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Mathematical Image Analysis Group | Saarland University

Classroom Work Assignment C12

Problem C12.1 (Directional Derivatives)

(a) In Assignment H9, Problem 1 we used $\partial_{\boldsymbol{v}} u = \boldsymbol{v}^\top \boldsymbol{\nabla} u$ with a normalised vector $\boldsymbol{v}$. Show that

$$\partial_{\boldsymbol{v}\boldsymbol{v}} u = \boldsymbol{v}^\top \mathbf{Hess}\,(u)\, \boldsymbol{v},$$

where $\mathbf{Hess}\,(u)$ is the Hessian of u.

(b) Let $\boldsymbol{v}$ and $\boldsymbol{w}$ be normalised 2-D vectors with $\boldsymbol{v} \perp \boldsymbol{w}$. Using (a), prove that the 2-D Laplacian Δu can be written as

$$\Delta u = \partial_{\boldsymbol{v}\boldsymbol{v}} u + \partial_{\boldsymbol{w}\boldsymbol{w}} u\,.$$

What does this tell you about the Laplacian?

Homework Assignment H12

Problem H12.1 (Curvature-Based Morphology) 6P

Prove the following equivalences for MCM:

$$\begin{aligned}\partial_t u &= \partial_{\xi\xi} u \\ &= \frac{u_y^2 u_{xx} - 2u_x u_y u_{xy} + u_x^2 u_{yy}}{u_x^2 + u_y^2} \\ &= \Delta u - \frac{1}{|\boldsymbol{\nabla} u|^2} \boldsymbol{\nabla} u^\top \mathbf{Hess}\,(u) \boldsymbol{\nabla} u \\ &= |\boldsymbol{\nabla} u| \, \mathbf{div} \left(\frac{\boldsymbol{\nabla} u}{|\boldsymbol{\nabla} u|} \right).\end{aligned}$$

Problem 12.2 (Affine Invariant Arc-length) 6P

Consider a curve $\boldsymbol{c}$ in $\mathbb{R}^2$ with the parametrisation $\boldsymbol{c}(p) = (x(p), y(p))^\top$, and an affine transformation $f : \boldsymbol{x} \mapsto \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}$, where $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$ satisfies $\det \boldsymbol{A} = 1$, and $\boldsymbol{b} \in \mathbb{R}^2$ is a translation vector.
Show that the affine arc-lengths of $\boldsymbol{c}$ and $f(\boldsymbol{c})$ are equal.

Problem H12.3 (Mean Curvature Motion) 2+1+1+2P

(a) Supplement the missing code in `mcm.c` such that it performs mean curvature motion.

(b) Compile the program as usual and use it for shape simplification of the image `tyre.pgm`. Useful parameters: $\alpha = 0.49$, $\gamma = 1$, $\tau = 0.4$, and 50/250/1000 iterations.

(c) Apply `mcm.c` to `schaeuble.pgm` with the parameters from (b). Why is `mcm.c` not useful for fingerprint processing?

(d) The images `check1.pgm` and `check2.pgm` are visually very similar. How do they evolve under MCM? Is this in contradiction to the theory?

Problem H12.4 (Corner Detection of Alvarez/Morales) 3+3P

The program `corner_detect.c` uses the ideas of Alvarez and Morales to investigate the corner evolution with the affine morphological scale-space.

(a) Compile it and apply the program to the images `corner01.pgm` and `corner02.pgm` with $\alpha = 0$, $\gamma = 1$, $\tau = 0.01$, and 10000 iterations. The exact corner angles are 90° for `corner01.pgm` and 53.13° for `corner02.pgm`. How good are the results with respect to different stopping times?

(b) To check the robustness of the method under noise, also try the images `corner01_50.pgm` and `corner02_50.pgm`. How is the result influenced by noise?

Submission: Please create a directory `Ex12_<your_name>` with the following files (and nothing else):

- a pdf file – which can also be a scanned handwritten solution – that contains
  - the names of all people working together for this assignment
  - the solutions of the theoretical Problems 1–2 and answers to the questions in Problem 3 and 4
- for Problem 3(a): the completed file `mcm-complete.c`,
- for Problem 3(b): the resulting images `tyre{50,250,1000}.pgm`
- for Problem 3(c): the resulting images `schaeuble{50,250,1000}.pgm`
- for Problem 3(d): the resulting images `check1-mcm.pgm` and `check2-mcm.pgm`
- for Problem 4: the estimated coordinates of the corner point and the corner angle.

Compress the directory to a zip file `Ex12_<your_name>.zip`.

Submit the file via CMS.
Deadline for submission is Friday, January 26, 14:00.