Supersonic freefall - A modern adventure as a topic for the physics class

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Supersonic freefall – A modern adventure as a topic for the physics class

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Abstract. This article investigates some of the physics behind the remarkable supersonic freefall of Felix Baumgartner accomplished with *Red Bull Stratos* project on October 14, 2012. The underlying problem is complex: A freefall with air resistance through a medium of changing density. We developed a model for the motion within the framework of classroom physics and present a numerical approach for the dynamic solution that can be implemented by students. Our approach provides both interesting qualitative insights and rather accurate quantitative results for the resulting speed.

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1. Introduction

On the 14th of October 2012, the *Red Bull Stratos* project culminated in a remarkable event: the unprecedented freefall jump from a height of more than 39,000 m, performed by Austrian BASE-jumper Felix Baumgartner. He jumped from a balloon, falling freely for more than four minutes and reached a peak speed of 1341 km/h – well beyond "*Mach 1*". In other words: Baumgartner's freefall broke both the sound barrier and the standing record set 1960 by American jet pilot Joe Kittinger [1, 2]. The recent event attracted significant media attention, thus generating both fascinating footage [3, 4] and value for its sponsor, the soft drink manufacturer *Red Bull*. Since Baumgartner's jump is spectacular both substantially and observably, it may thus invite physics students to deal with the complications and inconveniences coupled with the treatment of a "motion with air resistance". As it will turn out, it is possible to gain a deeper understanding of the relevant physics from rather simple arguments.

For our discussion we will assume air resistance to be proportional to v^2. For a wide range of motions with air resistance this assumption is reasonable since for significant air resistance the respective Reynolds numbers are far above the critical value. Whether such an approach works for the record-breaking freefall is to be demonstrated by our simulated results. Müller [5] discusses Kittinger's freefall with a similar approach but fails to reproduce the maximal speed. Our challenge was to treat stratospheric freefalls in a straightforward and coherent manner. Let us outline what we have done in our analysis.

An increasing air resistance with increasing speed implies that such a freefall is bound to reach an equilibrium velocity where the downward acceleration due to gravity and the upward acceleration due to air resistance cancel each other. The air resistance depends on air density which in turn depends on height. The problem here is to find an appropriate formula for the extremly high altitudes of Baumgartner's and Kittinger's jumps. Since the stratosphere is much thinner than the atmosphere at sea level significant air resistance occurs only at a relatively high speed. These speeds must be compared to the speed of sound c for the respective height. Here, the speed of sound depends on the air temperature or more precisely: c is proportional to $\sqrt{T}$ (where T is the absolute temperature). Since the temperature in the stratosphere is 10-20 % lower than in our common atmosphere, the speed of sound is accordingly lower compared to sea level.

The next two sections prepare the kinematic description of Baumgartner's freefall from these considerations. Section 4 derives the equations of motion. For a subsequent quantitative discussion of these equations we use a *numerical solution*. To that end, section 5 explains the method and section 6 suggests how to work out a numerical solution with students. Section 7 discusses quantitative results and section 8 concludes our inquiry. In the appendix we describe an alternative procedure for the numerical integration (Appendix A) and a fitting procedure for optimizing the numerical parameter describing the aerodynamics of the problem (Appendix B).

2. Falling with air resistance

It is a common experience that air resistance increases notably with speed: while cycling slowly needs virtually no physical effort a fast bicycle ride is a sweat-inducing affair even under cold conditions. Under most conditions motion with significant air resistance will involve turbulent air flow. A phenomenological description of such a motion goes back to Lord Rayleigh. For such a description one will sensibly expect the force F_R due to air resistance to be proportional to the air density ρ and the frontal area A. The proportionality constant C_d – the *drag coefficient* – will depend on the respective aerodynamic properties of the movin object. The question whether air resistance increases proportionally to the speed v or as v^2 may be answered by scientific consensus or decided by a simple class room experiment [6, Experiment 13].‡ Thus, the conventional formula for F_R is written as

