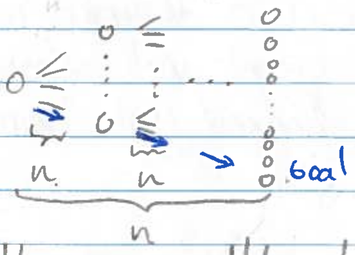
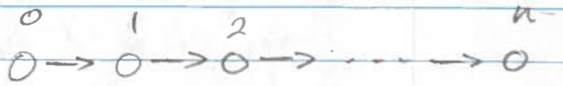
COMP 424 A#1 CAMILO GARCIA #260657037

Q2 a) When the branching factor $b = n$ and the goal depth $d = n$ and the goal state is found in the first expanded branch (very unlikely) then IDS is $O(n^2)$, DFS is $O(n)$



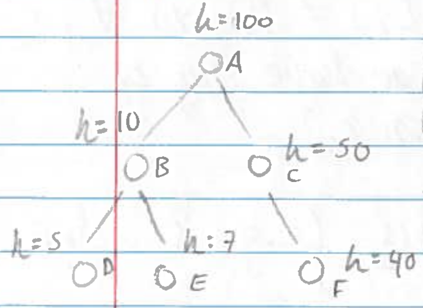


Another case, although less than $O(n^2)$ is a single linked list. In which case IDS would traverse the node n_i : $n-i$ times.
eg. $\underset{0}{o} \rightarrow \underset{1}{o} \rightarrow \underset{2}{o}$ node 0 : 3 times 1 : 2 times 3 : 1 time

b) BFS is indeed a special case of UCS where $f = (1 \cdot \text{depth of } n)$

Given a graph where all the edges have unit cost, then the "cheapest-first" heuristic of UCS will favor exploring siblings before exploring children

c) DFS is indeed a special case of Best FS :



In the case where the "cost-to-go" heuristic has the incremental pattern shown in the tree on the left, Best FS will expand the nodes in the same order as DFS would : A, B, D, E ; A, C, F.

generalizing: If $f = -\text{depth of } n$ is the heuristic for Best FS, it will behave as DFS

d) UCS is indeed a special case of A*:

A* will expand given f: cost-so-far + cost-to-go.
If the cost-to-go heuristic is 0 $\forall$ nodes in the graph, the A* run on the graph will behave the same as UCS with its "cheapest first" heuristic.

Q4 a) A CSP is comprised of:

- A set of Variables: $\underline{X} = \{R_1, R_2, \ldots, R_K\}$ where K does NOT represent a row or col, simply an identifier for K different rooks.

- A set of Domains: $\underline{D} = \{[:n, :n]\} \; \forall \; R_i \in X$ where $[:n, :n]$ represents the row, col position any rook could take.

- A set of Constraints: $\underline{C} = R_i[x] \neq R_j[x] \; \forall \; i \neq j$
  $R_i[y] \neq R_j[y] \; \forall \; i \neq j$

  In other words, no 2 rooks can have any of their components (x,y) the same.

  expanding the constraints yields: (e.g. K=2, n=2)

  $(R_1:(0,0), R_2:(1,1)), (R_1:(0,1), R_2:(1,0)),$
  $(R_1:(1,0), R_2:(0,1)), (R_1:(1,1), R_2:(0,0)).$

b) A:
U: R_1 R_2 R_3

ASSIGNED → A
UNASSIGNED → U

A: $R_1 = (0,0)$
U: R_2 R_3

} Select first unassigned variable find value that satisfies constraint move to next variable. Search through all D

A: $R_1 = (0,0)$ $R_2 = (1,1)$
U: R_3

} Same as above.
Notice the algorithm iterates over all D_2 (9) possible locations until it finds (1,1)

A: $R_1 = (0,0)$ $R_2 = (1,1)$ $R_3 = (2,2)$
U:

} Same as above, with D_3

GOAL STATE

c) A:
U: R_1 R_2 R_3

} $D_1 = D_2 = D_3 = [D]$

A: $R_1 = (0,0)$
U: R_2 R_3

} $D_2 = D_3 = [(1,1), (1,2), (2,1), (2,2)]$

A: $R_1 = (0,0)$ $R2 = (1,1)$
U: R_3

} $D_3 = [(2,2)]$
Because of forward checking, the amount of values we check are reduced as new variables are assigned values

Q1 b) Transaction cost = number of tile to be moved

for the heuristic to be admissible, it must underestimate the distance to the goal from every node.

Given the following exemple:

$$\begin{array}{|c|c|c|}\hline 1 & 4 & 2 \\ \hline 5 & 0 & 3 \\ \hline\end{array} \longrightarrow \overset{M}{\begin{array}{|c|c|c|}\hline 1 & 0 & 2 \\ \hline 5 & 4 & 3 \\ \hline\end{array}} \longrightarrow \begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 5 & 4 & 3 \\ \hline\end{array}$$

Manhattan distance heuristic:	$h: 2$	$h: 1$	$h: 0$
Transaction cost:	$h^*: 5$	$h^*: 1$	$h^*: 0$

Because Manhattan distance heuristic was an admissible consistent heuristic for unit cost n-puzzle and that the change in transaction cost increments the real shortest path cost h^*, Manhattan will remain an admissible heuristic. $h(n) \leq h^*(n)$ for all tiles, even the smallest valued one: 1.
In addition as seen in state M, even when moving the smallest cost tile Manhattan satisfies the definition of a consistant or monotone heuristic: $h(s) \leq c(s, s') + h(s')$

c) We are looking for a heuristic $h_2(n) \geq h_1(n)$ which still satisfies admissibility for A^*.

