GPA546
Robots industriels

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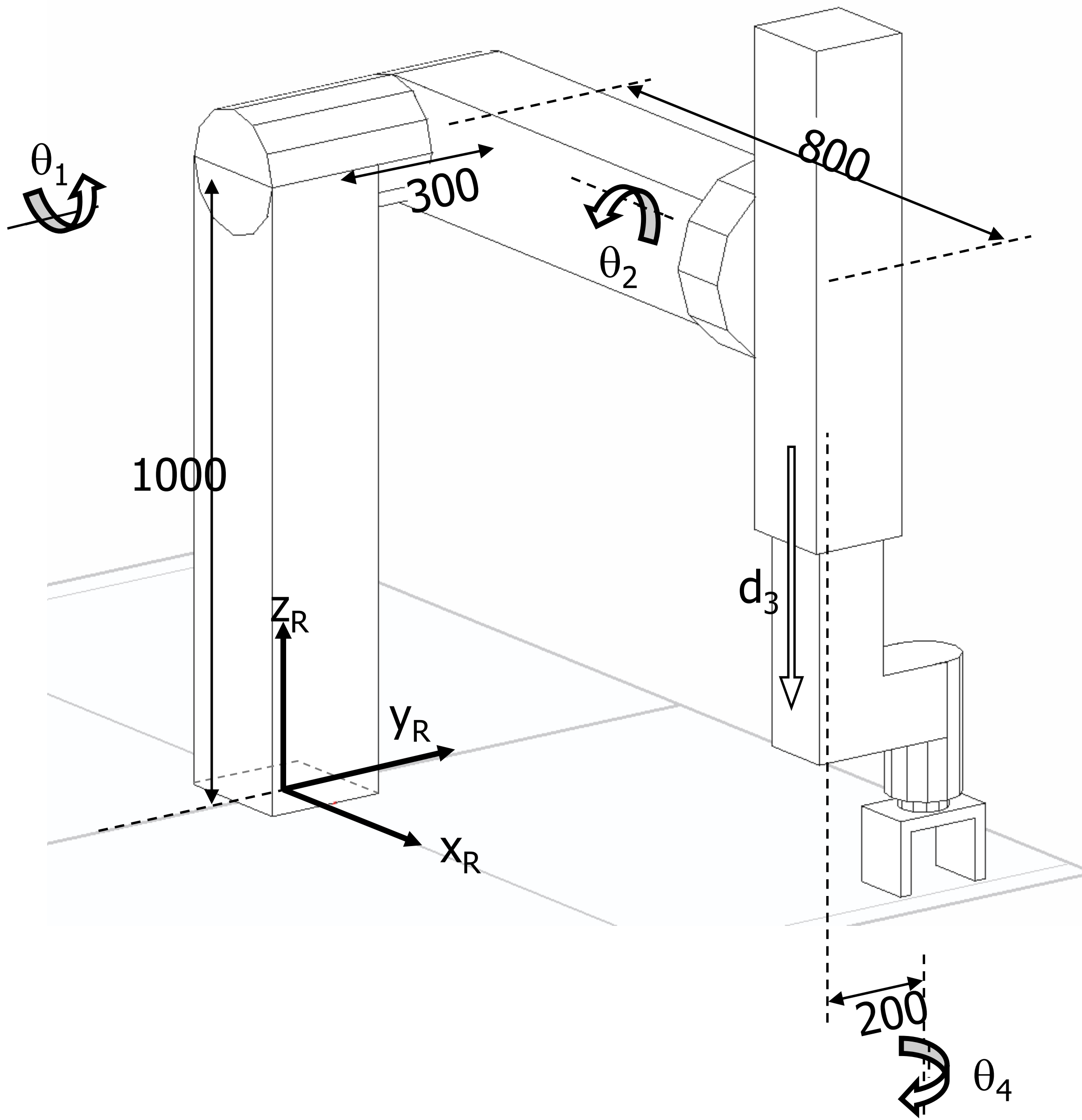
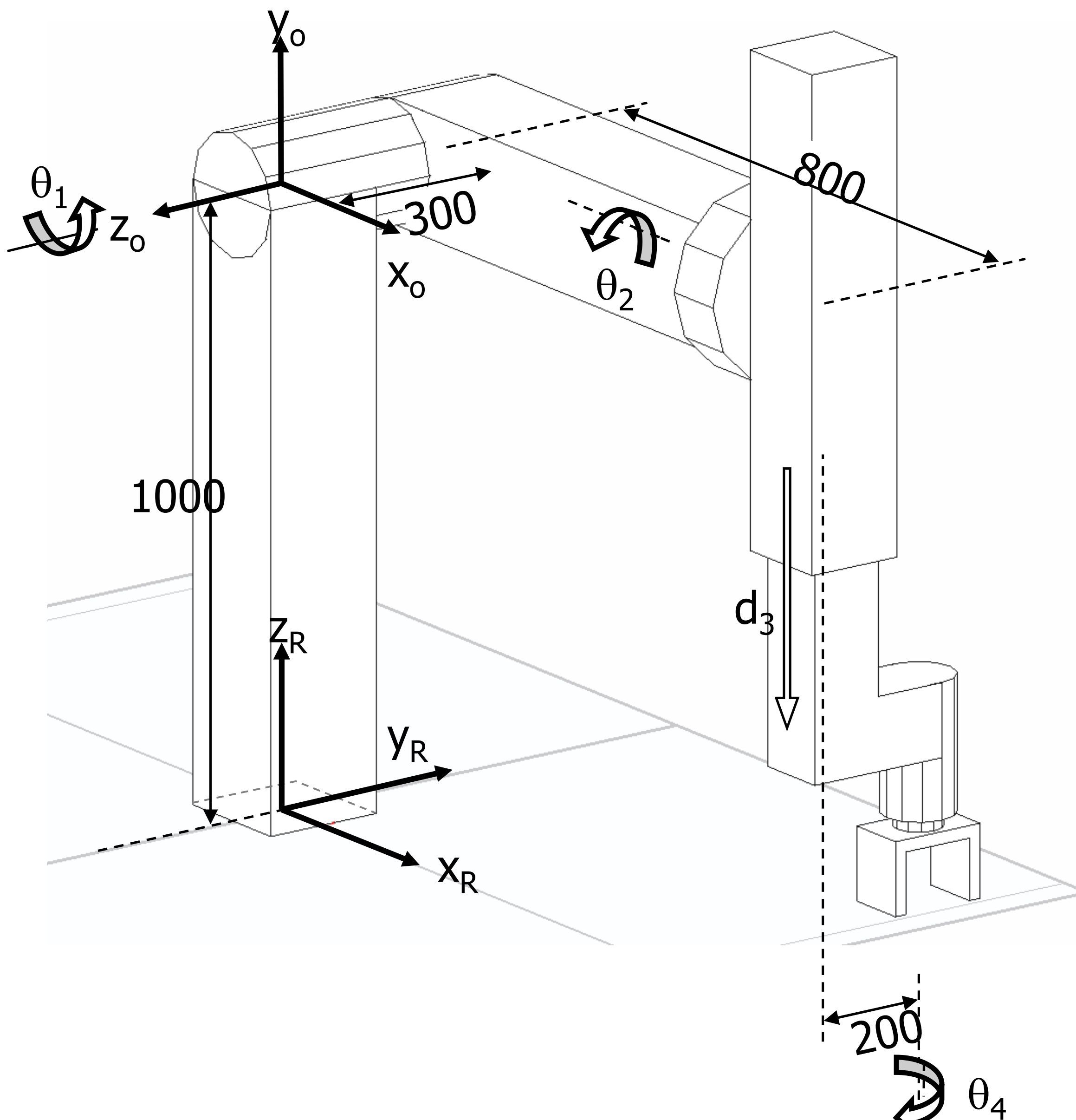
Professeur, ÉTS