$$F_R = \frac{1}{2} C_d \, \rho A v^2 \; . \tag{1}$$

A "freefall with air resistance" means concurrent action of gravity and aerodynamic drag. While gravity "pulls downwards" the aerodynamic drag will work in the opposite direction. Gravity itself does not vary significantly even for a fall of several kilometers. Since the downward acceleration due to gravity also increases the upward aerodynamic drag the two forces may cancel out at a certain falling speed v_{eq}. If $G = mg$ specifies weight we obtain from $F_R = G$ and eq. 1 the *equilibrium velocity* to

$$v_{eq} = \sqrt{\frac{2mg}{C_d \, \rho A}} \; . \tag{2}$$

This means that for constant air density ρ and a certain size and shape of the body v_{eq} will increase with increasing body mass m. On the other hand for given ρ and m a reduced frontal area A or better aerodynamic properties (i. e. smaller C_d) will also increase v_{eq}.

In order to understand the net effect of gravity and drag qualitatively for the most simple case of a fall with constant air density, frontal area and drag coefficient, we shall distinguish three situations:

- For a body falling with a speed $v = v_{eq}$ gravity is compensated by the aerodynamic drag – no acceleration will occur, the speed is constant.
- For a body falling with a speed $v < v_{eq}$ gravity is *partially* compensated by air resistance. The body will accelerate downward – it will accelerate more strongly in cases of weaker aerodynamic drag, i. e. slower motion and vice versa.
- For a body moving initially faster than v_{eq} but unpropelled§ the air resistance is stronger than the weight force – the motion of the body will slow down.

Fig. 1 shows this schematically in a diagram. Since the speed of falling asymptotically becomes v_{eq}, it is often also called *terminal speed.*

‡ For a beautiful demonstration unfortunately not suitable for a physics class see the first minutes of [7].
§ E. g. a bullet shot downwards.

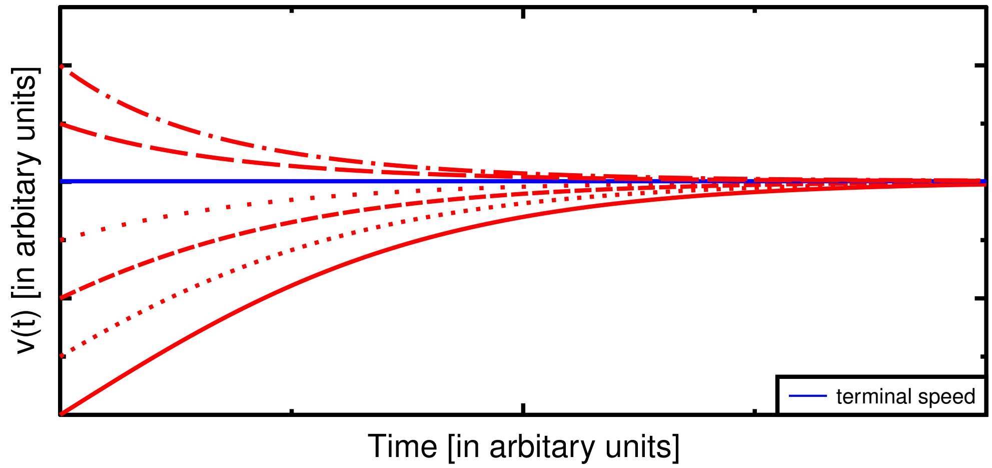

Figure 1. Schematic diagrams of falling speed v vs. time for aerodynamic drag according to eq. (1) in arbitrary units. The different initial values of v (red) correspond to 0 %, 25 %, 50 %,. . . , 150 % of the terminal speed v_{eq} (blue): accelerated motion starting at a speed slower than v_{eq} as well as unpropelled motion at a speed higherr than v_{eq} asymptotically approach the terminal speed.

3. Falling through the stratosphere: The influence of changing air density

A lift towards a mountain top or sometimes even an elevator ride often result in a noticeable feeling of pressure in the ear as the atmospheric pressure decreases with height. With pressure and air density being proportional, for a freefall of many kilometers, as envisaged by *Red Bull Stratos*, this pressure variation will be a crucial constraint. Indeed, the atmospheric pressure in the stratosphere is so low that the use of pressurized suits is absolutely necessary. On the other hand, it is clear from eq. (2) that for a given falling body the low air density allows a higher terminal speed: For a given ratio $p_1/p_2 = \rho_1/\rho_2$ we obtain

$$v_{eq(1)} = \sqrt{\frac{\rho_2}{\rho_1}} v_{eq(2)} \ . \tag{3}$$

This means that if air pressure is reduced by a factor of k the terminal speed v_{eq} is increased by a factor $\sqrt{k}$.