we are bound by state B where $h_1(B) = 1$ and $C(B)=1$ hence $h_2(B) = 1$. Lets visualize $h_1(n)$

$$\overset{A}{\begin{array}{|c|c|c|}\hline 1 & 4 & 2 \\ \hline 5 & 3 & 0 \\ \hline\end{array}} \longrightarrow \overset{B}{\begin{array}{|c|c|c|}\hline 1 & 4 & 2 \\ \hline 5 & 0 & 3 \\ \hline\end{array}} \longrightarrow \overset{C}{\begin{array}{|c|c|c|}\hline 1 & 0 & 2 \\ \hline 5 & 4 & 3 \\ \hline\end{array}} \longrightarrow \overset{D}{\begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 5 & 4 & 3 \\ \hline\end{array}}$$

$g: 0$	$g: 3$	$g: 7$	$g: 8$
$h_1: 3$	$h_1: 2$	$h_1: 1$	$h_1: 0$
$h^*: 8$	$h^*: 5$	$h^*: 1$	$h^*: 0$
$h_2: 4$	$h_2: 4$	$h_2: 1$	$h_2: 0$

So I propose $h_2(n)$ = cost of the largest tile that needs to be moved. One could call it Max_Manhattan.

e.g. If you look back at states A–D, h_2 represents:

A: max tile to be moved	$\rightarrow 4$	$h_2(n) \geq h_1(n)\ \forall n$
B:	$\rightarrow 4$	
C:	$\rightarrow 1$	
D:	$\rightarrow 0$	

d) $L/R = 2 \quad U/D = \frac{1}{2}$

In this new problem, we now have a transaction cost less than the unit cost. The following counter example showcases a scenario where Manhattan heuristic overestimates the distance to the goal:

5	1	2
0	4	3

$h_1 : 1$
$h^* : \frac{1}{2}$
$\Big\}\ h_1(n) > h^* \rightarrow$ inadmissible h.

Q3

January 30, 2018

0.1 Q3

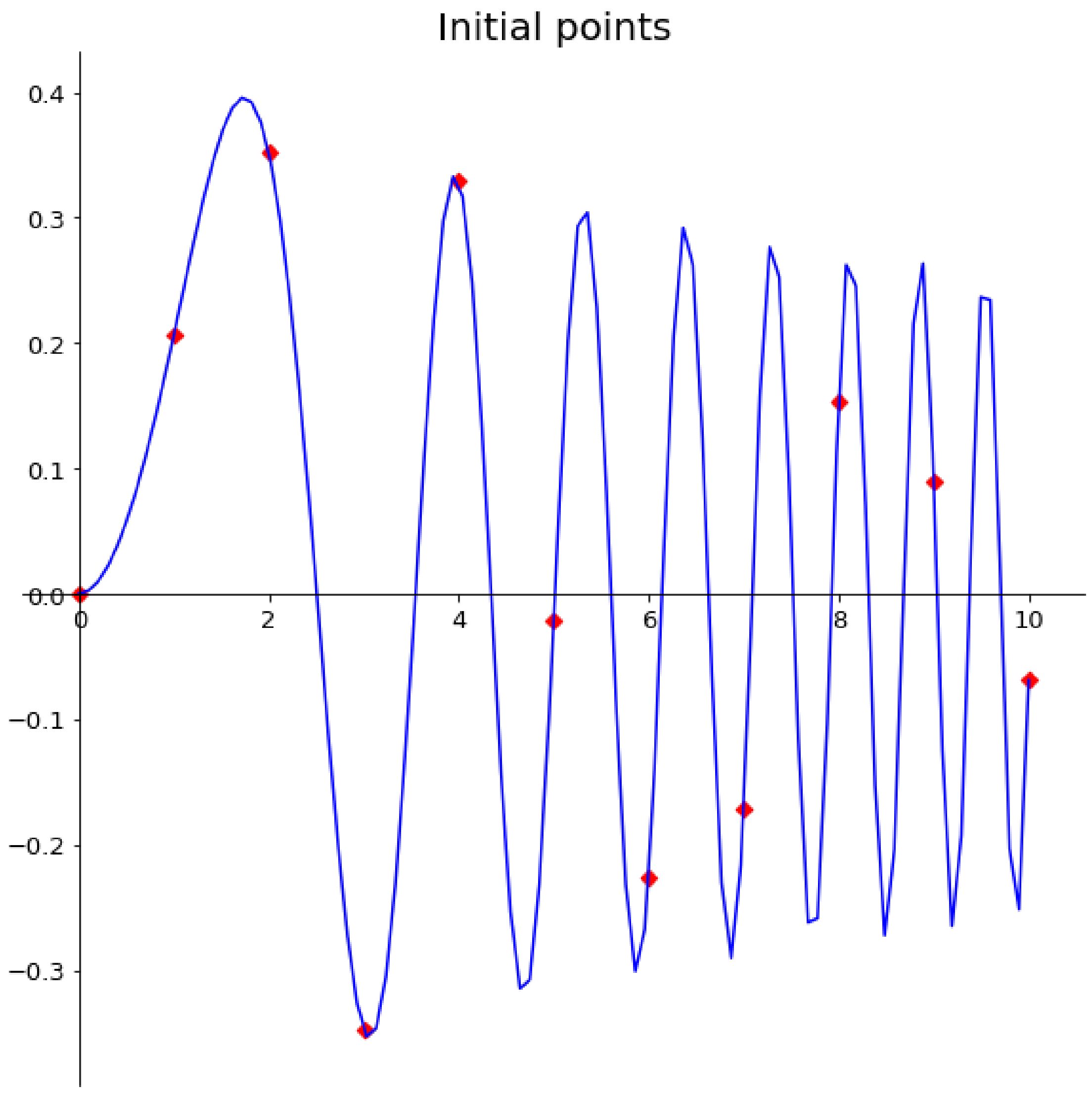
NOTE

The clear formatted output of the requested values (# iterations, final (x,y)) for each parameter (step-size, T, cooling coeff) can be found in **./Output**.