Cinématique directe d’un robot à 4 DDL

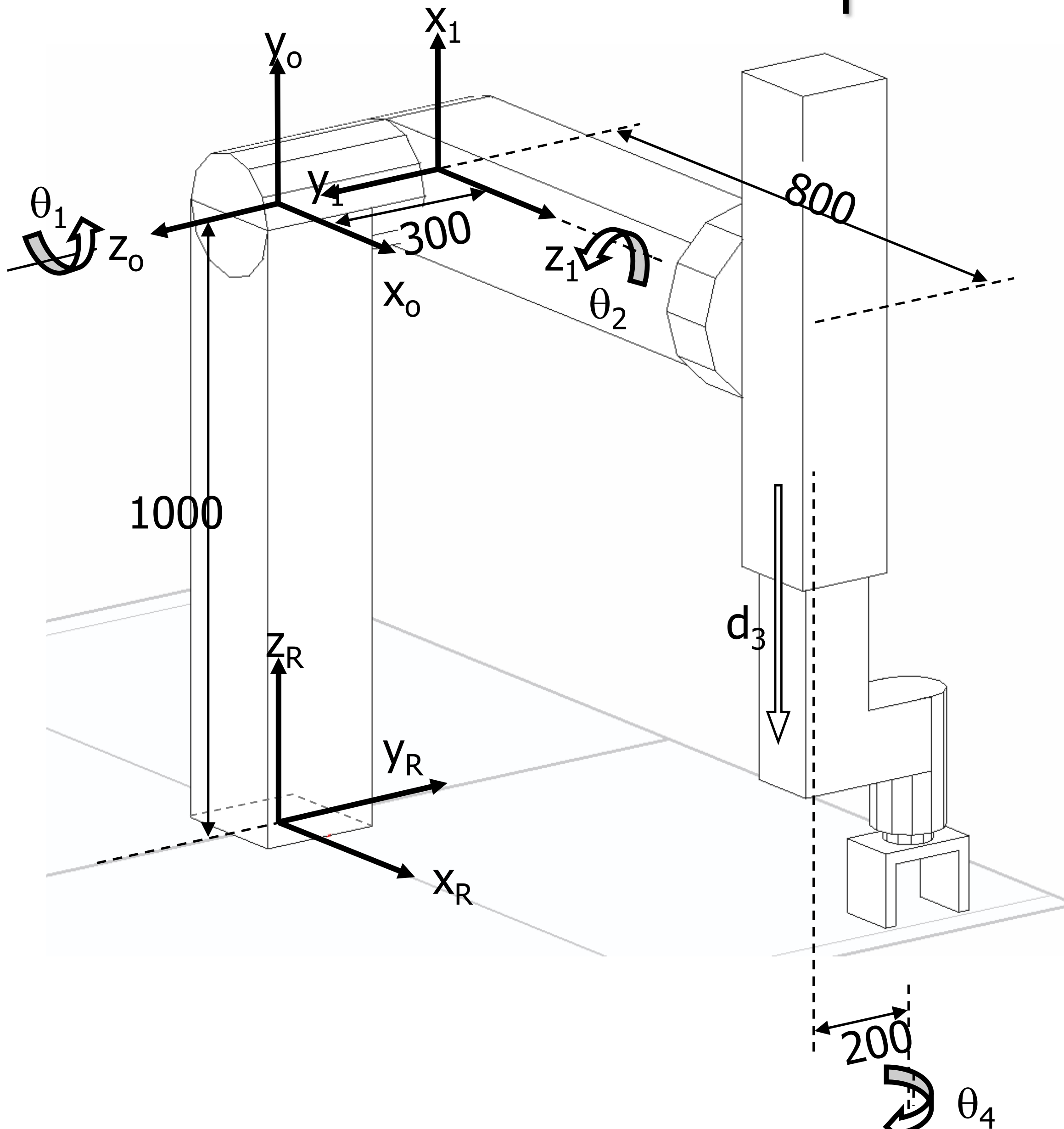
Dimensions du robot



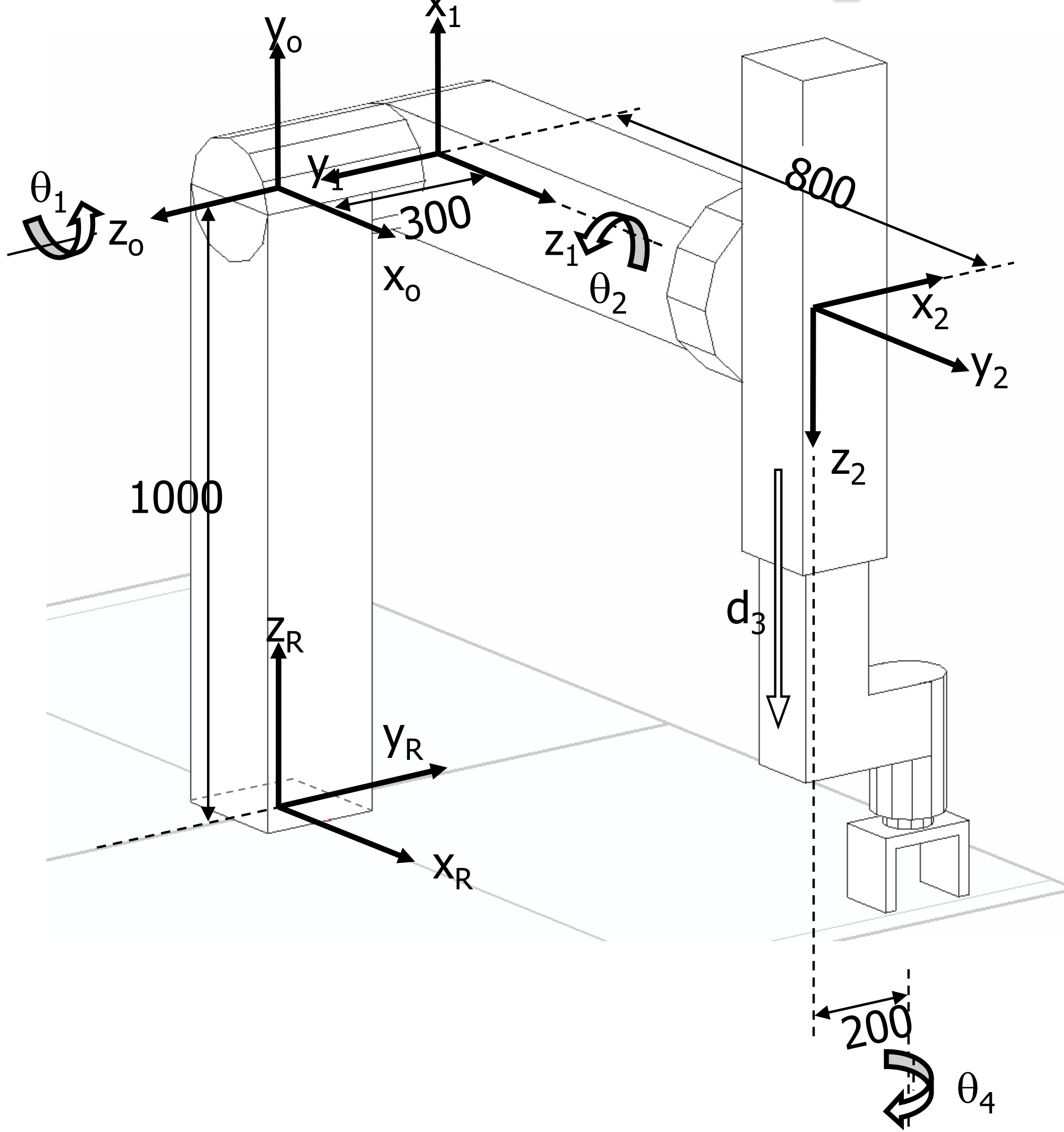
Référentiel F_0



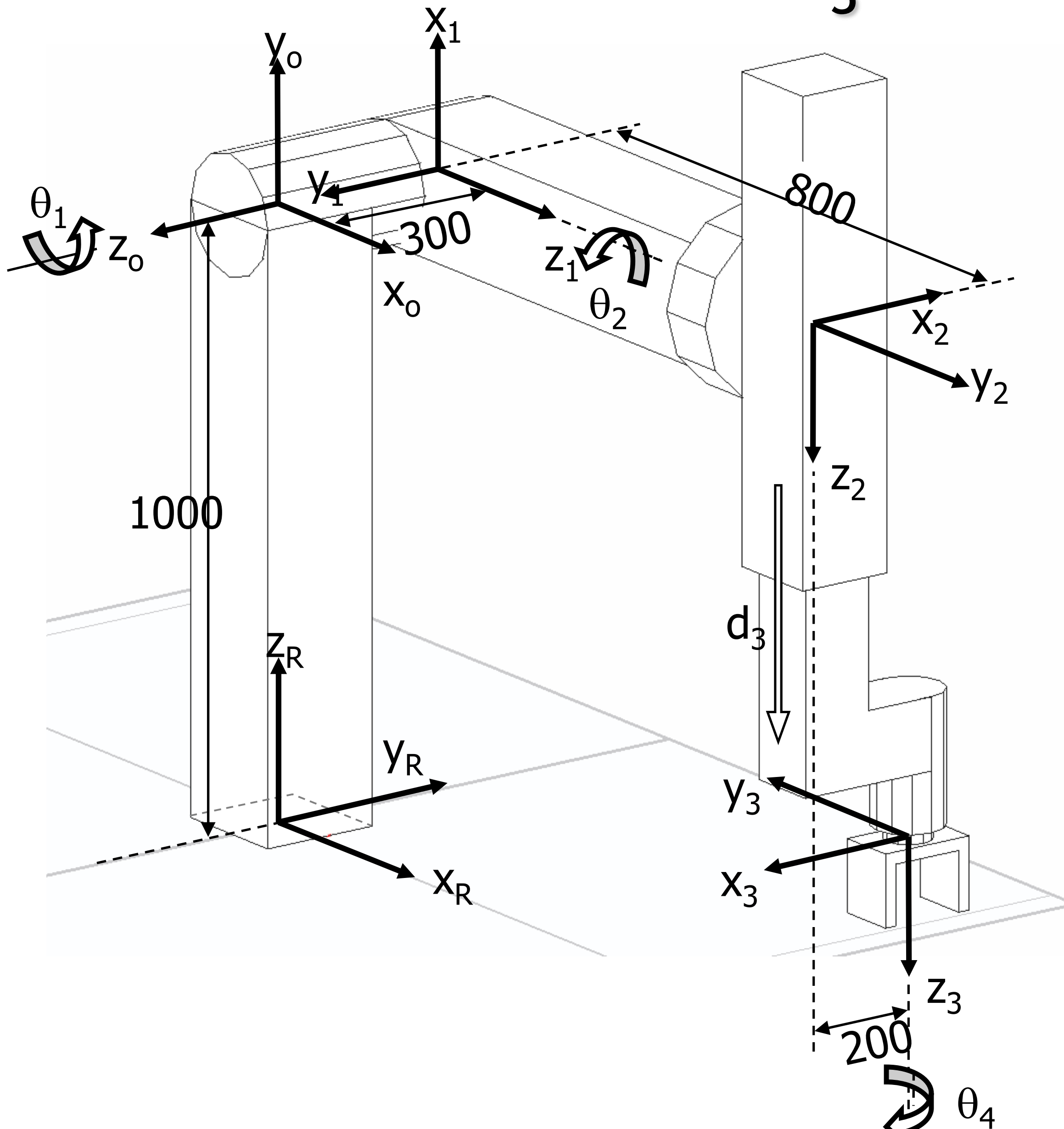
Référentiel F_1



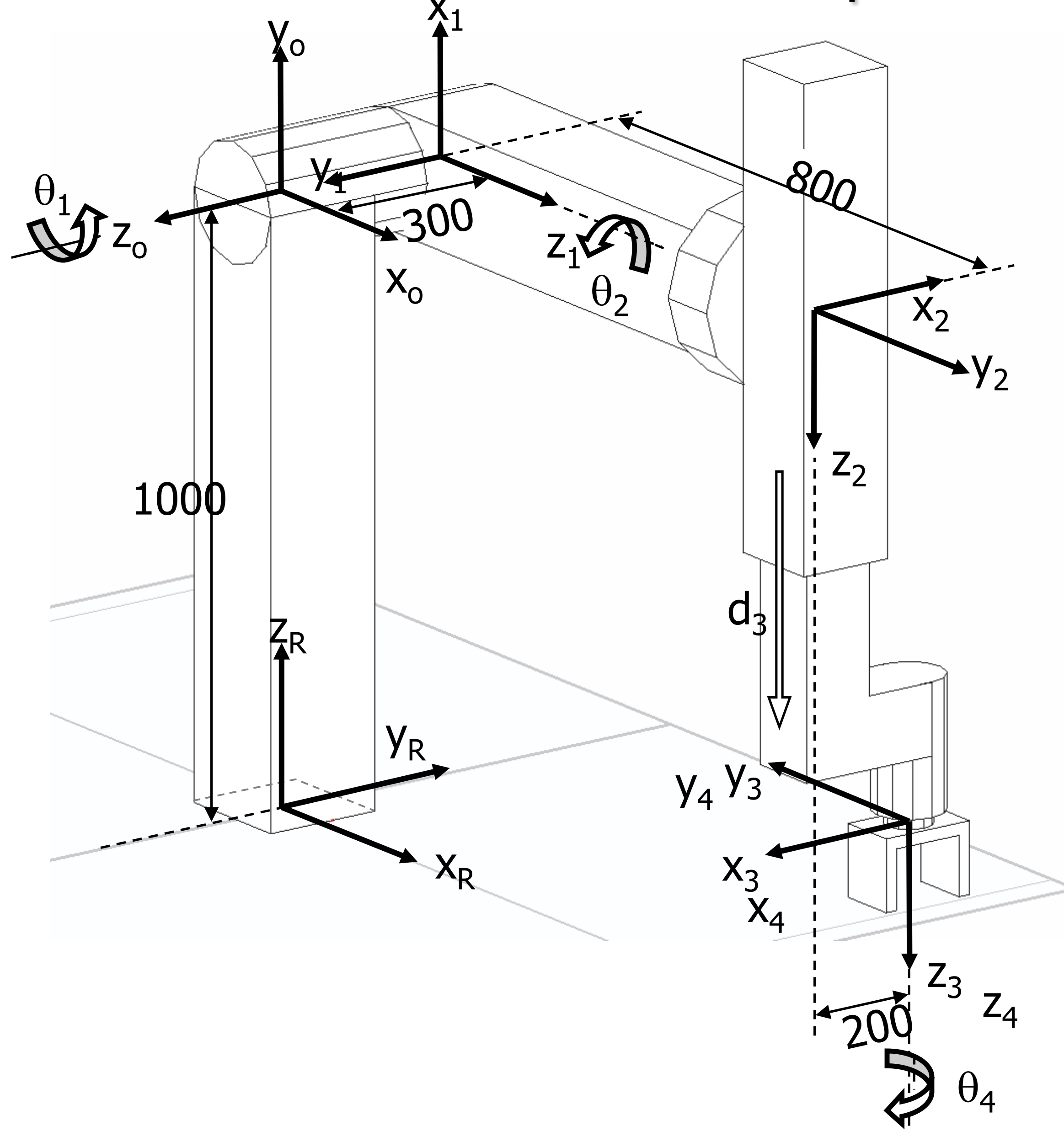
Référentiel F_2



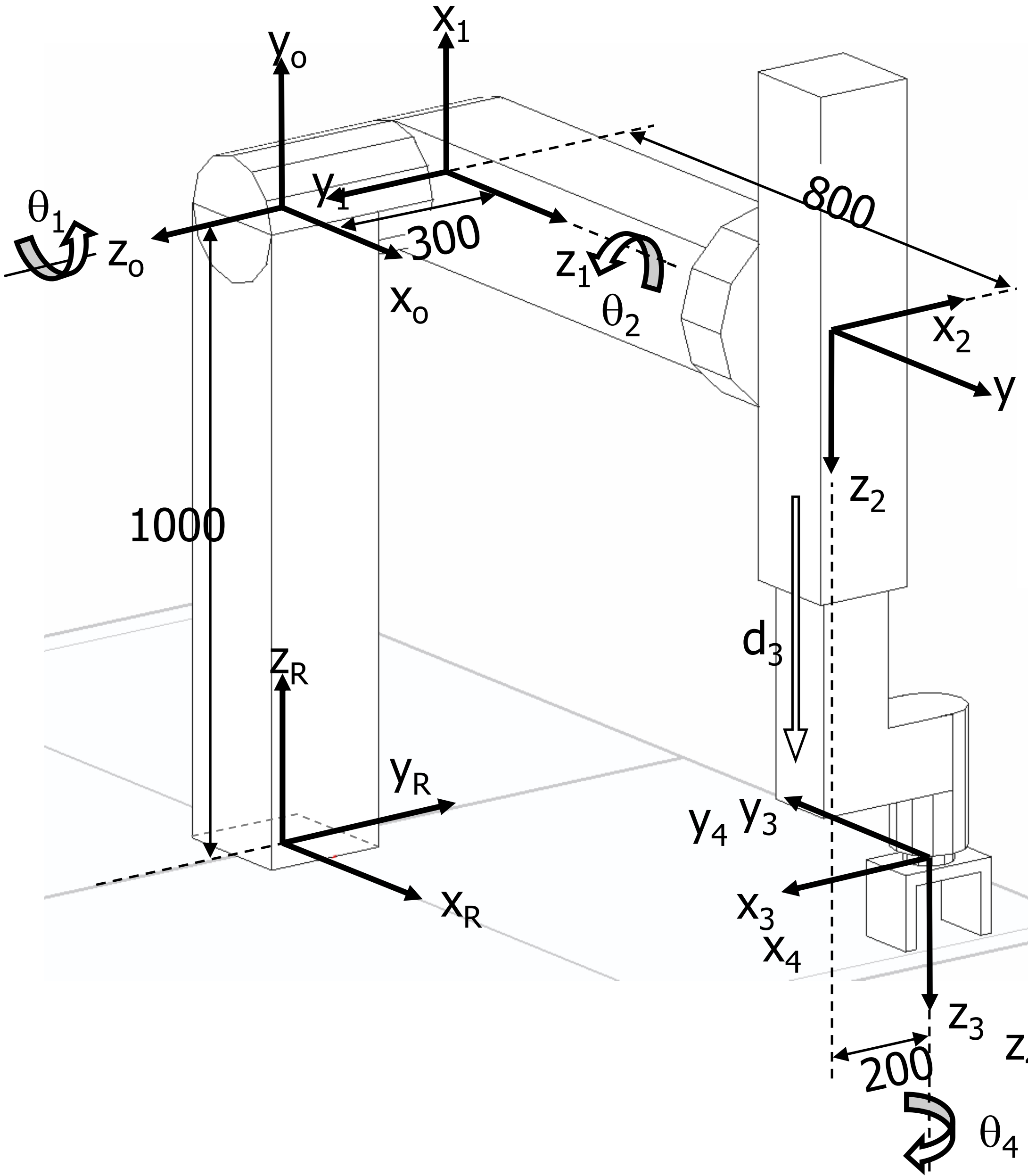
Référentiel F_3



Référentiel F_4



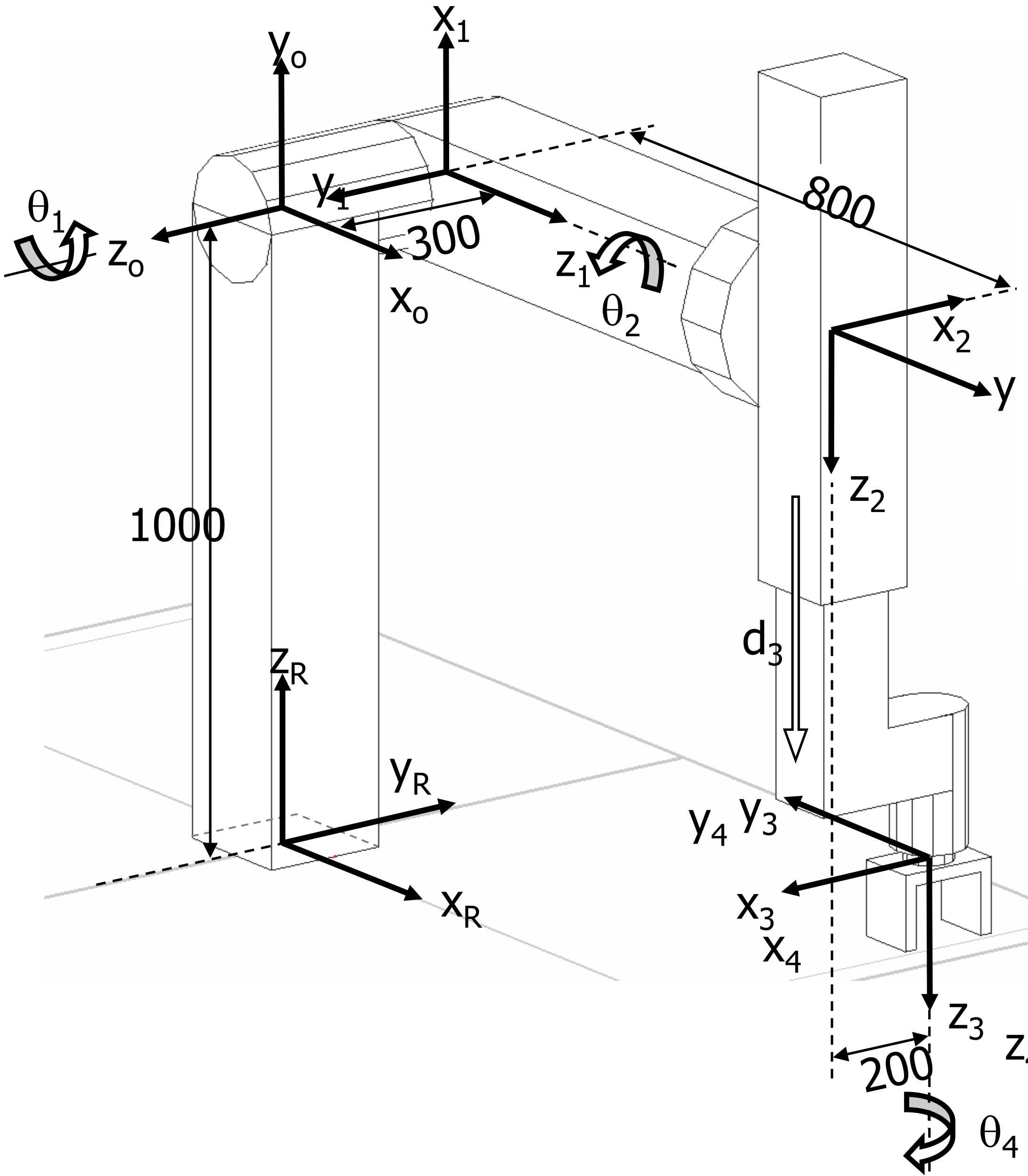
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	t_i
1	θ_1+90°			
2				
3				
4				

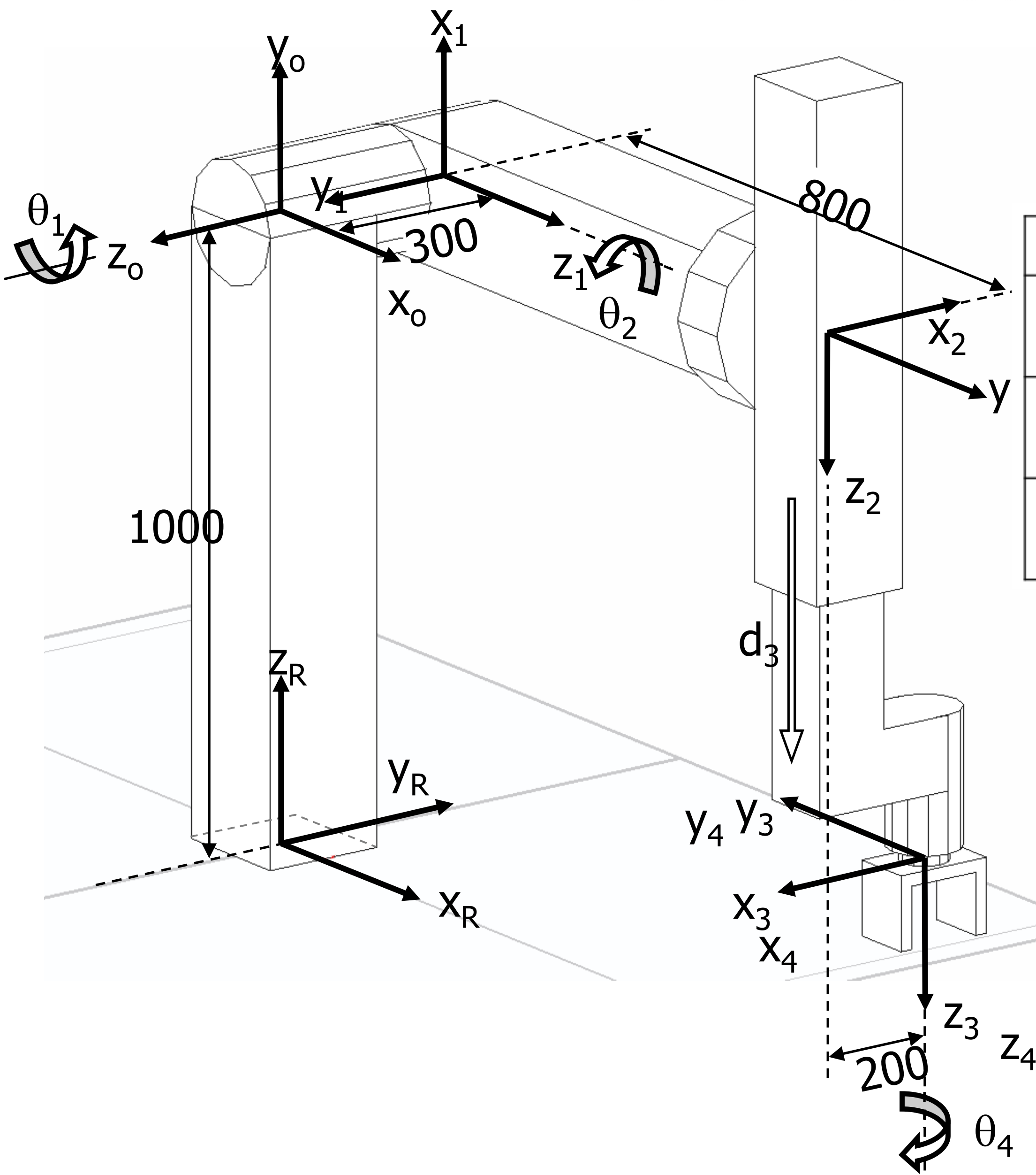
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	θ_1+90°			
2	θ_2-90°			