It is this fact that drives the idea of reaching the speed of sound in a freefall: For example, with a typical stratospheric pressure of some millibar – i. e. a factor $k \approx 100$ – the terminal speed can be $\sqrt{k} \approx 10$ times faster than at sea level. Unfortunately, the common barometric formula does not apply for the stratosphere since the stratospheric temperature is distinctly higher than assumed by the standard barometric formula. From [8] or similar sources we find (with h measured in meters)

$$\rho(h) \approx 1.5\,\mathrm{kg/m^3} \times \exp\left(\frac{-h}{6450\,\mathrm{m}}\right) \tag{4}$$

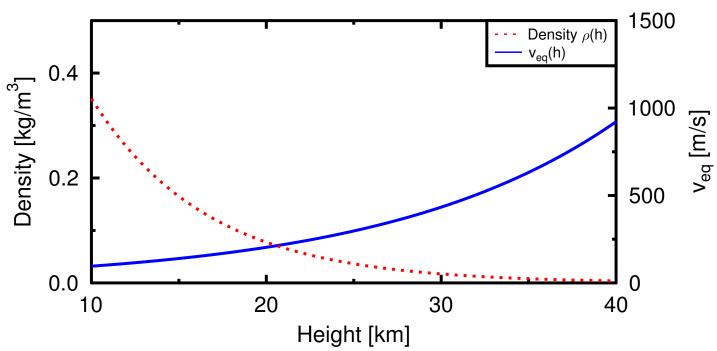

Figure 2. Atmospheric density (red) and terminal speed v_{eq} (blue) as functions of height h according to eqs. (4) and (5), respectively. At sea level, the atmospheric density is about $1.3\,\mathrm{kg/m^3}$.

as an estimate for the air density $\rho(h)$ at the relevant altitudes of 10-40 km. Note that the scale length 6450 m is much smaller than in the conventional barometric formula (where it is $\approx 8400\,\mathrm{m}$) and $\rho(0)$ does not match the actual value of about $1.2\,\mathrm{kg/m^3}$.

In this line of argument we may construct an estimate for the height dependence of the equilibrium speed: We write eq. (3) as $v_{eq}(h) = v_{eq}(0)\sqrt{\rho(0)/\rho(h)}$ and insert eq. (4) for $\rho(h)$. An Internet research gives us 180 km/h=50 m/s as a typical value for v_{eq} at sea level. However, we have to correct this value with a factor of $\sqrt{1.2/1.5}$ to account for the value of $\rho(0)$ in eq. (4). Thus, we obtain

$$v_{eq}(h) \approx 45\,\mathrm{m/s} \times \exp\left(\frac{h}{12900\,\mathrm{m}}\right) . \tag{5}$$

Figure 2 shows plots of eqs. (4) and (5). Note that as h increases the density *de*creases while $v_{eq}(h)$ *in*creases – after all, the terminal speed is higher for a lower density. Due to the square root in eq. (3), the scale lengths differ by a factor of 2.

4. The equations of motion

We may now straightforwardly derive the equations of motion. We start from Newton's Second Law which states that the rate $\dot{p}$ at which the momentum p of a body changes is equal to the sum of any forces acting on the body. For a falling object with air resistance this means $\dot{p} = G + F_R$. Since the height h increases upwards and weight acts downwards we write $G = -mg$, yet F_R has positive sign. With eq. (1) we obtain

$$\frac{d}{dt}p = m\dot{v} = m\ddot{h} = \frac{1}{2}C_d\,\rho(h)Av^2 - mg . \tag{6}$$

Here, the air density ρ depends explicitly on the height as described by eq. (4). Dividing by m, expanding the expression for F_R with g, and comparing this with eq. (2) yields a more transparent form:

$$\dot{v} = \ddot{h} = gv^2 \frac{C_d\,\rho(h)A}{2mg} - g \ = g\left(\frac{v^2}{v_{eq}^2(h)} - 1\right) \tag{7}$$

where $v_{eq}(h)$ is the terminal speed as a function of height h according to eq. (5) and fig. 2. Additionally, the relation $\dot{h} = v$ holds. We may read eq. (7) as an extended and quantitative formulation of the three freefall situations of section 2:

- For $v = v_{eq}(h)$ the acceleration $\dot{v} = 0$;
- for $v < v_{eq}$ the falling body experiences a downward acceleration $\dot{v} < 0$, for $v \ll v_{eq}$ even $\dot{v} \approx -g$; on the other hand, for $v \to v_{eq}$ we obtain $\dot{v} \to 0$;
- analogously for $v > v_{eq}$ the falling body experiences an *upward* acceleration $\dot{v} > 0$, i. e. the motion slows down and $\dot{v} \to 0$ for $v \to v_{eq}$.

However, for a stratospheric freefall the evolution of $v(t)$ according to equation (7) will differ from that discussed in section 2 since air density and the terminal speed $v_{eq}(h)$ will permanently vary during the fall. Thus, v_{eq} decreases as the height decreases, too – the equilibrium conditions vary with the evolution of $v(t)$. In this inherently dynamic situation, the falling speed $v(t)$ will increase as long as $v(t) < v_{eq}(h(t))$ holds. The maximum speed is obviously given by the condition

$$v(t) = v_{eq}(h(t)) \tag{8}$$

when $\dot{v}$ in eq. (7) changes its sign. After that moment, $v(t) > v_{eq}(h(t))$ will hold and $v(t)$ will decrease.

5. Solving the equations of motion numerically

Of course, an exact solution of eq. (7) is beyond the scope of a standard physics class. However, this does not mean that formulating the equations of motions is useless. It is both possible and instructive to obtain a *numerical* solution with students in a comprehensible way. With the possibility to calculate the maximal speed and to plot sufficiently good approximations of $v(t)$, our approach stands as an example for a more general, modern, and productive strategy of physical investigation: For a quantitative description of nature we may state the problem adequately and subsequently *solve it with the help of computers*. Thus, though we do not know the function $v(t)$ itself, we may perform a quantitative analysis of our problem.

The fundamental idea behind such a numerical solution is strikingly easy: Instead of an unknown continuous function $v(t)$ we evaluate a set $\{v_i\}$ of discrete values of the speed during the freefall. These values correspond to a set $\{t_i\}$ of instants of time where $v(t_i) \approx v_i$. For simplicity, these instants of time may be thought of as equidistant, i. e. $t_i := t_0 + i\Delta t$ with suitable starting time t_0 and time interval Δt. In any moment, the evolution of $v(t)$ (i. e. the value of $\dot{v}$) depends on the current value $v(t)$ itself and the actual height $h(t)$ since v_{eq} depends on the height. With a sufficiently small interval Δt (e. g. 0.1 s) the values of $v(t)$ and $h(t)$ will vary approximately linearly between two