The visual, simplified representation of the results is presented in this notebook for reference. The individual figures can be found under **./Figures**. ### A) Hill Climbing First we will display the function we will work with along with the points at which we will perform Hill Climbing:

```
In [1]: %matplotlib inline
        import math
        import numpy as np
        import matplotlib.pyplot as plt

        fig_counter  = 0
        MIN_X, MAX_X = 0, 10

        def increment_counter():
            global fig_counter
            fig_counter += 1
            return fig_counter

        Y = lambda x: math.sin(math.pow(x, 2) / 2) / math.log((x + 4), 2)

        def plot_model(x_axis, y_model, xs, es, title):
            fig, ax = plt.subplots(figsize=(10,10))
            plt.rc('xtick',labelsize=13)
            plt.rc('ytick',labelsize=13)

            ax.plot(x_axis, y_model, 'b')
            ax.scatter(xs, es, marker='D', color='r')

            ax.spines['right'].set_color('none')
            ax.spines['top'].set_color('none')
            ax.spines['left'].set_position(('data',0))
            ax.spines['bottom'].set_position(('data',0))

            plt.title(title, fontsize=20)
```

```
    fig.savefig('./Figures/fig_{}.png'.format(increment_counter()))

    plt.show()

x_axis = np.linspace(MIN_X, MAX_X, 100)
y_model = [Y(x) for x in x_axis]

init_x = np.linspace(MIN_X, MAX_X, 11)
init_e = [Y(x) for x in init_x]

plot_model(x_axis, y_model, init_x, init_e, 'Initial points')
```



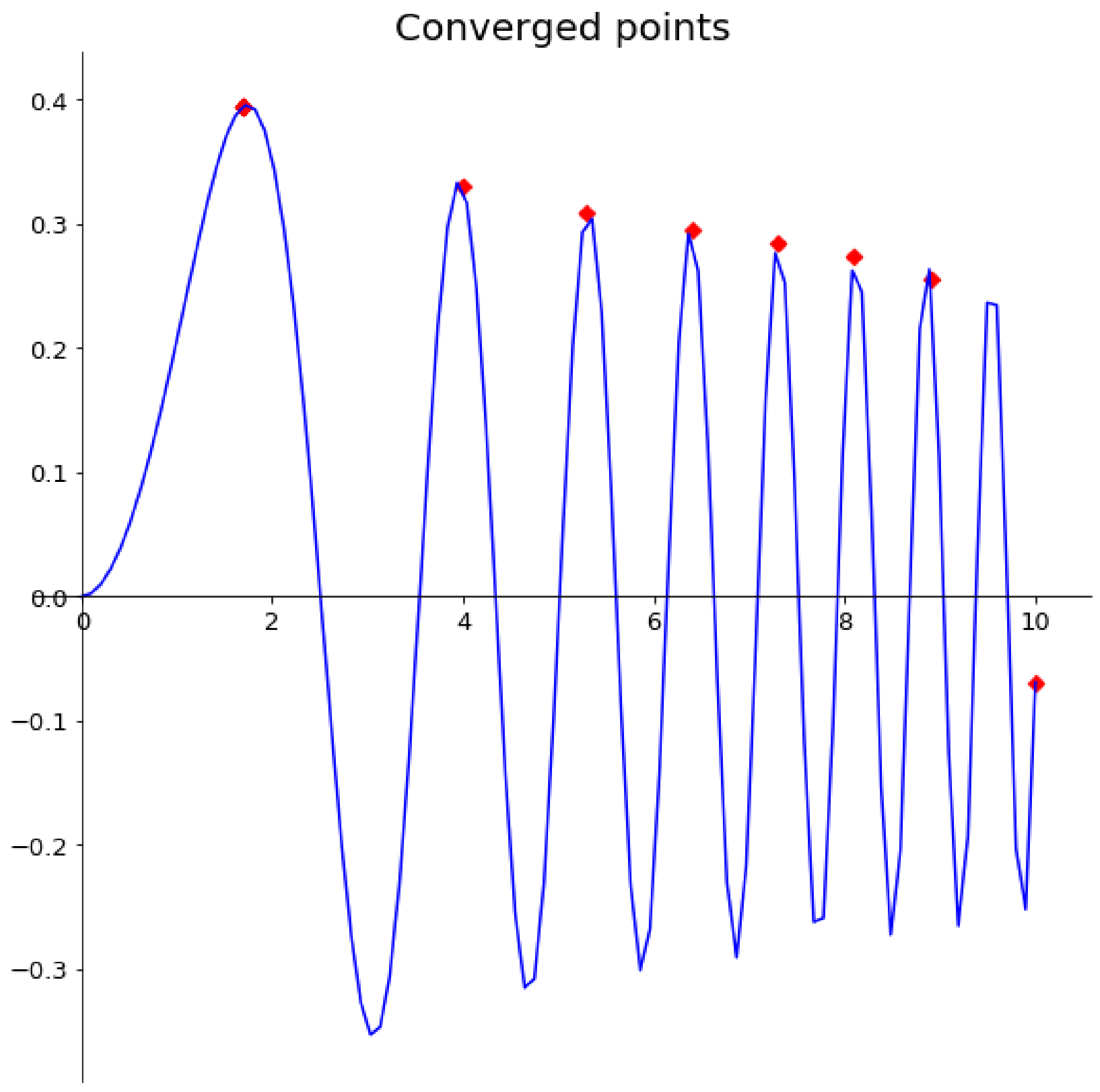
Now we will perform hill climbing for each of the initial states with step sizes $\in [0.01, 0.1]$:

```
In [2]: # given an initial state and a step size,
        # return the local max x, its energy and the number of iterations it took to reach
        def hill_climbing(x_0, step):
            curr_x = x_0
            iterations = 0

            while True:
                iterations += 1
                left_x  = curr_x - step
                right_x = curr_x + step

                # only take into consideration points inside domain
                left_e  = Y(left_x) if left_x >= MIN_X else -float('inf')
                right_e = Y(right_x) if right_x <= MAX_X else -float('inf')

                max_x, max_e = (left_x, left_e) if left_e > right_e else (right_x, right_e)

                curr_e = Y(curr_x)
                if curr_e < max_e:
                    curr_x = max_x
                else:
                    break

            return curr_x, curr_e, iterations


        step_sizes = np.linspace(0.01,0.1,10)
        avg_iters = []


        with open('./Output/Q3a.txt', 'w') as f:
            for step in step_sizes:
                f.write('\n\n---- STEP SIZE: {:.2f} ----\n'.format(step))
                xs, es, iterations = [], [], []
                for x_0 in init_x:
                    x, e, i = hill_climbing(x_0, step)
                    xs.append(x)
                    es.append(e)
                    iterations.append(i)
                    f.write('Initial X = {:.0f}\tFinal (X,Y) = ({:.2f},{:.2f})\tIterations: {}\n

                avg_i = sum(iterations) / len(iterations)
                f.write('Average iterations to converge: {:.1f}\n'.format(avg_i))
                print('step size {:.2f} took in avg {:.1f} iterations to converge.'.format(step,
                avg_iters.append(avg_i)
```

```
step size 0.01 took in avg 49.4 iterations to converge.
step size 0.02 took in avg 25.1 iterations to converge.
step size 0.03 took in avg 17.2 iterations to converge.
step size 0.04 took in avg 13.1 iterations to converge.
step size 0.05 took in avg 10.6 iterations to converge.
step size 0.06 took in avg 8.9 iterations to converge.
step size 0.07 took in avg 8.1 iterations to converge.
step size 0.08 took in avg 7.2 iterations to converge.
step size 0.09 took in avg 6.3 iterations to converge.
step size 0.10 took in avg 5.7 iterations to converge.
```