3				
4				

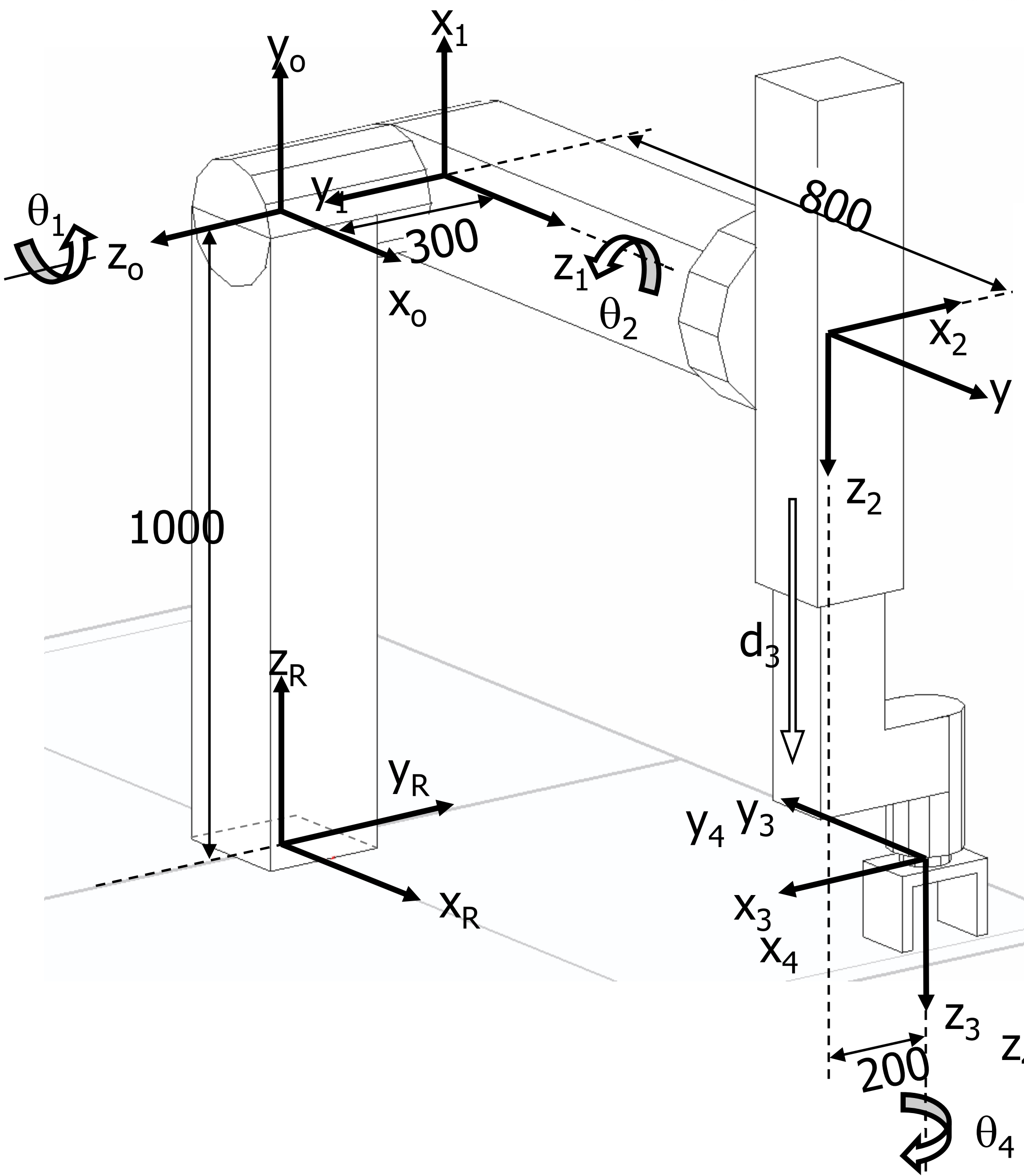
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowleft x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowleft z_1, x_1$
2	$x_1 \circlearrowleft x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowleft z_2, x_2$
3	$x_2 \circlearrowleft x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowleft z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$			
2	$\theta_2-90°$			
3	$180°$			
4				

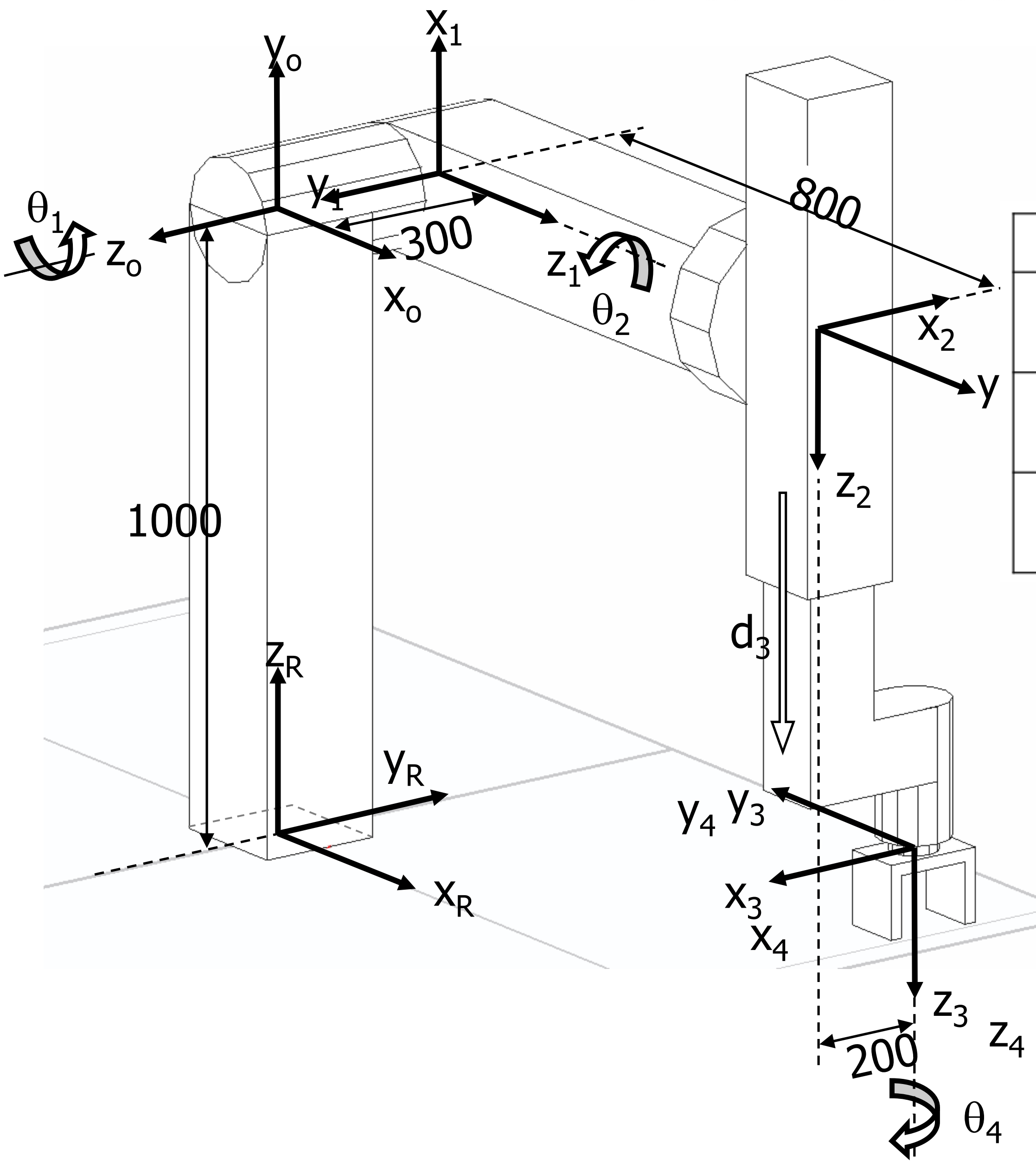
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$			
2	$\theta_2-90°$			
3	$180°$			
4	θ_4			