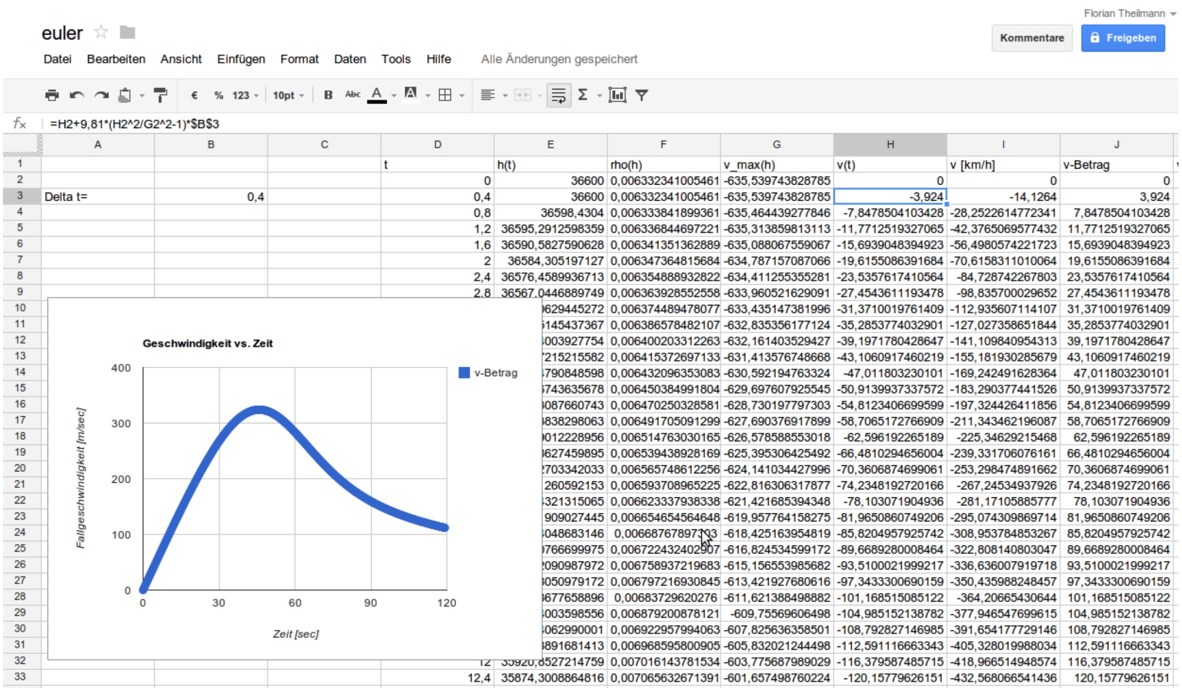

Figure 3. Screen shot of the procedure described in section 6 implemented in Google docs. The columns contain the sequences t_i, h_i, $\rho(h_i)$ etc. Subsequent values of h_i and v_i are computed from cell entries in row $(i-1)$. It is possible to plot results (left) and to directly inspect the impact of changes in Δt or the starting height h_0.

consecutive instants of time t_i and t_{i+1}. Thus, we may calculate v_{i+1} from v_i and the value of $h_i := h(t_i)$. To that end, we set $\dot{v} \approx (v_{i+1} - v_i)/\Delta t$ and rewrite eq. (7):

$$v_{i+1} = v_i + \Delta t \cdot g \left(\frac{v_i^2}{v_{eq}^2(h_i)} - 1 \right) \; . \tag{9}$$

Analogously, we obtain

$$h_{i+1} \approx h(t_{i+1}) \approx h_i + v_i \Delta t \; . \tag{10}$$

This approach of numerically solving of ordinary differential equations is known as the "*Euler method*". Virtually all of the more sophisticated schemes extend and vary this idea [9, pp. 701 – 745].

As discussed above, for a concrete computation of the evolution of $v(t)$ we need to specify $v_{eq}(h)$. If we consider the height dependence in eq. (5) as prespecified by atmospheric data, the only adjustable parameter is the estimate $v_{eq}(0)$ for the speed limit at sea level. A "trial and error" approach quickly narrows this parameter down to values slightly above 40 m/s. Appendix B discusses a possible fitting procedure for that value – below, we will use a value of 40.51 m/s from this procedure.

6. A spreadsheet implementation of Euler's method

Equations (9) and (10), i. e. a numerical integration of eq. (7) using Euler's method, may be implemented in a standard spreadsheet software (Fig. 3). Our procedure was as follows:

- We define Δt, typically in the range of 0.1-1.0 s (cell `B3` in fig. 3),
- we define the set of t_i (column `D` in fig. 3, e. g. from setting `D2` to 0, setting `D3` to `D2+$B$3`, and "dragging" the expression along column `D`; `$B$3` absolutely addresses cell `B3`, i. e. the address will not change for other cells of column `D`),
- we define the values of $v(0)$ (or v_0) and $h(0)$ (or h_0) in the cells `E2` and `H2`,
- we calculate the corresponding air density $\rho(h_0)$ and terminal speed $v_{eq}(h_0)$ in cells `F2` (as `=1,5*EXP(-E2/6450)`) and `G2` (as `=-40,51*exp(E2/12900)`).

At this point all necessary information to start the procedure is available. Thus,

- we define h_1 from eq. (10), i. e. cell `E3` as `E2+$B$3*H2`, and v_1 from eq. (9), i. e. cell `H3` as `H2+9,81*(H2^2/G2^2-1)*$B$3` (again using the expression "`$B$3`" for Δt) and
- drag the definitions of $\rho(h_i)$ and $v_{eq}(h_i)$ from the cell `F2` and `G2` to `F3` and `G3`, respectively.

