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Saarland University

Classroom Work Assignment C9

Problem C9.1 (Data Terms for Large Displacements)

(a) Write down the optic flow constraints that model the combined brightness and gradient constancy of a grey value image sequence, if one refrains from Taylor linearisation.

(b) How does the corresponding quadratic data term look like?

(c) One can transform the previous quadratic data term into a nonquadratic formulation, by applying robust functions Ψ_j in different ways on the quadratic data term. Discuss the different possibilities and their respective advantages and disadvantages. Also state scenarios where the different formulations make sense.

Homework Assignment H9

Problem H9.1 (Joint Image- and Flow-driven Regularisation)

2+2+2+3P

(a) Let $\boldsymbol{v}$ be a vector, normalised to 1, and let $\partial_{\boldsymbol{v}} u := \boldsymbol{v}^\top \boldsymbol{\nabla} u$ denote the derivative in $\boldsymbol{v}$ direction. Show that the quadratic optic flow regulariser $S = |\partial_{\boldsymbol{v}} u_1|^2 + |\partial_{\boldsymbol{v}} u_2|^2$ creates the diffusion tensor $\boldsymbol{D} = \boldsymbol{v}\boldsymbol{v}^\top$ in the gradient descent equations.

(b) Analogously, show that the nonquadratic optic flow regulariser $S = \Psi\left(|\partial_{\boldsymbol{v}} u_1|^2 + |\partial_{\boldsymbol{v}} u_2|^2\right)$ yields the diffusion tensor $\Psi'\left(|\partial_{\boldsymbol{v}} u_1|^2 + |\partial_{\boldsymbol{v}} u_2|^2\right) \boldsymbol{v}\boldsymbol{v}^\top$.

(c) Use the previous results to construct a diffusion tensor that uses eigenvectors parallel and perpendicular to the spatial edges of the image sequence f. Its eigenvalues should be given by a decreasing function of the squared flow derivatives along these eigenvectors.

(d) What would be the corresponding optic flow regulariser for the diffusion tensor in (c)? Justify your answer.

Note: The method in (c) is a joint image- and flow-driven method, since its eigenvectors depend on the image sequence f, and its eigenvalues on the flow field $\boldsymbol{u}$. It benefits from the nice edge structure of f, while its actual amount of smoothing is reduced at flow discontinuities. Hence, it does not suffer from the typical oversegmentation problems of pure image-driven optic flow methods.

Problem H9.2 (Algorithms for the Parabolic and Elliptic Problem)

2+2+1P

Consider the energy functional

$$E(u,v) = \frac{1}{2} \int_\Omega \left(\boldsymbol{w}^\top \boldsymbol{J} \boldsymbol{w} + \alpha \left(|\boldsymbol{\nabla} u|^2 + |\boldsymbol{\nabla} v|^2 \right) \right) dx\, dy$$

where $\boldsymbol{w} = (u, v, 1)^\top$ denotes the optic flow, and $\boldsymbol{J} = (J_{i,k}) : \Omega \to \mathbb{R}^{3\times 3}$ is some motion tensor field.

Derive two iterative numerical algorithms for $u_{i,j}^{k+1}$ and $v_{i,j}^{k+1}$ that solve the corresponding Euler-Lagrange equations at inner positions (i, j). Proceed in the following steps:

(a) Discretise the parabolic problem. Use the modified explicit scheme with an implicitly stabilised data term. How does the iterative scheme for $u_{i,j}^{k+1}$ and $v_{i,j}^{k+1}$ look like?

(b) Discretise the elliptic problem. Use the Jacobi method to derive equations for $u_{i,j}^{k+1}$ and $v_{i,j}^{k+1}$.

(c) Compare the two methods for some inner pixel (i, j). What do you notice?

Use central differences for approximating second order derivatives and write down the equations for the grid sizes $h_1 = h_2 = h$.

Problem H9.3 (Flow-driven Isotropic Optic Flow) 4+6P

(a) Supplement the program `optic.c` with the missing code such that it becomes a modified explicit scheme for flow-driven isotropic optic flow estimation with the Charbonnier diffusivity

$$g(s^2) \;=\; \Psi'(s^2) \;=\; \frac{1}{\sqrt{1 + s^2/\lambda^2}} \,.$$

(b) Use it for computing the optic flow between the images `ole8.pgm` and `ole9.pgm` with the following three parameter settings:

- $\alpha = 500$ and $\lambda = 0.01$,
- $\alpha = 5000$ and $\lambda = 0.01$,
- $\alpha = 500$ and $\lambda = 10000$.

In all cases, use 20000 iterations with time step size $\tau = 0.2$.
How does α influence the result? What is the role of λ? Which optic flow model is approximated for the third setting?

Submission: Please create a directory `Ex09_<your_name>` with the following files (and nothing else):

- a pdf file – which can also be a scanned handwritten solution – that contains
  - the names of all people working together for this assignment
  - the solutions of the theoretical Problems 1–2 and answers to the questions in Problem 3
- for Problem 3: the completed file `optic-complete.c` and the flow fields `ole-of1.ppm`, `ole-of2.ppm`, and `ole-of3.ppm` for the three parameter settings in Problem 3(b)

Compress the directory to a zip file `Ex09_<your_name>.zip`.

Submit the file via CMS.
Deadline for submission is Friday, January 5, 14:00.