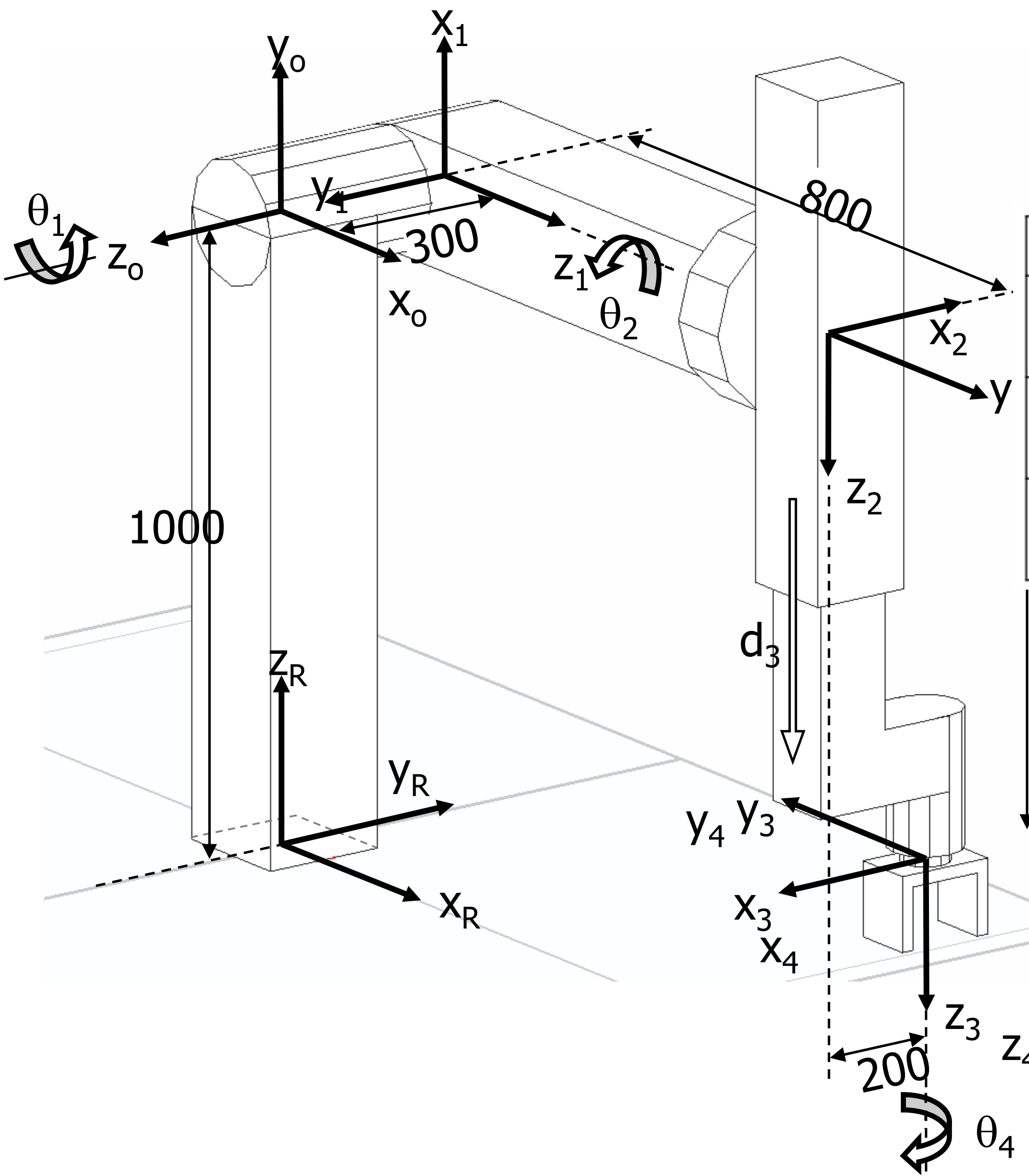
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	θ_1+90°	−300		
2	θ_2−90°			
3	180°			
4	θ_4			

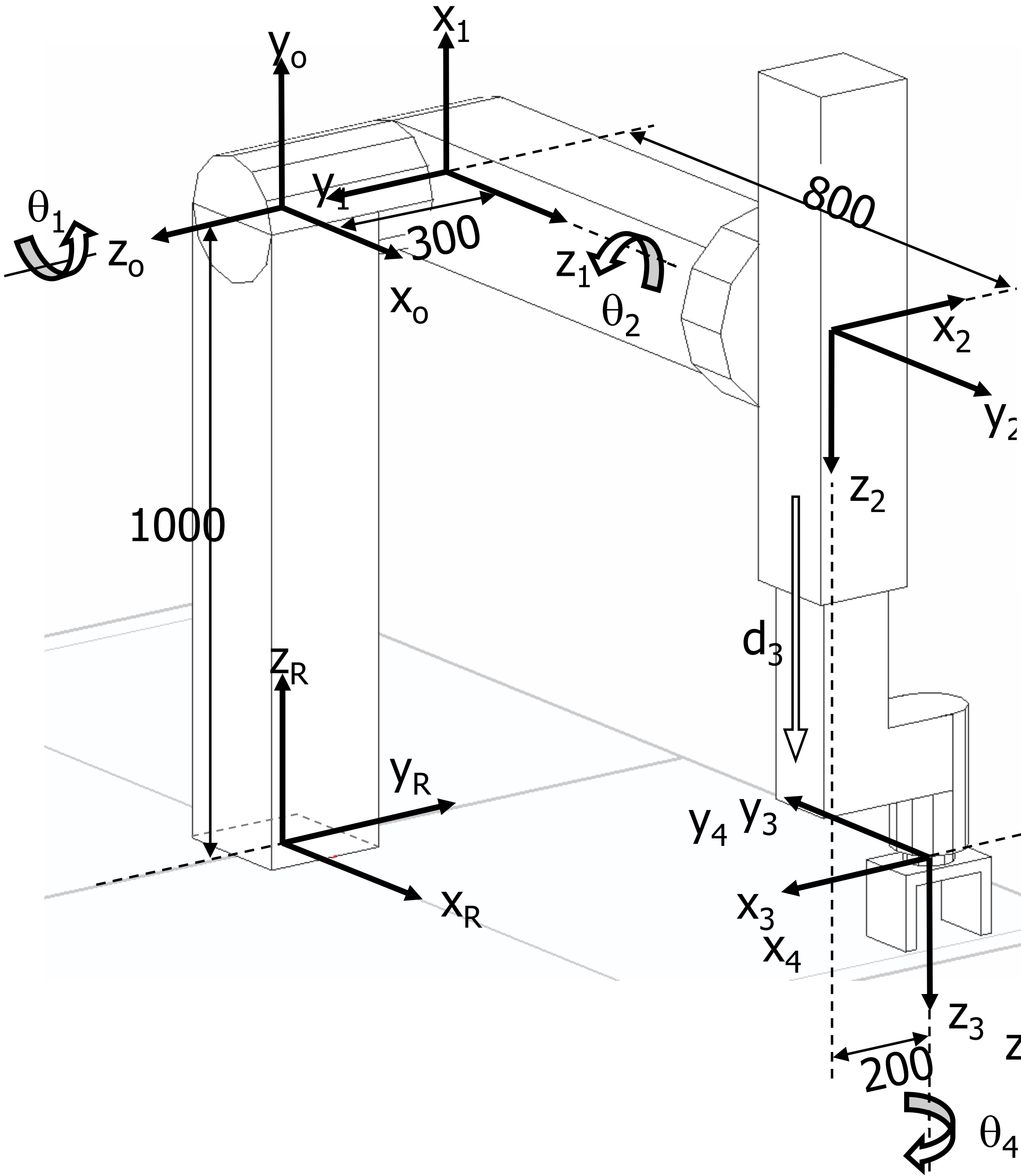
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowleft x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowleft z_1, x_1$
2	$x_1 \circlearrowleft x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowleft z_2, x_2$
3	$x_2 \circlearrowleft x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowleft z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	θ_1+90°	−300		
2	θ_2−90°	800		
3	180°			
4	θ_4			

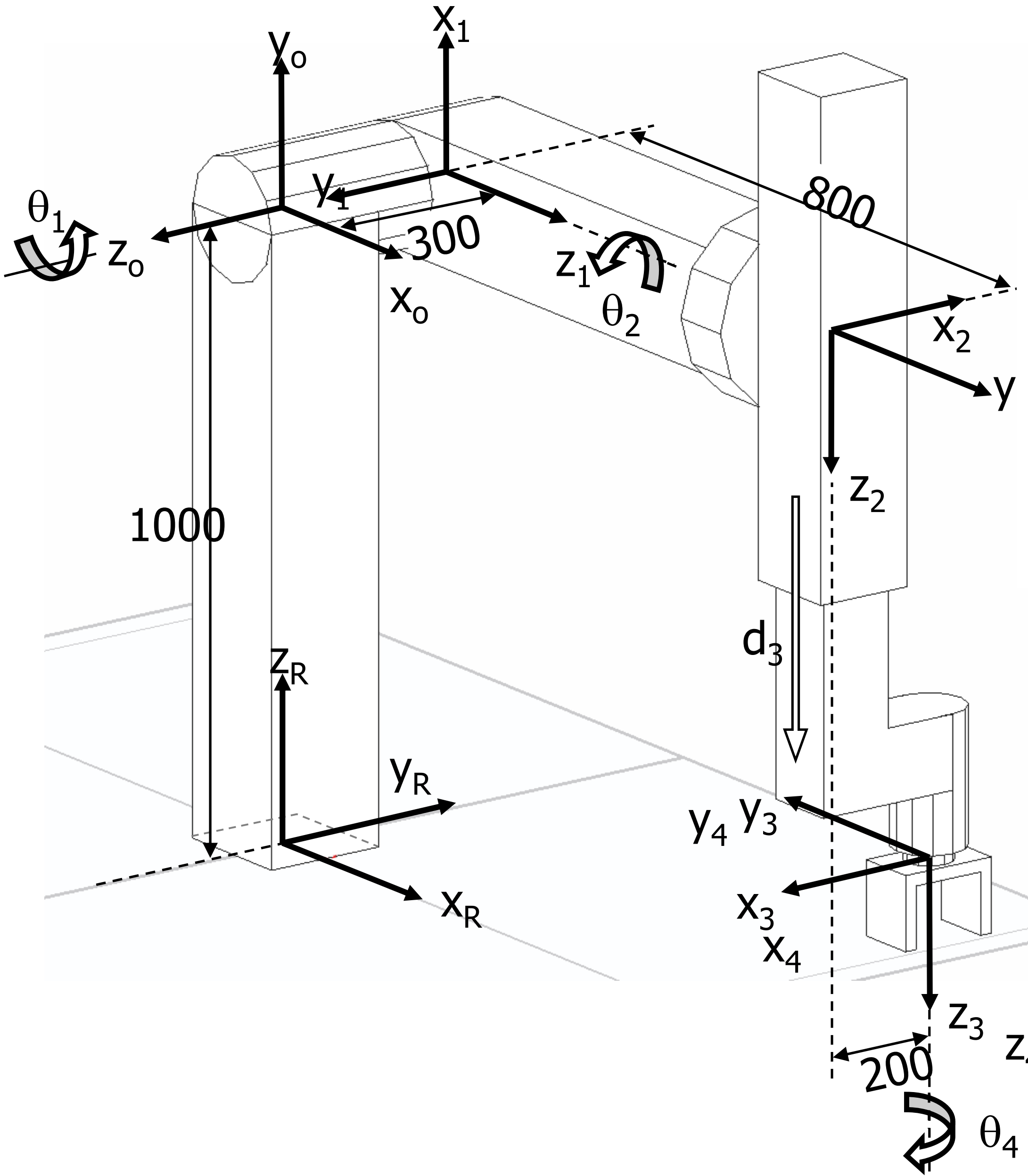
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \to x_1, z_0$	$z_0 \to z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \to x_2, z_1$	$z_1 \to z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \to x_3, z_2$	$z_2 \to z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	θ_1+90°	−300		
2	θ_2−90°	800		
3	180°	d_3+690		
4	θ_4			