Finally we just mark the cells `E3` to `G3` and drag the area to the desired time step t_i in column `D`. The procedure calculates sequences of h_i and v_i with Euler's method. If we choose SI-units for the calculation, it is convenient to add further columns for calculating v_i in km/h and also the absolute value of the speed during the freefall (v_i is negative since the values of h_i decrease during the flight).

7. Results and discussion

For the quantitative discussion we used a professional routine from [10] for the numerical integration of eq. (7). Generally, the benefits of a more sophisticated calculation are higher accuracy and better stability. While the integration of eqs. (9) and (10) is not critical in this respect the results from this approach may serve as a "save reference point" for the results from the spreadsheet and for different values of Δt. Appendix A describes the use of the program.

Figure 4 shows simulations of the three freefalls from the *Red Bull Stratos* project and of Joe Kittinger's jump from 1960. The $v(t)$-curves (green and red) are compared to an ideal freefall without air resistance ($v = gt$, blue) and the measured speed maxima (grey). The deviation of v_i from an ideal freefall occurs typically for about 40 % of the maximum speed v_{max}. Up to this point, the quotient v^2/v_{eq}^2 in eq. (7) is sufficiently small. This holds because $v < v_{eq}$ and the higher momentary values of v_{eq} (in the initial phase of falling the body is higher than for the condition of eq. (8). The actual speed maxima [3, Section "The Mission"] are reproduced with errors of a few percent. The phase of acceleration lasts from ca. 30 s for the first test jump (height 21828 m) to about 50 s for the record jump. After reaching the maximum speed, the falling body decelerates in an increasingly dense atmosphere.

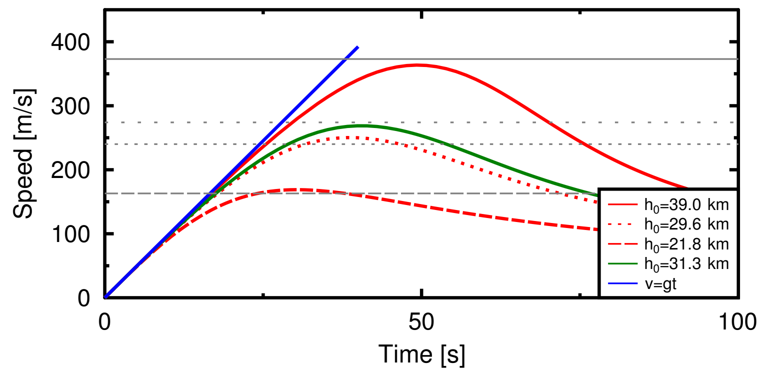

Figure 4. The simulation of Kittinger's jump (green), together with simulations of the *Red Bull Stratos* test jumps and of the record event (red). The green and red curves illustrate the simulated falling speed as a function of time $v(t)$ for the respective jump heights of 31.3 km (Kittinger) and 21.8 km, 29.6 km, and 39 km (*Red Bull Stratos*). The blue line is the falling speed $v(t) = gt$ *without* aerodynamic drag. The grey lines mark the officially announced maximum speed values for the respective freefalls.

