathbf{R}_i$'s can be performed exactly[6] and leads to

$$\langle P_{\text{dis}}(\mathbf{r}, \mathbf{R}_i) \rangle = \exp\left[-\rho \sum_{n \geq 1} (-1)^{n-1} \beta^n \frac{1}{n!} \prod_{j=1}^{n} \int_0^S ds_j \prod_{j=1}^{n-1} \delta^d(\mathbf{r}(s_j) - \mathbf{r}(s_n))\right], \tag{3}$$

where ρ is the impurity density N/V. In (3) the annealed site disorder has induced an *attractive* $n=2$ two-body interaction[6] $-\rho\beta^2$, and *repulsive* $n=3$ three-body one $+\rho\beta^3$. If we multiply (1) by (3), the resulting model has an $n=2$ coefficient[5,6] $b - \rho\beta^2$, and for $n=3$, $\rho\beta^3$. One is then tempted to conclude that there is a standard Θ-point transition at $b - \rho\beta^2 = 0$. However, the (ultraviolet) contributions of higher orders $n \geq 3$ in a regularized model must be taken into account since they all induce lower effective interactions, in particular at the level $n=2$, and change the location of the Θ point, if any. This can be done[6] in the case of the recurrent model (3) and a standard Θ transition does indeed take place at some value of the impurity density ρ. One ends up then with a standard Θ-tricritical model with only two infrared-dominant interactions $n=2$ and $n=3$ for $2 < d < 4$. At $d=2$, as in usual field theory, all interactions in (3) are infrared relevant, and have the same magnitude, hence possibly leading to new instabilities. In two dimensions, the next study undertaken will be that of the universality of the SAW-percolation model against change of lattice. In particular, if it were true that the hexagonal lattice leads, because of a very peculiar set of nn and nnn attractions, to a threefold unstable Θ' point, a (e.g.) square lattice model should fall in a different universality class. On the contrary, if the percolation model is stable, this would reinforce the belief that it describes indeed the Θ point.

B. Duplantier and H. Saleur
Service de Physique Théorique
Centre d'Etudes Nucléaires de Saclay
91191 Gif-sur-Yvette Cedex, France

Received 23 December 1987
PACS numbers: 36.20.Ey, 64.60.Ak, 64.60.Kw, 75.40.Cx

[1] B. Duplantier and H. Saleur, Phys. Rev. Lett. **59**, 539 (1987).
[2] P. H. Poole, A. Coniglio, N. Jan, and H. E. Stanley, preceding Comment [Phys. Rev. Lett. **60**, 1203 (1988)].
[3] A. Coniglio, N. Jan, I. Majid, and H. E. Stanley, Phys. Rev. B **35**, 3617 (1987).
[4] B. Nienhuis, private communication.
[5] D. Thirumalai, Phys. Rev. A **37**, 269 (1988).
[6] B. Duplantier, Centre d'Etudes Nucléaires de Saclay Report No. SPT/88-001, 1988 (to be published).