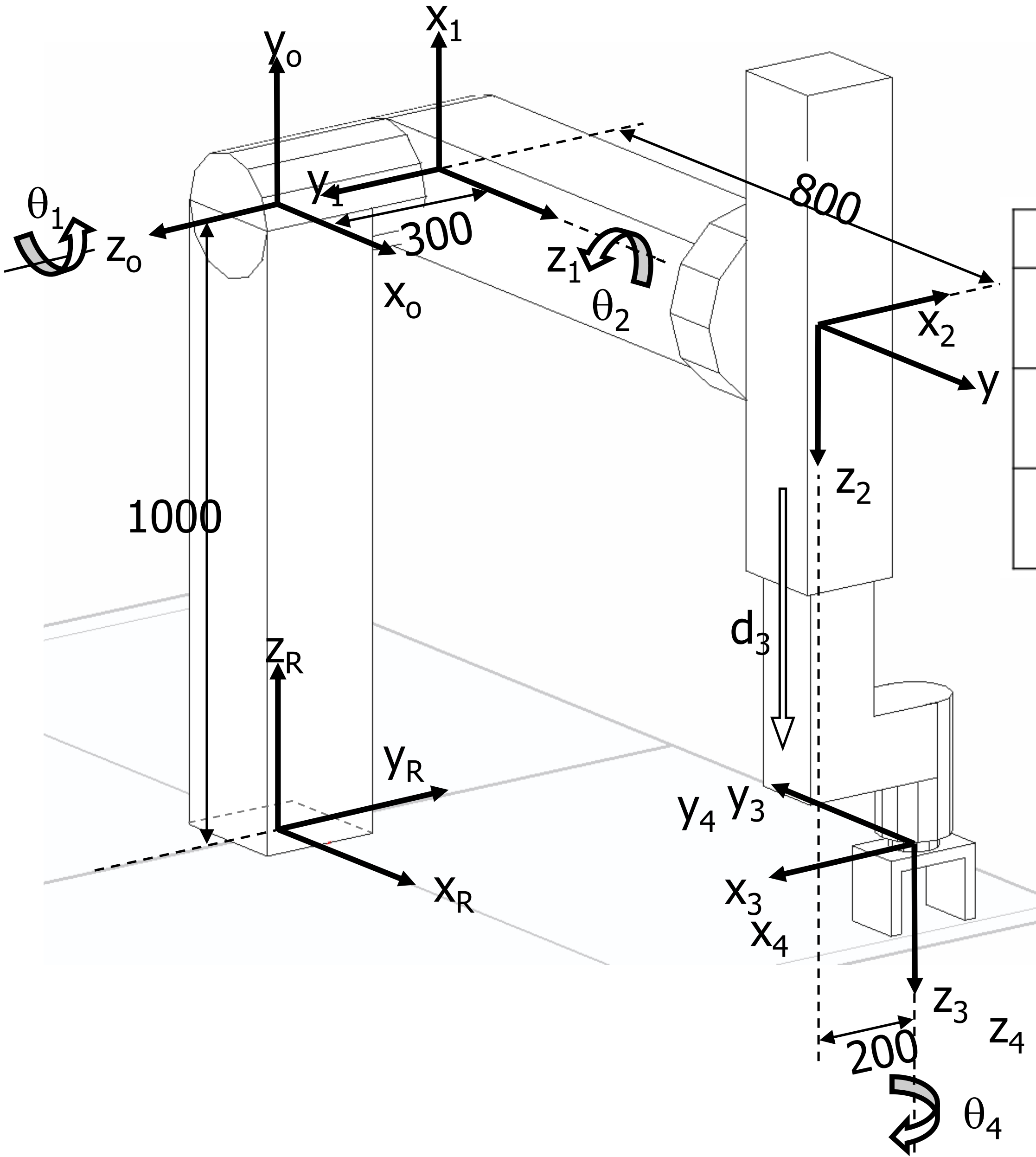
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowleft x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowleft z_1, x_1$
2	$x_1 \circlearrowleft x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowleft z_2, x_2$
3	$x_2 \circlearrowleft x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowleft z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$	-300		
2	$\theta_2-90°$	800		
3	180°	d_3+690		
4	θ_4	0		

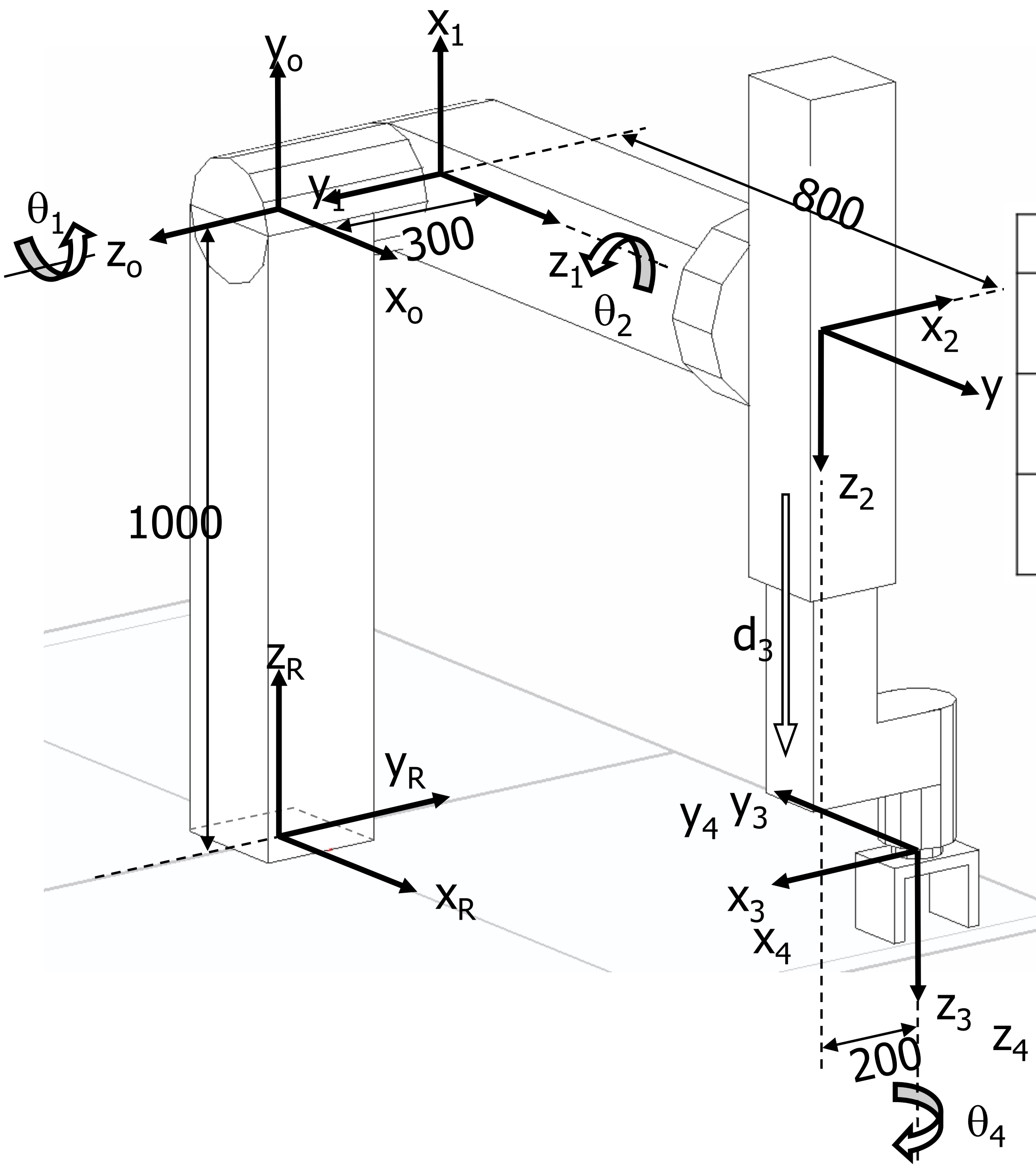
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \to x_1, z_0$	$z_0 \to z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \to x_2, z_1$	$z_1 \to z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \to x_3, z_2$	$z_2 \to z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$	-300	0	
2	$\theta_2-90°$	800		
3	$180°$	d_3+690		
4	θ_4	0		

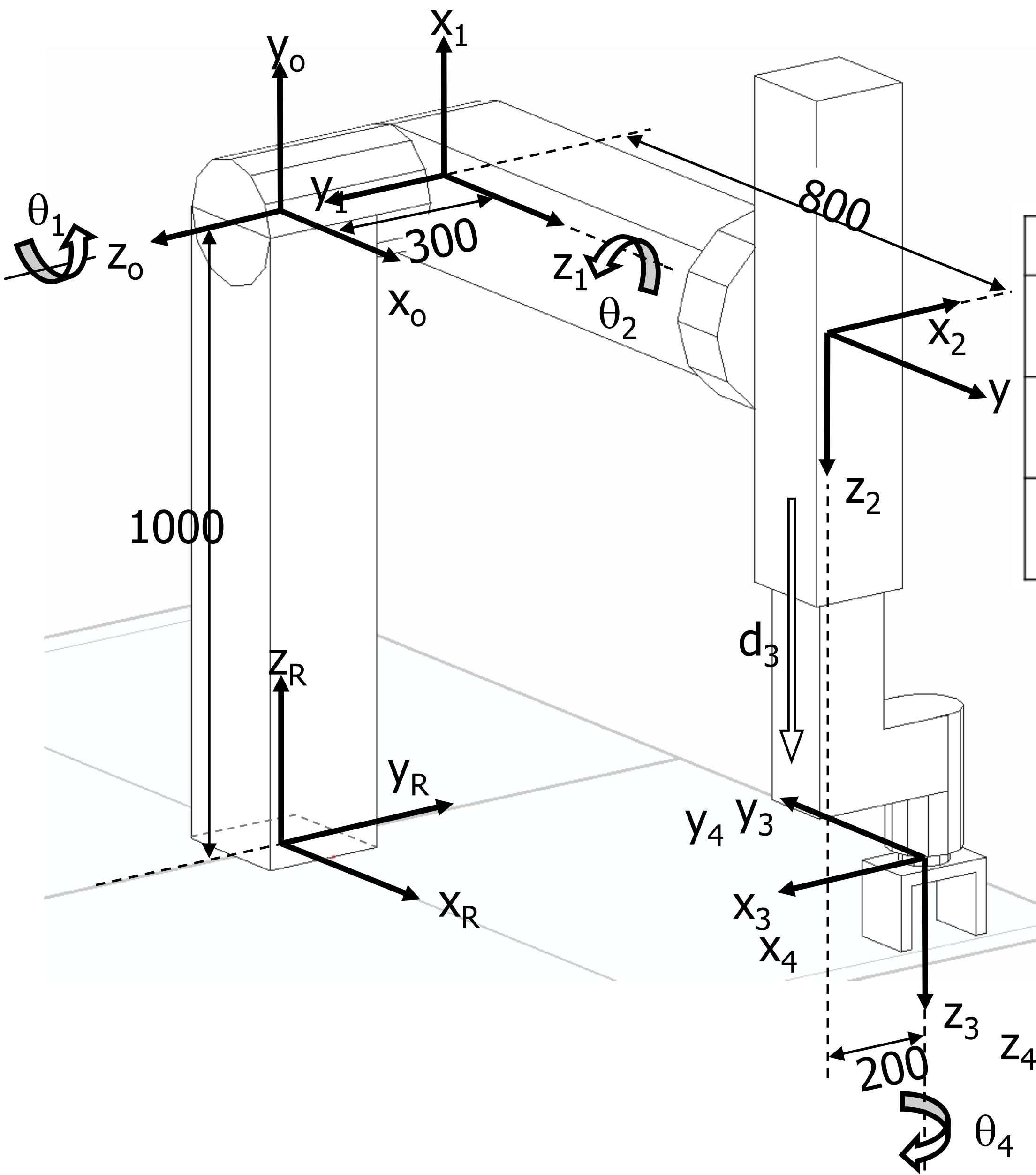
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowleft x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowleft z_1, x_1$
2	$x_1 \circlearrowleft x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowleft z_2, x_2$
3	$x_2 \circlearrowleft x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowleft z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	θ_1+90°	−300	0	
2	θ_2−90°	800	0	
3	180°	d_3+690		
4	θ_4	0		

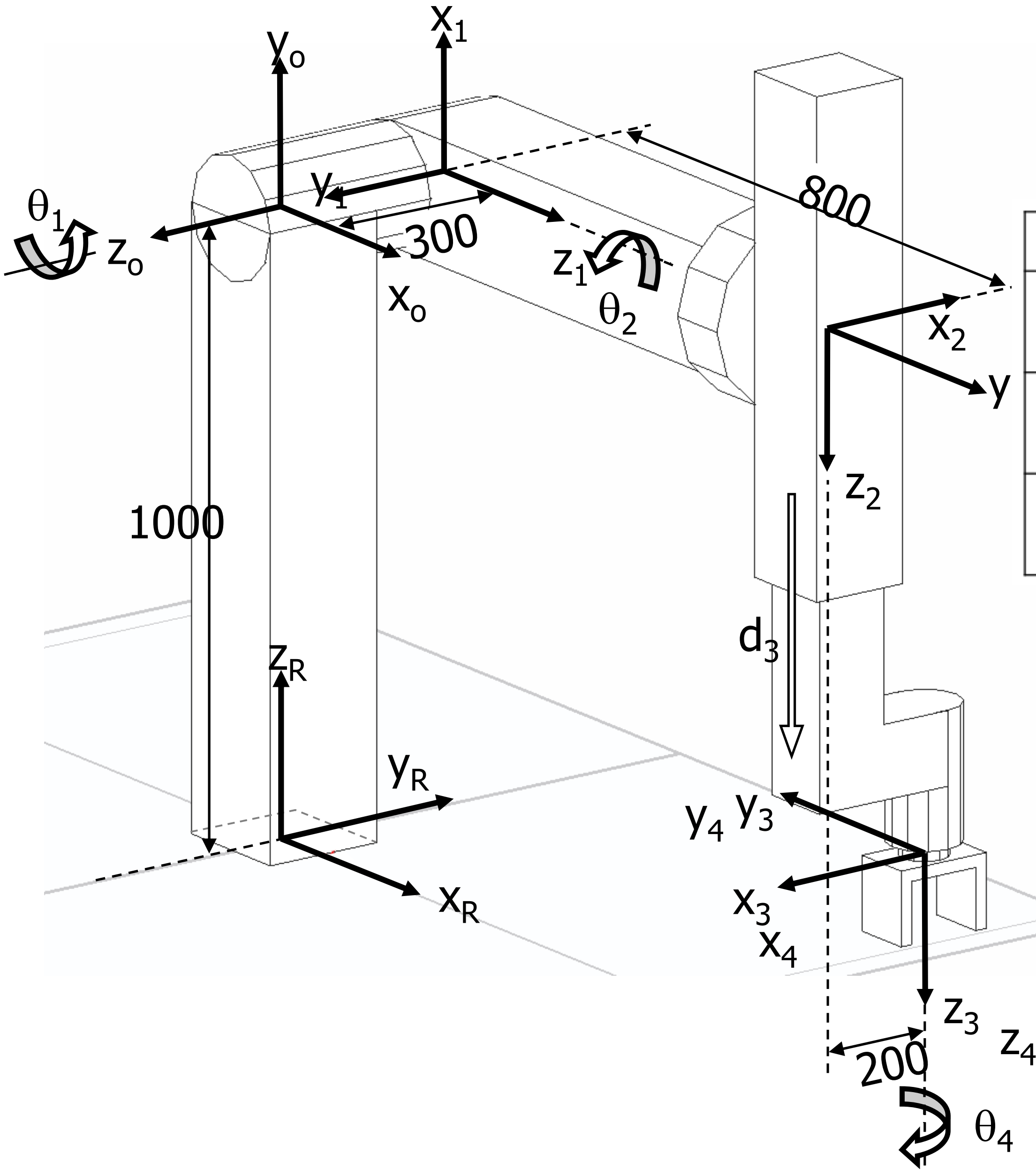
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowleft x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowleft z_1, x_1$
2	$x_1 \circlearrowleft x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowleft z_2, x_2$
3	$x_2 \circlearrowleft x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowleft z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$	-300	0	
2	$\theta_2-90°$	800	0	
3	180°	d_3+690	-200	
4	θ_4	0		