The local maximum points found are shown below:

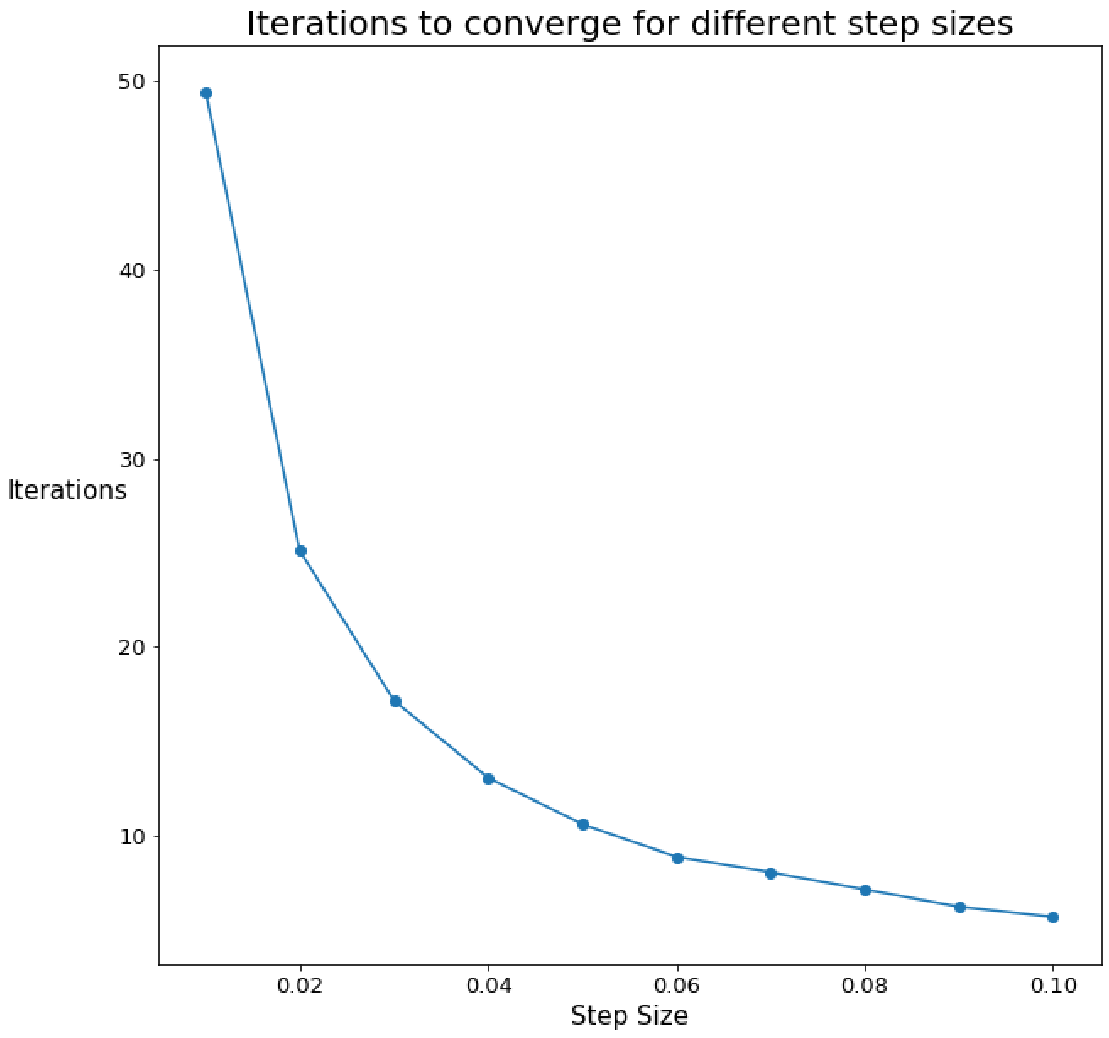
```
In [3]: title = 'Converged points'
        plot_model(x_axis, y_model, xs, es, title)
```