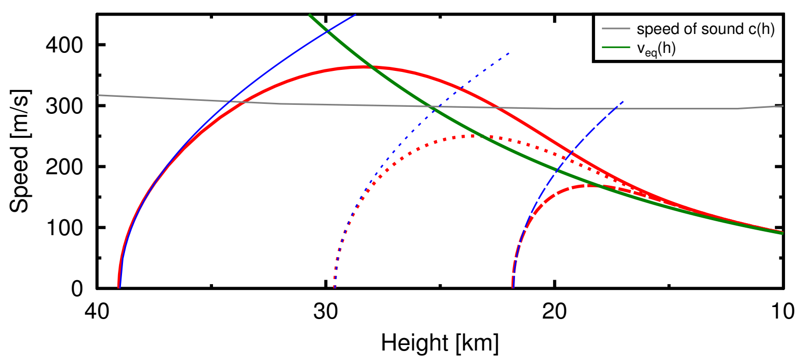

Figure 5. The simulation of the the two test jumps and of the record event of the *Red Bull Stratos* project. To align the time arrow of the process with fig. 4 ("left to right") the x-axis is inverted: height *de*creases during the fall. The red curves correspond to the simulated falling speed as a function of height $v(h)$ for the respective jump heights of 21.8 km, 29.6 km, and 39 km. The blue lines are the respective falling speed $v(h) = \sqrt{2(h - h_0)/g}$ *without* aerodynamic drag. The grey line marks the speed of sound as a function of height. The green line is the function $v_{eq}(h)$ obtained from an optimized estimate for the terminal speed at sea level, cf. text.

For the three freefalls from the *Red Bull Stratos* project, figure 5 shows the simulated speed as a function of height. The $v(h)$-curves (red) are again compared to ideal freefalls from the respective jump height without air resistance ($v = \sqrt{2(h_0 - h)/g}$, blue), the speed of sound $c(h)$ (grey) as a function of altitude, and the equilibrium speed $v_{eq}(h)$ (green) from eq. (5). As is clearly visible, with his jump from an altitude of 39 km Baumgartner exceeded Mach 1 significantly, and in fact for a distance of more than 10 km. As already deduced qualitatively from eq. (7) the speed maxima occur for

$v(h) = v_{eq}(h)$ (cf. eq. (8)). In each case, for the rest of the freefall the respective momentary speed exceeds $v_{eq}(h)$. However, due to the exponential character of the air density $\rho(h)$ (eq. (4)) the excess speed causing the deceleration tends to become increasingly higher for higher altitudes. Towards a height of $h = 10\,\mathrm{km}$ the falling speed tends asymptotically to $v_{eq}(h)$ (as it tends asymptotically to a constant v_{eq} for a constant air density).

8. Conclusion

The main result of our discussion is that atmospheric freefalls and even the record breaking freefall of the *Red Bull Stratos* project can be successfully treated quantitatively as a fall with aerodynamic drag proportional to v^2. In that respect, eq. (1) proves to be a rather robust description of air resistance over an impressive speed range. This is somewhat surprising since the aerodynamics of a spinning skydiver is fairly complex and we would not expect to give eq. (1) satisfying results beyond the speed of sound. However, our approximation of stratospheric freefalls works quite well. From this perspective, the topic lends itself for enquiry-based learning in the physics classroom

Additionally, the physical benefit of our model is not limited to reproducing the speed maxima more or less exactly. The model describes the transition from an (approximately) ideal fall without significant air resistance to the equilibrium between gravity and aerodynamic drag. The balance of driving forces, be it gravity or external propulsion, and counteracting air resistance is a fundamental problem encountered in science and everyday life. On that account, we consider it worthwhile to discuss various aspects of the problem.

Discussing the physics behind Baumgartner's freefall also reveals methodological threads of interest for physics classes. As mentioned above, the approach of identifying the relevant physics, building the governing equations, and solving these equations numerically, if necessary, may be an important and innovative variation of the classical treatment of comprehensible yet sterile problems in the classroom. Additionally, the process of generalizing simplified conditions (neglect of aerodynamic drag, constant air density) encourages students to use imagination and scientific reasoning. The numerical procedure is an obvious field for student activities and autonomous inquiry. We would be grateful to hear of concrete experiences and improvements in this regard.

Appendix A. Using more advanced numerics

The program package `plotutils` [10] from the GNU project is freely available for a variety of platforms and provides programs for plotting data and converting graphic file formats, as well as a program for numerical integration of typical ordinary differential equations. We quote from the online manual of this last program, `ode`:

> **ode** is a tool that solves, by numerical integration, the initial value problem for a specified system of first-order ordinary differential equations. Three

distinct numerical integration schemes are available: Runge-Kutta-Fehlberg (the default), Adams-Moulton, and Euler. The Adams-Moulton and Runge-Kutta schemes are available with adaptive step size. The operation of ode is specified by a program, written in its input language.