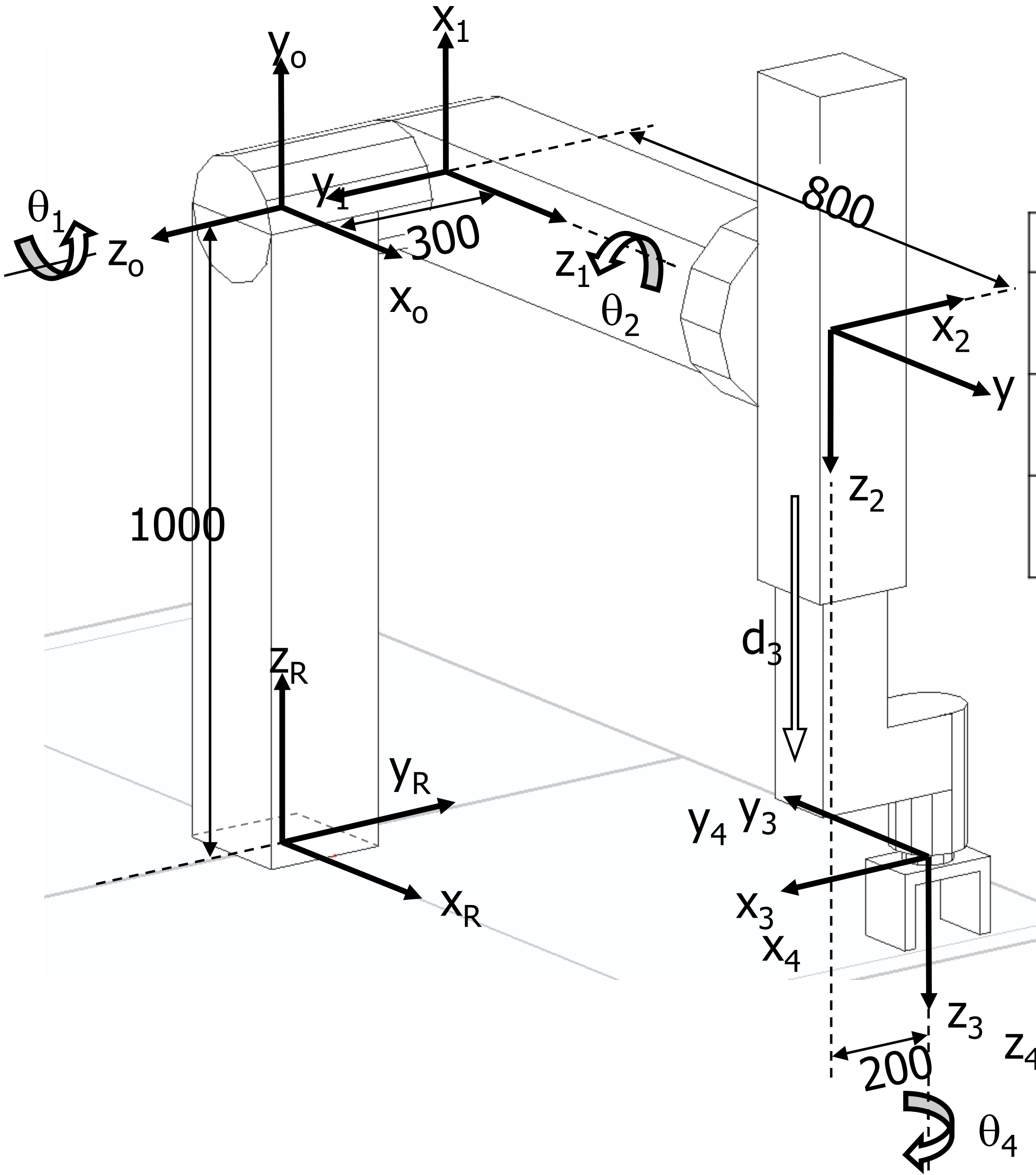
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$	-300	0	
2	$\theta_2-90°$	800	0	
3	180°	d_3+690	-200	
4	θ_4	0	0	

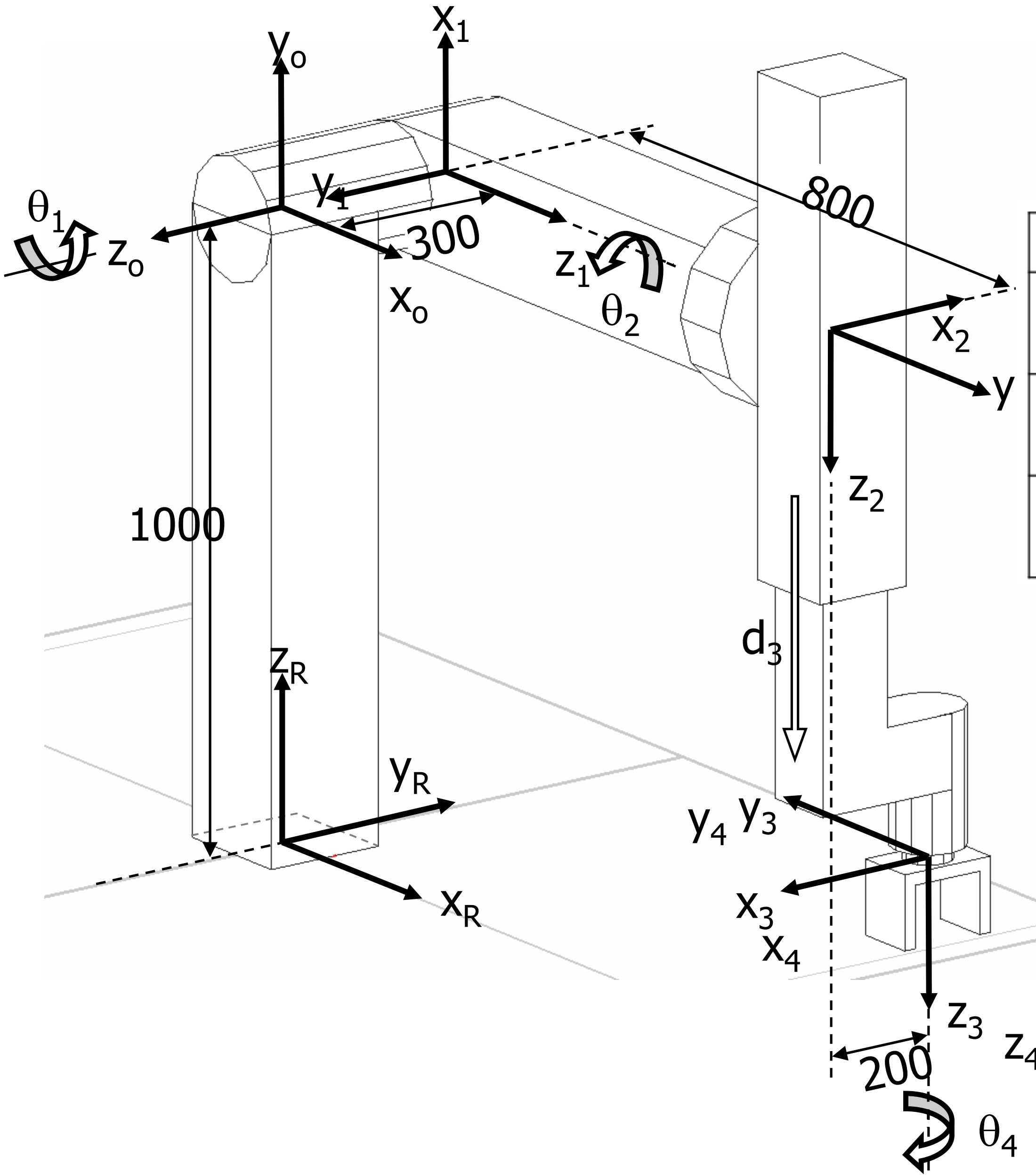
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	θ_1+90°	−300	0	90°
2	θ_2−90°	800	0	
3	180°	d_3+690	−200	
4	θ_4	0	0	

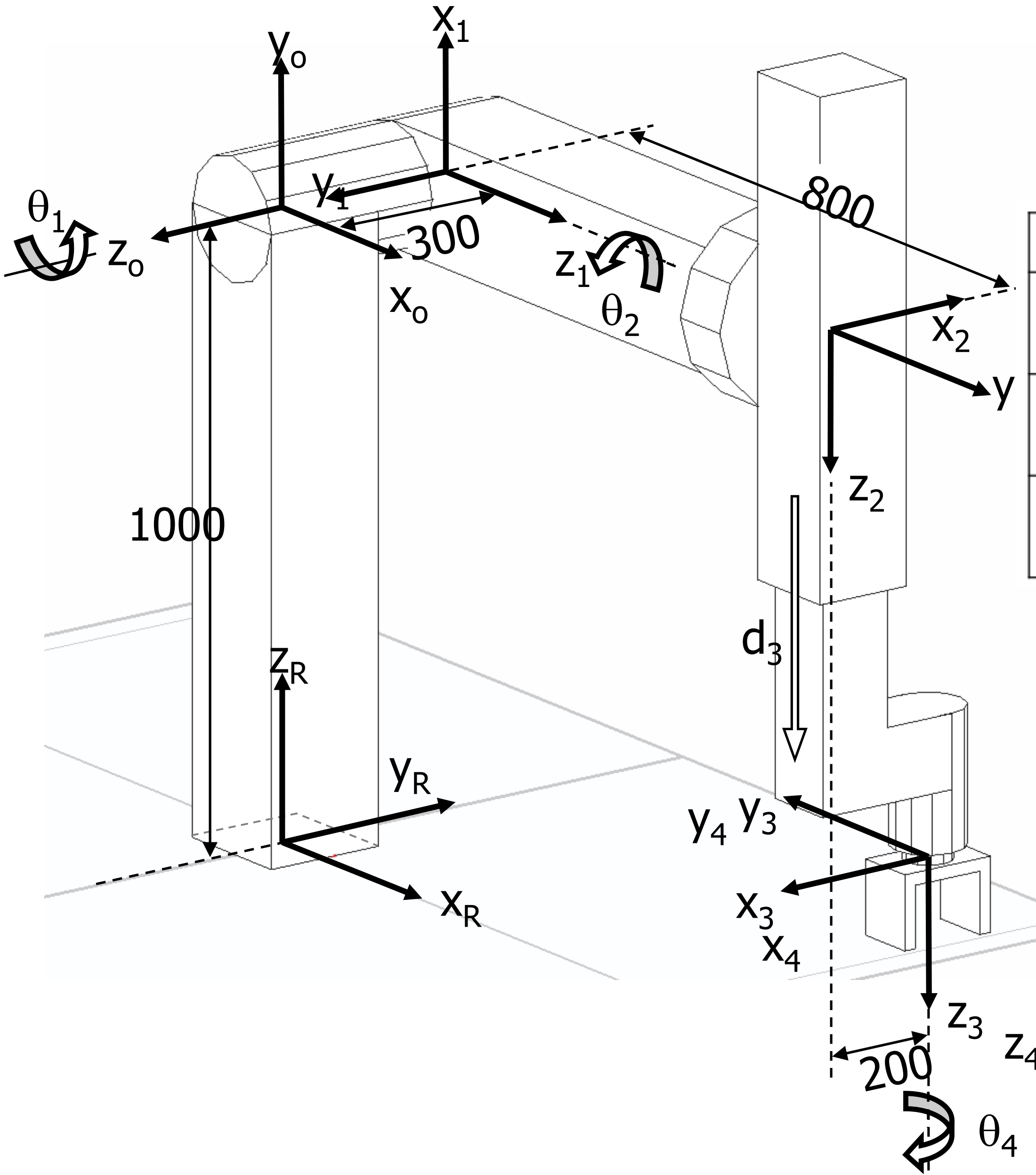
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowleft x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowleft z_1, x_1$
2	$x_1 \circlearrowleft x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowleft z_2, x_2$
3	$x_2 \circlearrowleft x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowleft z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	θ_1+90°	−300	0	90°
2	θ_2−90°	800	0	90°
3	180°	d_3+690	−200	
4	θ_4	0	0	