The corresponding Y values found for each initial state can be found in the text file **./Output/Q3a.txt**

Next we plot how many iterations in average it took for each set of initial points to converge given different step sizes:

```
In [4]: def plot_convergence(x, y, title, x_label, y_label):
            fig, ax = plt.subplots(figsize=(10,10))
            ax.plot(x, y)
            ax.scatter(x, y)
            plt.title(title, fontsize=20)
            plt.xlabel(x_label, fontsize=15)
            plt.ylabel(y_label, fontsize=15, rotation=0, labelpad=30)

            fig.savefig('./Figures/fig_{}.png'.format(increment_counter()))
            if print:
                plt.show()

        title = 'Iterations to converge for different step sizes'
        x_label = 'Step Size'
        y_label = 'Iterations'
        plot_convergence(step_sizes, avg_iters, title, x_label, y_label)
```



Based on the above graph we see that a step size of 0.1 yield the minimal amount of iterations while still finding all the local maxima:

```
In [5]: min_iters_idx = avg_iters.index(min(avg_iters))
        optimal_step = step_sizes[min_iters_idx]
        print('Optimal step size: {}'.format(optimal_step))
```

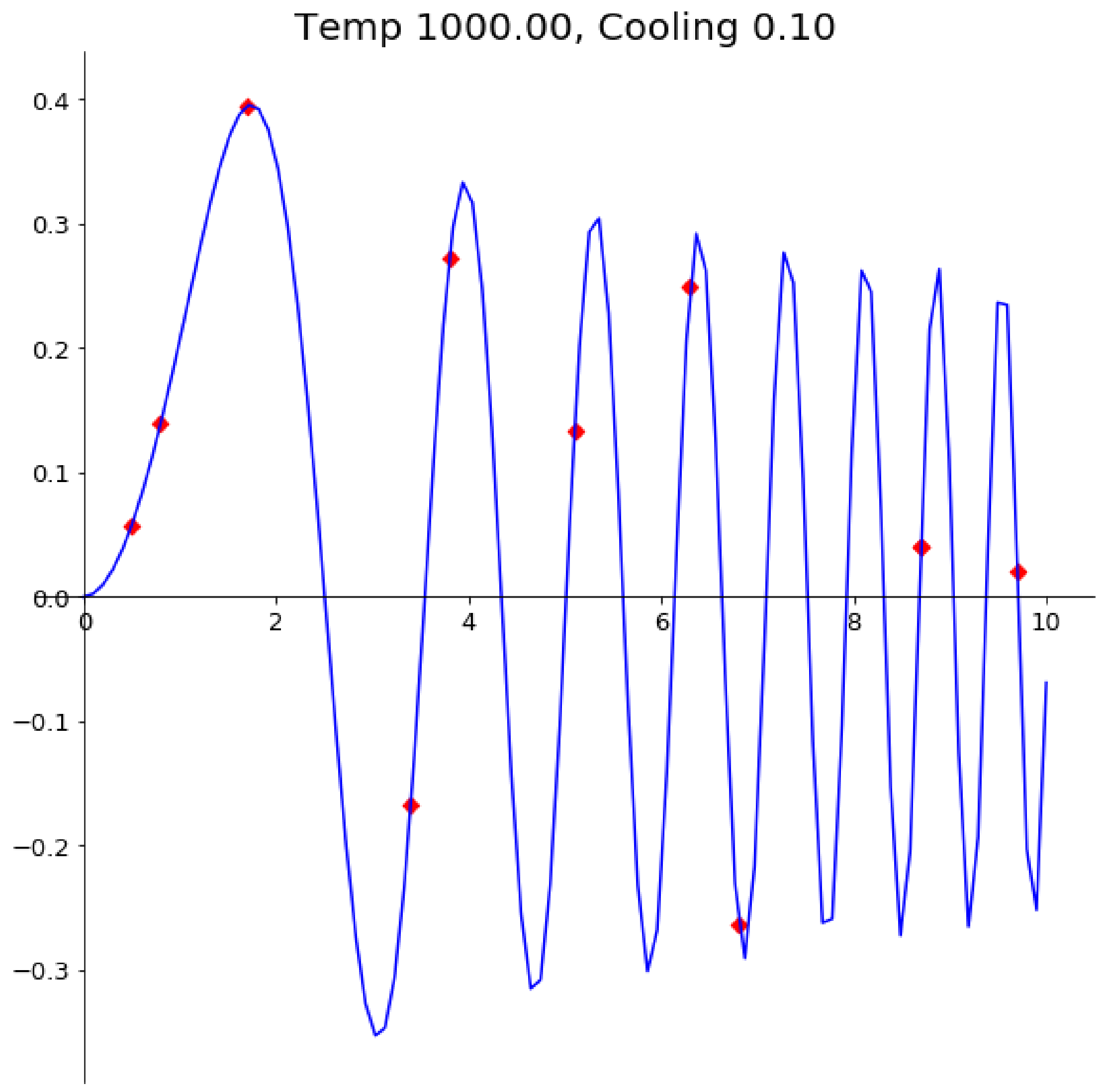
```
Optimal step size: 0.1
```

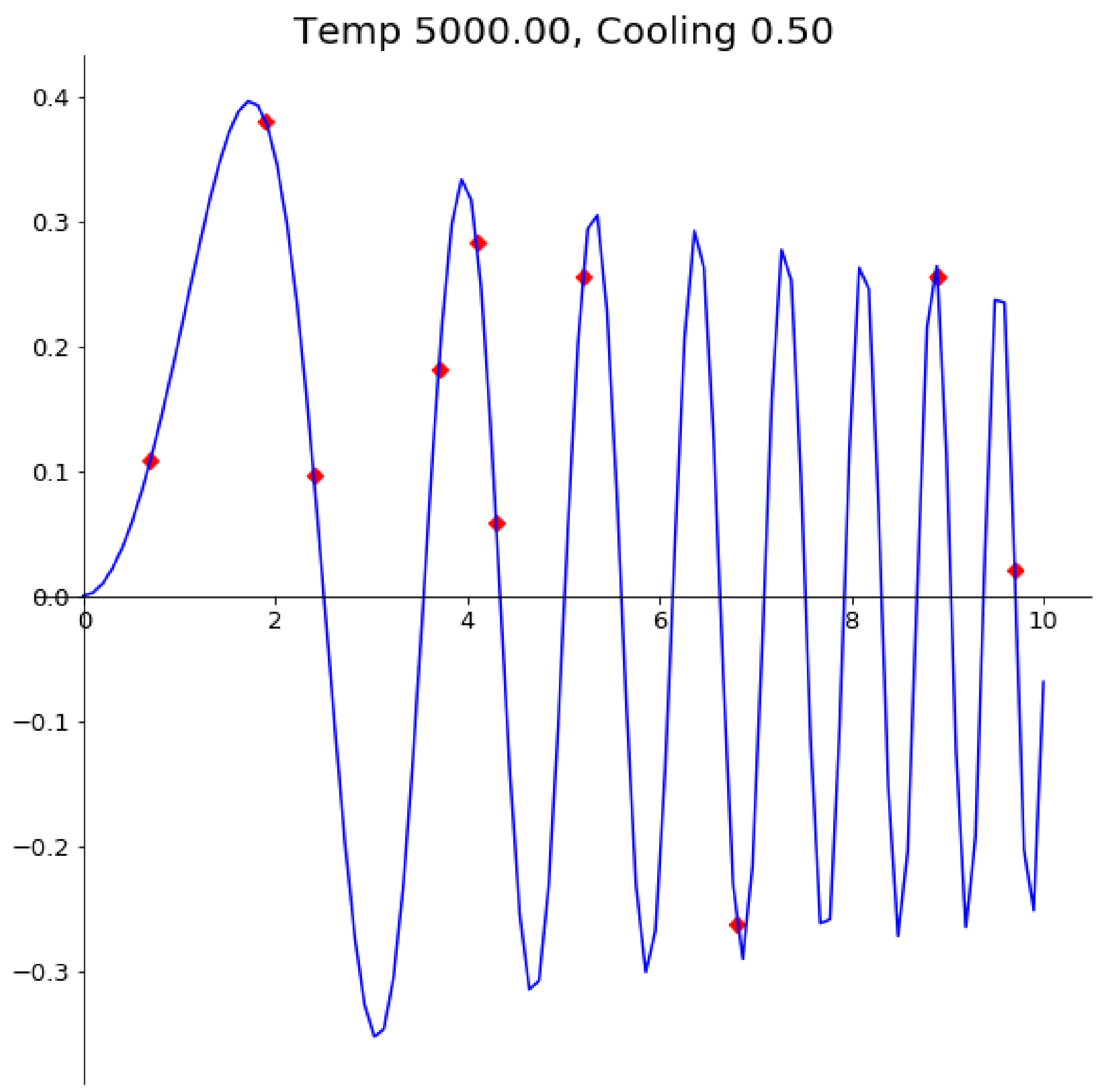
0.1.1 B) Simulated Annealing

The corresponding Y values found for each initial Temperature and Cooling coefficient can be found in the text file **./Output/Q3b.txt**

```
In [6]: import random
```