The specification of operation for our problem is rather straightforward and best explained with our source code of `ode`'s input language:

```
# Comments start with a hash ('#')
h' = v
v' = 9.81*(v^2/(40.51)^2/exp(h/6450)-1)
#h = 21820
#h = 29610
h = 39045
v = 0
print  t, h, v
step 0, 200
```

The first line is a comment. The next two lines specify the equations to be solved. We then set the initial values h_0 and v_0. The `print` statement effects printing t_i, h_i and v_i for every time step as text to the standard output (and can be redirected to a text file). The `step` statement specifies the time interval of integration. The program can be contained in a text file *file* invoked by the command "`ode -f` *file*". Typing "." on the standard input stops `ode`.

`ode` comes with documentation and examples and may be used with students in an open inquiry, a problem based learning environment or with a video projector during instruction. Alternatively, one may consider interactive websites as [11, 12].

Appendix B. A systematic fitting procedure for the numerical parameter $v_{eq}(0)$

In order to optimize the value for $v_{eq}(0)$ used in the simulation of Baumgartner's stratospheric freefalls in a more strict sense, we may use the condition for the maximum speed in eq. (8). This equation states that speed maxima occur as $v(h) = v_{eq}(h)$ holds. The function $v_{eq}(h)$ is written as

$$v_{eq}(h) = a \cdot \exp\left(\frac{h}{12900\,m}\right) \tag{B.1}$$

We may choose a first estimate of a and then iterate the following steps:

(i) Calculate $v(h)$ for Baumgartner's three freefalls (h_0 =21828 m, 29610 m, and 39045 m);

(ii) obtain the three values $h_i^{(1)}$, $h_i^{(2)}$, and $h_i^{(3)}$ for which the speed maxima $v_{max}^{(1)}, v_{max}^{(2)}$ and $v_{max}^{(3)}$occur;

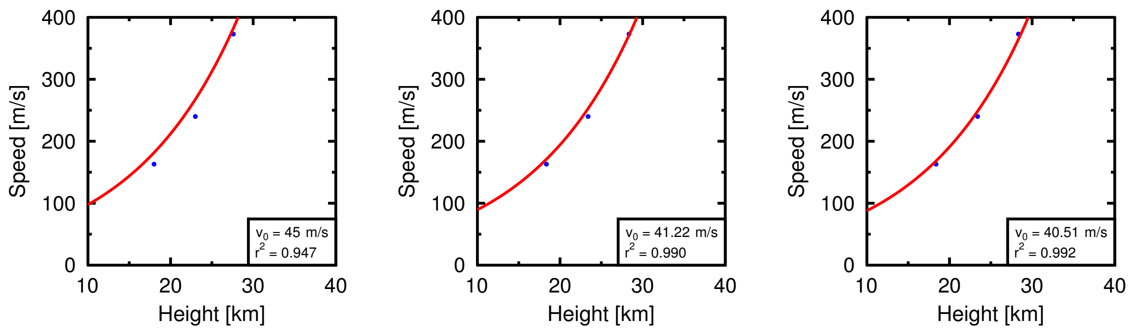

Figure B1. Three steps of the fitting procedure for $v_{eq}(0)$ in eq. (5). For the indicated value of $v_0 := v_{eq}(0)$ the blue markers represent the *real* speed maxima from Baumgartner's three freefalls assigned to the *simulated* heights for which the corresponding speed maxima are obtained. The red curves are the plots of $v_{eq}(h)$ from eq. (5) with the respective value of $v_{eq}(0)$. The fits optimize the matching of actual speed maxima and the integrated description of the aerodynamic drag in eq. (5).

(iii) fit a in eq. (B.1) to the three data points $(h_i^{(1)}, v_{max}^{(1)})$, $(h_i^{(2)}, v_{max}^{(2)})$, and $(h_i^{(3)}, v_{max}^{(3)})$, i. e. to the *simulated* height and the *real* maximum speed (163 m/s, 240 m/s, and 373 m/s);

(iv) if a varies significantly, repeat the procedure.

We used the freely available program `gle` [13] for fitting (as well as for the graphics). Figure B1 shows our fit. For each guess of the value of $v_{eq}(0)$ in eq. (5) we simulate the freefalls from the *Red Bull Stratos* project. Then we compare the actual speed maxima (assumed to occur for the simulated height) with $\rho(h)$ from eq. (5). The fitting procedure suggests an improved value for $\rho(0)$ and the procedure may be repeated.

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