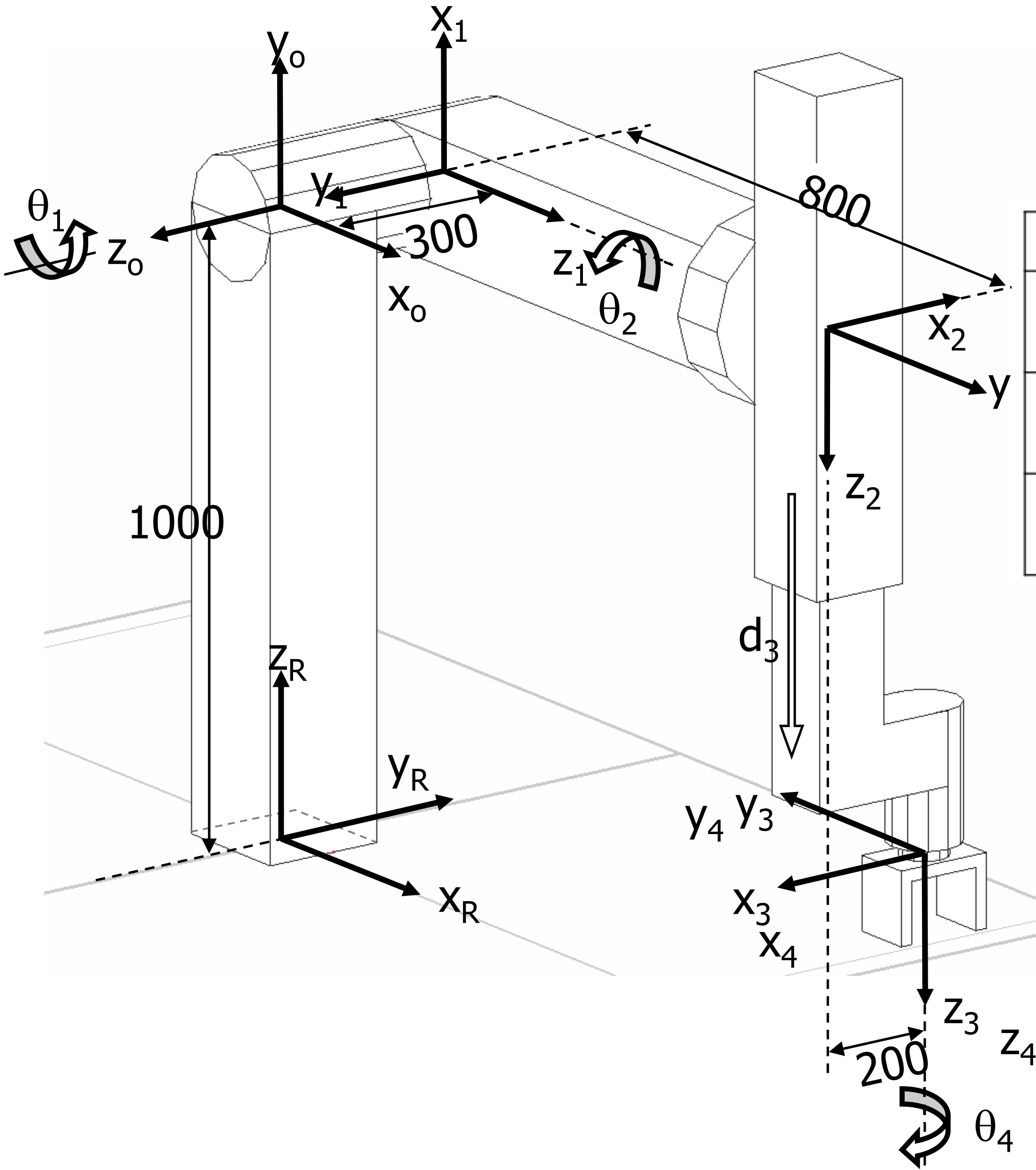
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowright x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowright z_1, x_1$
2	$x_1 \circlearrowright x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowright z_2, x_2$
3	$x_2 \circlearrowright x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowright z_3, x_3$

i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$	-300	0	90°
2	$\theta_2-90°$	800	0	90°
3	180°	d_3+690	-200	0°
4	θ_4	0	0	

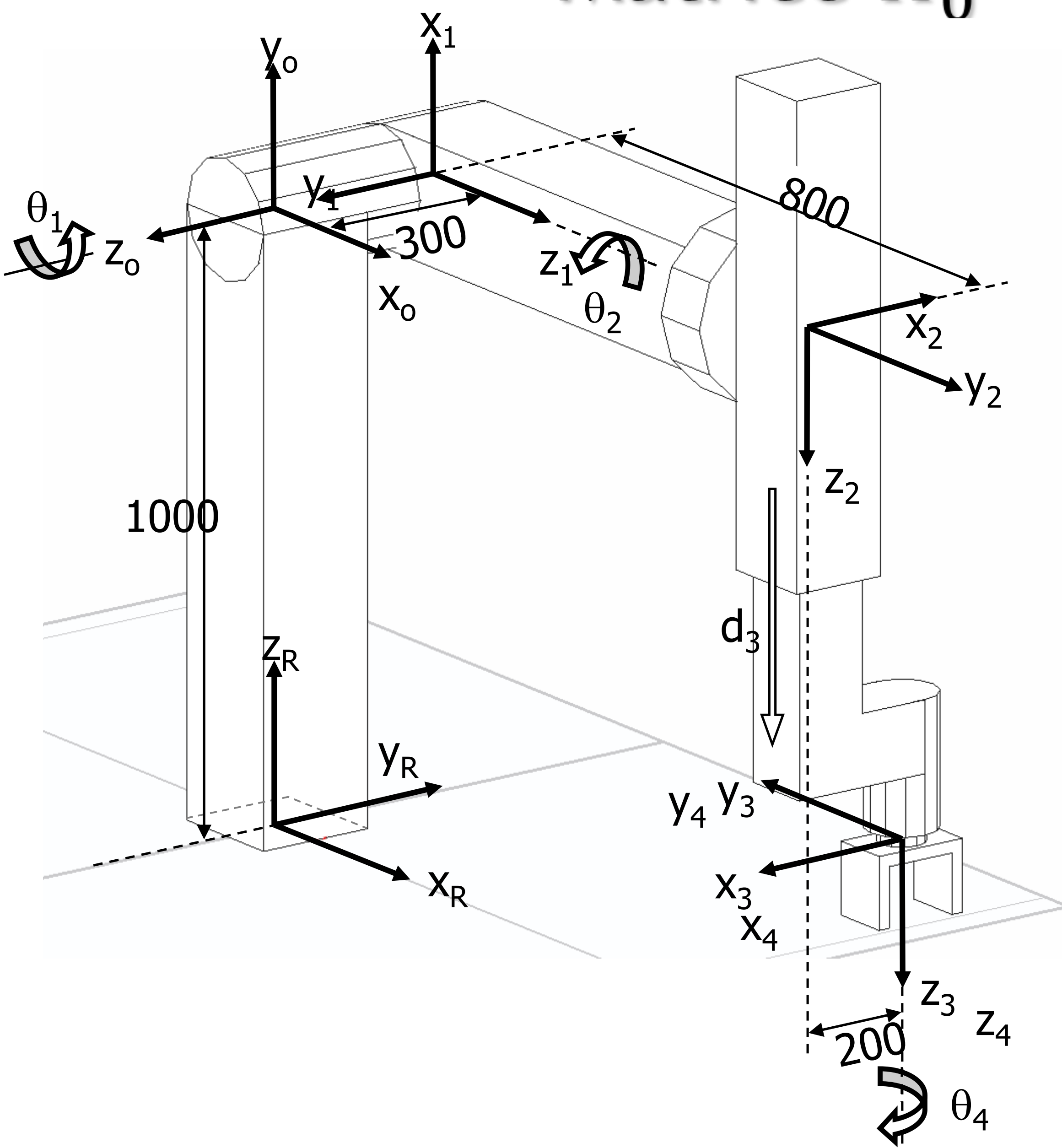
Paramètres D-H



i	θ_i	d_i	a_i	α_i
1	$x_0 \circlearrowleft x_1, z_0$	$x_0 \rightarrow x_1, z_0$	$z_0 \rightarrow z_1, x_1$	$z_0 \circlearrowleft z_1, x_1$
2	$x_1 \circlearrowleft x_2, z_1$	$x_1 \rightarrow x_2, z_1$	$z_1 \rightarrow z_2, x_2$	$z_1 \circlearrowleft z_2, x_2$
3	$x_2 \circlearrowleft x_3, z_2$	$x_2 \rightarrow x_3, z_2$	$z_2 \rightarrow z_3, x_3$	$z_2 \circlearrowleft z_3, x_3$

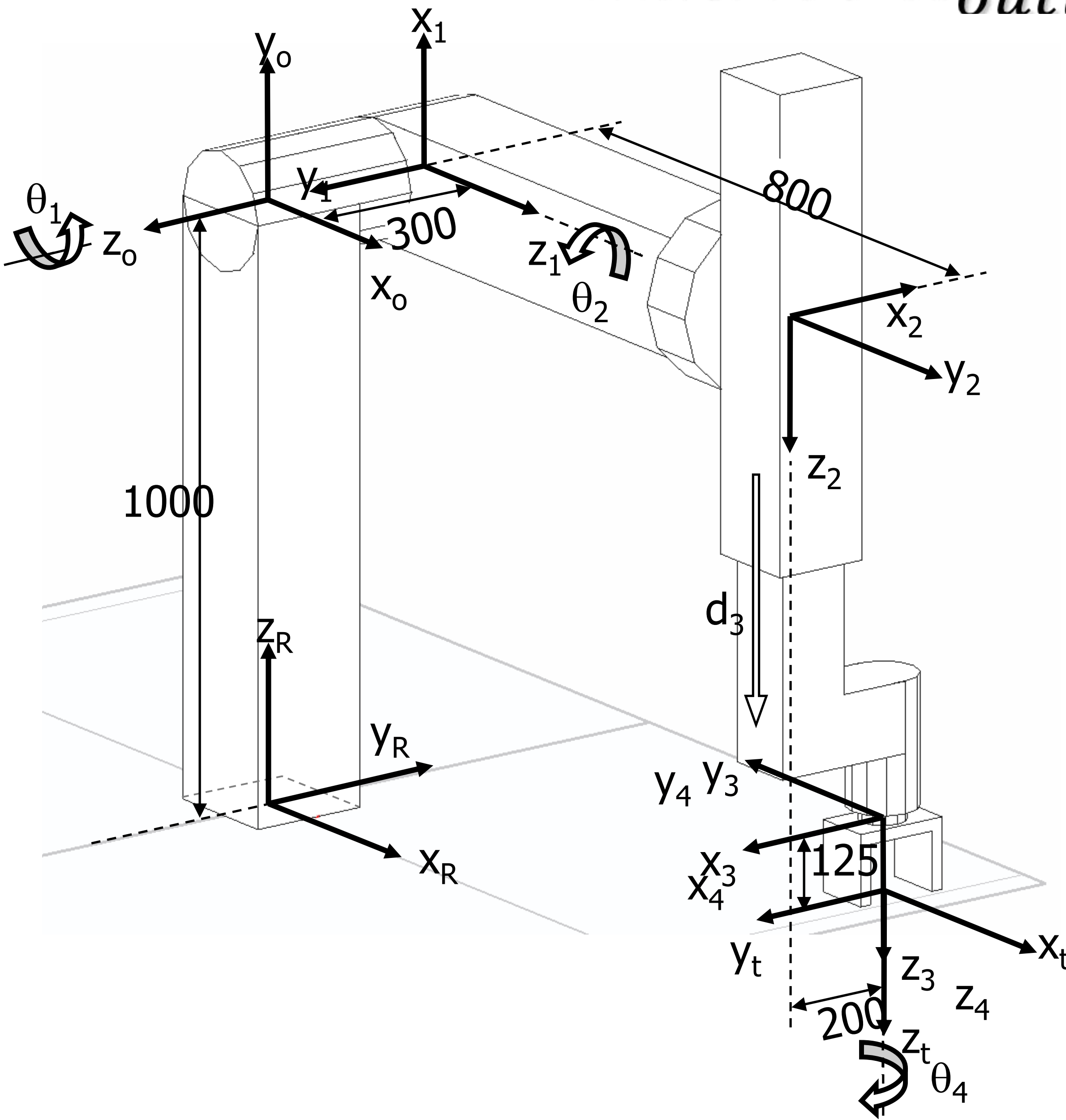
i	θ_i	d_i	a_i	α_i
1	$\theta_1+90°$	-300	0	90°
2	$\theta_2-90°$	800	0	90°
3	180°	d_3+690	-200	0°
4	θ_4	0	0	0°

Matrice $H_0^{atelier}$



$$H_0^{atelier} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrice H^4_{outil}



$$H^4_{outil} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 125 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Équation de la cinématique directe

$$H_{outil}^{atelier} = H_0^{\text{atelier}} H_1^0 H_2^1 H_3^2 H_4^3 H_{\text{outil}}^4$$

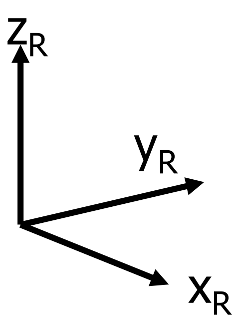
$$= \begin{bmatrix} s_1 s_2 s_4 + c_1 c_4 & s_1 s_2 c_4 - c_1 s_4 & s_1 c_2 & 125 s_1 c_2 - 200 s_1 s_2 + s_1 c_2 (d_3 + 690) + 800 c_1 \\ -c_2 s_4 & -c_2 c_4 & s_2 & 300 + 125 s_2 + 200 c_2 + s_2 (d_3 + 690) \\ -c_1 s_2 s_4 + s_1 c_4 & -c_1 s_2 c_4 - s_1 s_4 & -c_1 c_2 & 1000 - 125 c_1 c_2 + 200 c_1 s_2 - c_1 c_2 (d_3 + 690) + 800 s_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

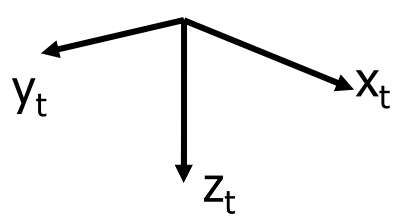
i	θ_i	d_i	a_i	α_i
1	θ_1+90°	−300	0	90°
2	θ_2−90°	800	0	90°
3	180°	d_3+690	−200	0°
4	θ_4	0	0	0°

