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Supersonic Jump

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• *The Question: What height is necessary to fall faster than the speed of sound?*

On October 14, 2012, Felix Baumgartner, an Austrian skydiver, set some new world records for his discipline. Jumping from a height of about 39 km, he reached a top speed of 1342 km/h, becoming the first human being to break the sound barrier in free fall. In order to understand some essential physics aspects of this remarkable feat, we wonder why his start height had to be that high (when the tremendous effort that was necessary for leaping from such a height required $50 million, as reported in the press). More precisely, can you give an estimate for the minimal start height—which we will call the Baumgartner limit, z_B—of a skydiver who wants to break the sound barrier in free fall?

• *Approximate treatment: Free-fall terminal velocity and atmospheric density*

At the basis of our problem are two main physical factors, the free-fall terminal velocity in a resistive medium, which depends on the density of the medium, and atmospheric density, which in turn depends on height. The combination of these dependencies will allow us to determine the limit height z_B we are interested in. To begin with, we recall the result for terminal velocity of an object moving through a resistive medium[1] (see also Refs. 2 and 3 for an extended treatment in this journal):

$$v_t = v_t(z) = \sqrt{\frac{2mg(z)}{\rho(z)AC_d}}\,, \qquad (1)$$

with m, A, C_d being the mass, area, and drag coefficient of the falling object, ρ the density of the medium through which the object falls, and g the gravitational acceleration. The terminal velocity $v_t = v_t(z)$ depends on height z through $\rho = \rho(z)$ and $g = g(z)$, but we can safely neglect the latter dependence for the approximate treatment intended here.[4] The ratio of terminal velocities at height z and at height 0 (Earth surface) is then

$$\frac{v_t(z)}{v_{t,0}} \approx \sqrt{\frac{\rho_0}{\rho(z)}} \qquad (2)$$

(here and in the following, all quantities with a subscript 0 refer to ground level, $z = 0$).

Next, we have to insert $\rho(z)$; for the approximate treatment we are interested in here, we can use the well-known "barometric formula" (or "exponential atmosphere" model) for the model of an isothermal atmosphere[5]

$$\rho(z) = \rho_0 e^{-z/h_s}\,, \qquad (3)$$

with the scale height $h_s = k_B T/Mg$ (k_B = Boltzmann constant, T, M = average temperature and molar mass of atmosphere). Combining Eqs. (2) and (3) and solving for z, we obtain

$$z = 2\ln\left(\frac{v_t(z)}{v_{t,0}}\right)h_s\,. \qquad (4)$$

for the height at which a given terminal velocity can be reached. Requiring that this terminal velocity be the velocity of sound, i.e. $v_t(z) = v_s$, we arrive at

$$z'_B = 2\ln\left(\frac{v_s}{v_{t,0}}\right)h_s\,. \qquad (5)$$

This is the height above which the terminal velocity can get larger than sound velocity, but it is not the Baumgartner limit z_B yet (that is why it carries a prime), because we have to add the free-fall distance over which v_s is actually obtained:

$$s_F = \frac{1}{2}v_s^2/g \qquad (6)$$

(of course, from $s = ½gt^2$ and $t = v/g$); see below for a comment on the approximation inherent in this approach). Equations (5) and (6) together yield our final result:

$$z_B = z'_B + s_F = 2\ln\left(\frac{v_s}{v_{t,0}}\right)h_s + \frac{1}{2}v_s^2/g \qquad (7)$$
$$\approx 27\text{ km}\,,$$

with $z'_B \approx 22$ km and $s_F \approx 5$ km as intermediate results and sound velocity $v_s = 330$ m/s, scale height $h_s = 8.4$ km (at $T = 288$ K terrestrial mean surface temperature), and terminal velocity at ground level $v_{t,0} = 90$ m/s as input data. Other values of these quantities are frequently used, but do not introduce an essential change for the approximate result we are interested in.

Discussion

The largest uncertainty on the level of the *input data* probably concerns $v_{t,0}$. A recent value was taken from Wikipedia[6] in our calculation, but more dependable sources[7] report somewhat lower values. Such a lower value of $v_{t,0}$ would lead to a greater value of the height necessary to reach v_s (i.e., of the Baumgartner limit z_B).[8] For the approximations on the level of the *model*, a first one is the usage of the density dependence $\rho(z)$ for the isothermal atmosphere instead of more realistic models (correcting z'_B). But the crudest approximation resides in using the free-fall model for s_F. While air resistance is low near the start height, it is not zero (check for s_F with the actual fall time before reaching sound velocity, as easily obtainable in the news reports). Note that the two major corrections on the input data and model level both lead to an increase of z_B given here, i.e., in the direction of the actual jump height.

In view of the various approximations and inaccuracies

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involved in the above result, we retain a value of

$$z_B \approx 30 \text{ km} \tag{8}$$

for the "Baumgartner limit," a value falling short of the actual jump height by about 25%, which is reasonable for an approximate treatment, and its corrections point in the right direction.

References

1. D. Halliday, and R. Resnick, J. Walker, *Fundamentals of Physics* (Wiley, New York, 2007), Chap. 6.4.
2. I. Lovatt and B. Innes, "Resistance is not futile: Air resistance in an algebra-based course," *Phys. Teach.* **43**, 544 (Nov. 2005).
3. N. Shea, "Terminal speed and atmospheric density," *Phys. Teach.* **31**, 176 (March 1993).
4. The reason for this is left to the interested reader herself.
5. Ref. 1, Chap. 25.1.
6. en.wikipedia.org/wiki/Terminal_velocity.
7. G. Elert, "The Physics Factbook," hypertextbook.com/facts/JianHuang.shtml.
8. Sound velocity is understood here as absolute measure of velocity, as usual e.g. for transsonic jets, with a fixed reference value (at normal conditions); of course the local value depends on p and T, but this is not considered here.

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