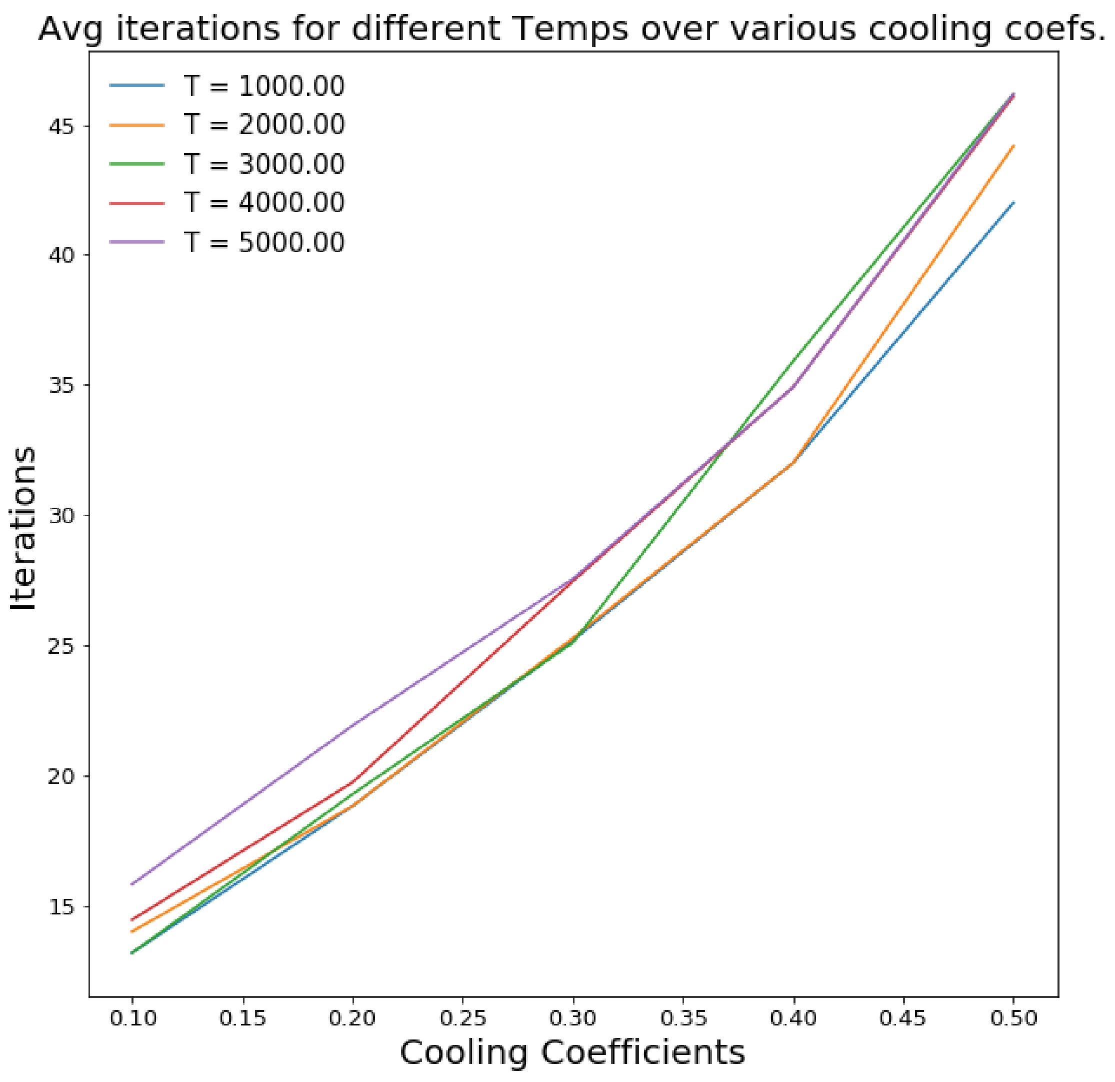
```python
def sim_annealing(x_0, step, T, cooling_coef):
    curr_x = x_0
    iterations = 0

    while True:
        if float('{:.3f}'.format(T)) <= 0: break

        left_x  = curr_x - step
        right_x = curr_x + step

        left_e = Y(left_x)
        right_e = Y(right_x)

        # choose the next direction keeping into consideration domain limits
        if left_x <= MIN_X:
            next_x, next_e = (right_x, right_e)
        elif right_x >= MAX_X:
            next_x, next_e = (left_x, left_e)
        else:
            direction = random.choice(['left', 'right'])
            next_x, next_e = (left_x, left_e) if direction == 'left' else (right_x, righ

        curr_e = Y(curr_x)
        delta_e = curr_e - next_e
        if delta_e > 0:
            curr_x = next_x
        else:
            if random.random() < (math.exp(-(delta_e) / T)):
                curr_x = next_x
            T *= cooling_coef

        iterations += 1

    return curr_x, Y(curr_x), iterations

temperatures  = np.linspace(1000,5000, 5)
cooling_coefs = np.linspace(0.1, 0.5, 5)

iters = []
with open('./Output/Q3b.txt', 'w') as f:
    for t in temperatures:
        f.write('\n\n---- T: {:.0f} ----\n'.format(t))
        avg_iters = []
        for c in cooling_coefs:
            f.write('\n\n---- Cooling Coeff: {:.2f} ----\n'.format(c))
            xs, es, iterations = [], [], []
            for x_0 in init_x:
                x, e, i = sim_annealing(x_0, optimal_step, t, c)
                xs.append(x)
                es.append(e)
                iterations.append(i)
                f.write('Initial X = {:.0f}\tFinal (X,Y) = ({:.2f},{:.2f})\tIterations:

            # plot the new location of eacth agent
            plot_model(x_axis, y_model, xs, es, 'Temp {:.2f}, Cooling {:.2f}'.format(t,c
            avg_i = sum(iterations) / len(iterations)
            avg_iters.append(avg_i)
            f.write('Average iterations to converge: {:.1f}\n'.format(avg_i))

        iters.append(avg_iters)
```





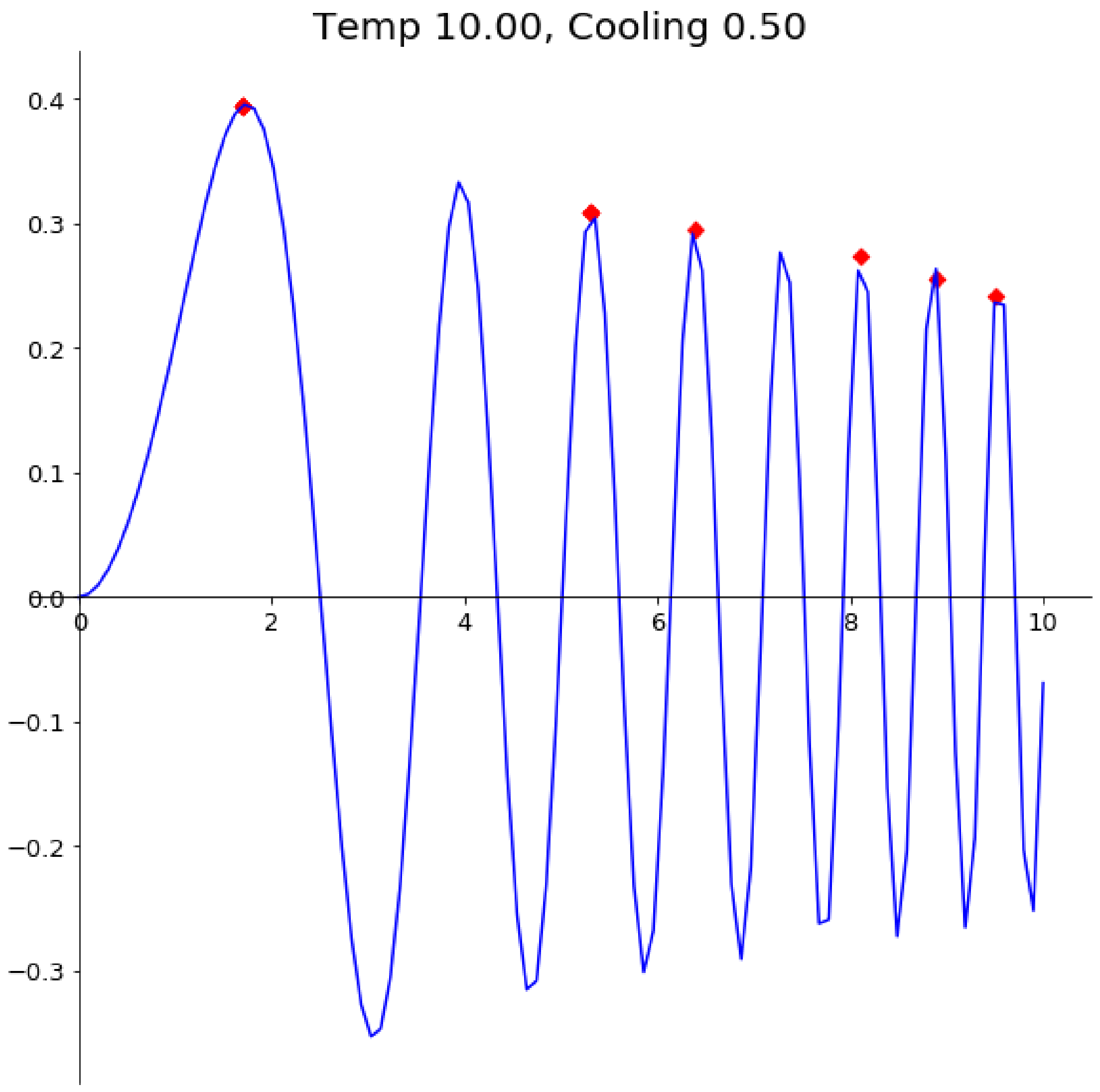
It is easier to visualize the changes in number of iterations to converge with the following graph:

```
In [7]: fig, ax = plt.subplots(figsize=(10,10))
        for idx, t in enumerate(iters):
            ax.plot(cooling_coefs, t, label='T = {:.2f}'.format(temperatures[idx]))

        plt.title('Avg iterations for different Temps over various cooling coefs.', fontsize=20)
        plt.xlabel('Cooling Coefficients', fontsize=20)
        plt.ylabel('Iterations', fontsize=20)
        plt.legend(loc='best', prop={'size':15}, frameon=False)

        fig.savefig('./Figures/fig_{}.png'.format(increment_counter()))

        plt.show()
```



Note that if we had let the algorithm run until it found a neighbour wiht smaller Energy as the current node (i.e. Hill Climbing) after T = 0, All the points would have ended in a local maxima.
An example of such algorithm is presented next:

```
In [8]: import random

        def sim_annealing(x_0, step, T, cooling_coef):
            curr_x = x_0
            iterations = 0

            while True:
                left_x  = curr_x - step
                right_x = curr_x + step

                left_e = Y(left_x)
                right_e = Y(right_x)

                # choose the next direction keeping into consideration domain limits
                if left_x <= MIN_X:
                    next_x, next_e = (right_x, right_e)
                elif right_x >= MAX_X:
                    next_x, next_e = (left_x, left_e)
                elif float('{:.3f}'.format(T)) > 0:
                    direction = random.choice(['left', 'right'])
                    next_x, next_e = (left_x, left_e) if direction == 'left' else (right_x, righ
                else:
                    next_x, next_e = (left_x, left_e) if left_e > right_e else (right_x, right_e

                curr_e = Y(curr_x)
                delta_e = curr_e - next_e
                if float('{:.3f}'.format(T)) > 0:
                    if delta_e > 0:
                        curr_x = next_x
                    else:
                        if random.random() < (math.exp(-(delta_e) / T)):
                            curr_x = next_x
                        T *= cooling_coef
                elif curr_e < next_e:
                    curr_x = next_x
                else:
                    break

                iterations += 1

            return curr_x, Y(curr_x), iterations

        temperatures  = np.linspace(10, 1000, 5)
        cooling_coefs = np.linspace(0.5, 0.5, 1)

        iters = []
        for t in temperatures:
            avg_iters = []
            for c in cooling_coefs:
                xs, es, iterations = [], [], []
                for x_0 in init_x:
                    x, e, i = sim_annealing(x_0, optimal_step, t, c)
                    xs.append(x)
                    es.append(e)
                    iterations.append(i)

                # plot the new location of eacth agent
                plot_model(x_axis, y_model, xs, es, 'Temp {:.2f}, Cooling {:.2f}'.format(t,c))
```


Temp 10.00, Cooling 0.50
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0
2
4
6
8
10