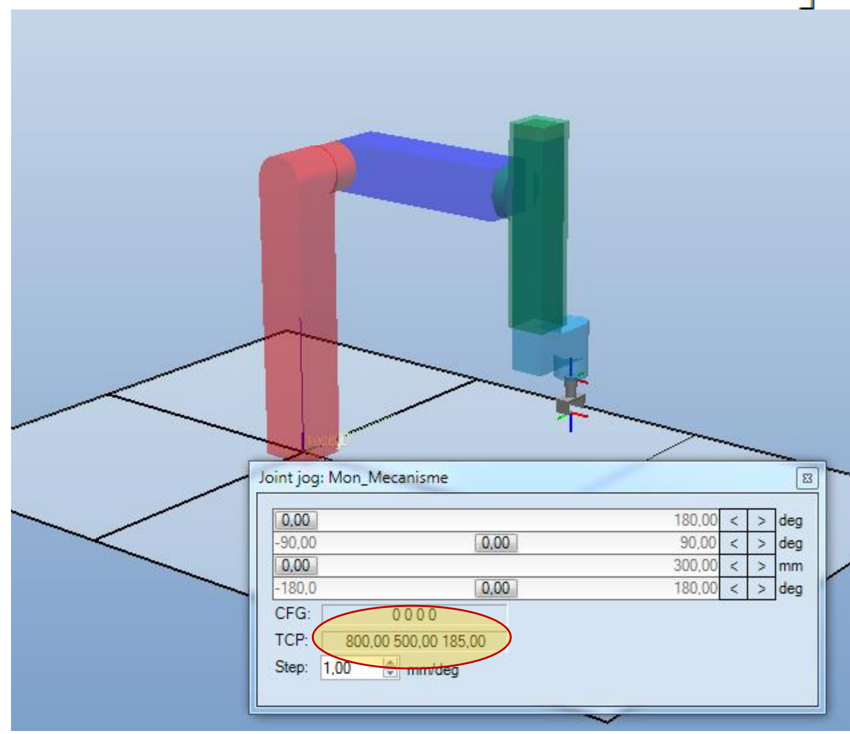
Validation pour la configuration zéro

$$H_{outil}^{atelier} = H_0^{atelier} H_1^0 H_2^1 H_3^2 H_4^3 H_{outil}^4$$

$$= \begin{bmatrix} s_1 s_2 s_4 + c_1 c_4 & s_1 s_2 c_4 - c_1 s_4 & s_1 c_2 & 125 s_1 c_2 - 200 s_1 s_2 + s_1 c_2 (d_3 + 690) + 800 c_1 \\ -c_2 s_4 & -c_2 c_4 & s_2 & 300 + 125 s_2 + 200 c_2 + s_2 (d_3 + 690) \\ -c_1 s_2 s_4 + s_1 c_4 & -c_1 s_2 c_4 - s_1 s_4 & -c_1 c_2 & 1000 - 125 c_1 c_2 + 200 c_1 s_2 - c_1 c_2 (d_3 + 690) + 800 s_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

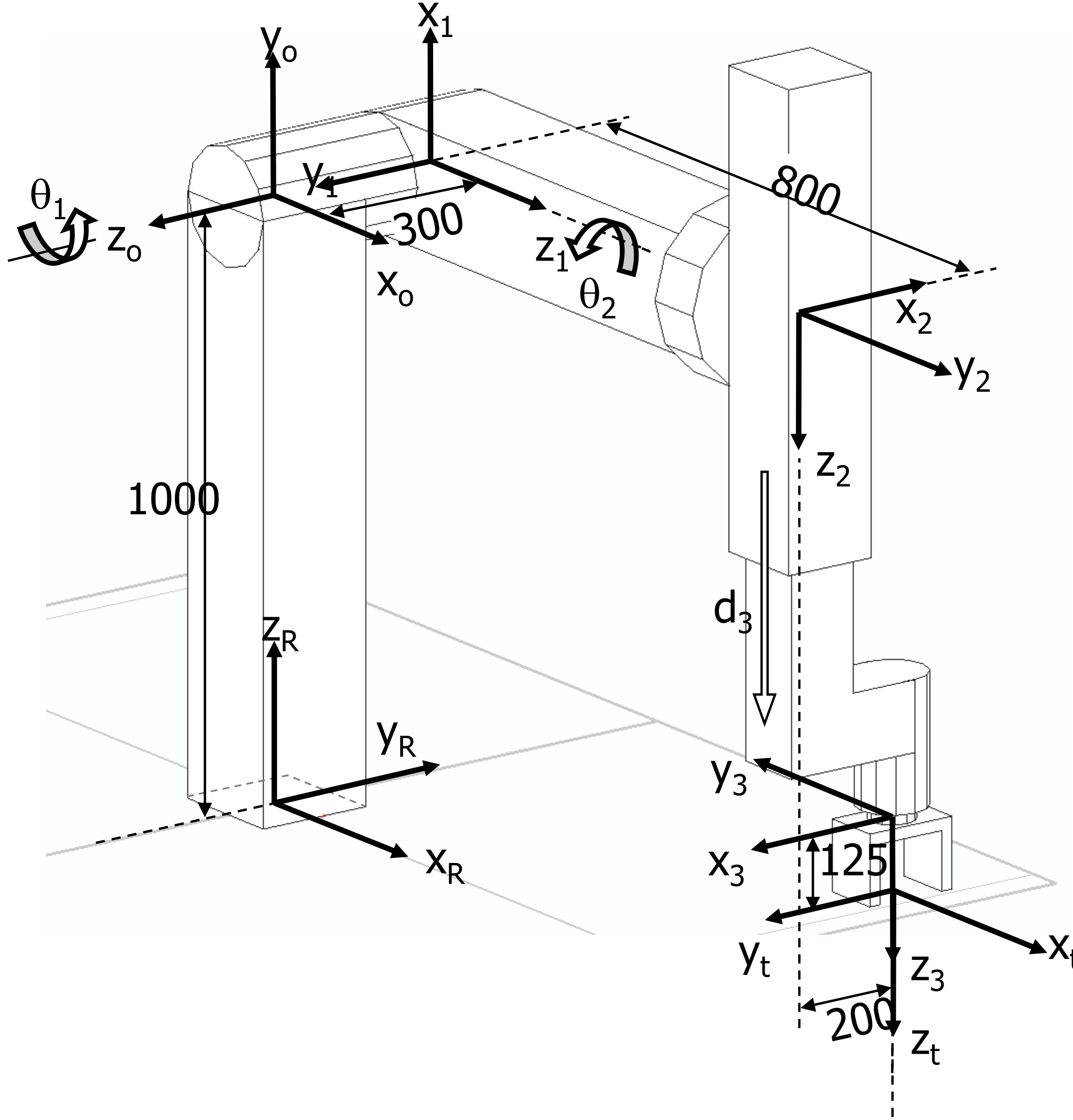






Cinématique inverse du robot à 4 DDL

Référentiels



les inconnues à trouver

pose désirée

variables connues

$$H_4^0 = \left(\mathbf{H}_0^{atelier}\right)^{-1} \mathbf{H}_{outil}^{atelier} \left(H_{outil}^4\right)^{-1} \equiv \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{H}_1^0 \mathbf{H}_2^1 \mathbf{H}_3^2 \mathbf{H}_4^3 = \mathbf{P}$$

Première équation matricielle

$$\mathbf{H}_1^0\mathbf{H}_2^1\mathbf{H}_3^2\mathbf{H}_4^3 = \begin{bmatrix} s_1s_2c_4 - c_1s_4 & -s_1s_2s_4 - c_1c_4 & s_1c_2 & -200s_1s_2 + s_1c_2(d_3+690) + 800c_1 \\ -c_1s_2c_4 - s_1s_4 & c_1s_2s_4 - s_1c_4 & -c_1c_2 & 200c_1s_2 - c_1c_2(d_3+690) + 800s_1 \\ c_2c_4 & -c_2s_4 & -s_2 & -300 - 200c_2 - s_2(d_3+690) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{P}$$

D'ici, on peut trouver s_2, mais cela ne nous permet pas de trouver une seule solution pour θ_2 (on n'a pas c_2).

les inconnues à trouver

pose désirée

variables connues

$$\mathbf{H}_n^0 = \left(\mathbf{H}_0^{atelier}\right)^{-1}\mathbf{H}_{outil}^{atelier}\left(\mathbf{H}_{outil}^{n}\right)^{-1} \equiv \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Deuxième équation matricielle

$$\mathbf{H}_2^1\mathbf{H}_3^2\mathbf{H}_4^3 = \begin{bmatrix} -s_2c_4 & s_2s_4 & -c_2 & -200s_2 - c_2(d_3+690) \\ c_2c_4 & c_2s_4 & -s_2 & -200c_2 - s_2(d_3+690) \\ -s_4 & -c_4 & \boxed{0} & \boxed{800} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_1n_x + c_1n_y & -s_1o_x + c_1o_y & -s_1a_x + c_1a_y & -s_1p_x + c_1p_y \\ n_z & o_z & a_z & p_z + 300 \\ s_1n_y + c_1n_x & s_1o_y + c_1o_x & \boxed{s_1a_y + c_1a_x} & \boxed{s_1p_y + c_1p_x} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left(\mathbf{H}_1^0\right)^{-1}\mathbf{P}$$

D'ici, on peut trouver s_1 et c_1, donc θ_1

$$\boxed{\theta_1 = \operatorname{atan2}\left(\frac{800a_x}{a_xp_y - a_yp_x}, \frac{-800a_y}{a_xp_y - a_yp_x}\right)}$$

Deuxième équation matricielle

$$\mathbf{H}_2^1\mathbf{H}_3^2\mathbf{H}_4^3 = \begin{bmatrix} -s_2c_4 & s_2s_4 & -c_2 & -200s_2 - c_2(d_3+690) \\ c_2c_4 & c_2s_4 & -s_2 & -200c_2 - s_2(d_3+690) \\ -s_4 & -c_4 & \boxed{0} & \boxed{800} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_1n_x + c_1n_y & -s_1o_x + c_1o_y & -s_1a_x + c_1a_y & -s_1p_x + c_1p_y \\ n_z & o_z & a_z & p_z + 300 \\ s_1n_y + c_1n_x & s_1o_y + c_1o_x & \boxed{s_1a_y + c_1a_x} & \boxed{s_1p_y + c_1p_x} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left(\mathbf{H}_1^0\right)^{-1}\mathbf{P}$$

$$\boxed{\theta_1 = \text{atan}2\left(\frac{800a_x}{a_xp_y - a_yp_x}, \frac{-800a_y}{a_xp_y - a_yp_x}\right)}$$

$$\boxed{\theta_4 = \left(-s_1n_y - c_1n_x, -s_1o_y - c_1o_x\right)}$$

$$\boxed{\theta_2 = \left(-a_z, s_1a_x - c_1a_y\right)}$$

Troisième équation matricielle

$$\mathbf{H}_3^2 = \begin{bmatrix} -1 & 0 & 0 & 200 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \boxed{d_3 + 690} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \boxed{\left(s_1 p_x - c_1 p_y\right) c_2 - s_2 (p_z + 300)} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left(\mathbf{H}_2^1\right)^{-1} \left(\mathbf{H}_1^0\right)^{-1} \mathbf{P} \left(\mathbf{H}_4^3\right)^{-1}$$

$$\boxed{d_3 = \left(s_1 p_x - c_1 p_y\right) c_2 - s_2 (p_z + 300) - 690}$$

Deuxième équation matricielle, cas spécial $a_x = a_y = 0$

Si $a_z = \pm 1$ on a aussi $n_z = o_z = 0$.

$$\mathbf{H}_2^1\mathbf{H}_3^2\mathbf{H}_4^3 = \begin{bmatrix} -s_2c_4 & s_2s_4 & \boxed{-c_2} & -200s_2 - c_2(d_3+690) \\ c_2c_4 & c_2s_4 & \boxed{-s_2} & -200c_2 - s_2(d_3+690) \\ -s_4 & -c_4 & 0 & 800 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_1n_x + c_1n_y & -s_1o_x + c_1o_y & \boxed{0} & -s_1p_x + c_1p_y \\ 0 & 0 & \boxed{a_z} & p_z + 300 \\ s_1n_y + c_1n_x & s_1o_y + c_1o_x & 0 & s_1p_y + c_1p_x \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left(\mathbf{H}_1^0\right)^{-1}\mathbf{P}$$

D'ici, on peut trouver s_2 et c_2, donc θ_2 :

$$\boxed{\theta_2 = \operatorname{atan2}\left(-a_z, 0\right)}$$

Quatrième équation matricielle, cas spécial $a_x = a_y = 0$

$$\mathbf{H}_3^2\mathbf{H}_4^3 = \begin{bmatrix} -c_4 & s_4 & 0 & \boxed{200} \\ -s_4 & -c_4 & 0 & \boxed{0} \\ 0 & 0 & 1 & d_3+690 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -c_1a_zn_y + s_1a_zn_x & -c_1a_zo_y + s_1a_zo_x & 0 & \boxed{\left(s_1p_x - c_1p_y\right)a_z} \\ s_1n_y + c_1n_x & s_1o_y + c_1o_x & 0 & \boxed{s_1p_y + c_1p_x - 800} \\ 0 & 0 & 1 & a_z\left(p_z+300\right) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left(\mathbf{H}_1^0\mathbf{H}_2^1\right)^{-1}\mathbf{P}$$

D'ici, on peut trouver s_1 et c_1, donc θ_1 (le dénominateur est toujours non-zéro) :

$$\boxed{\theta_1 = \text{atan2}\left(\frac{800a_zp_y + 200p_x}{a_z\left(p_x^2+p_y^2\right)}, \frac{800a_zp_x - 200p_y}{a_z\left(p_x^2+p_y^2\right)}\right)}$$

Ensuite on peut trouver le reste des variables articulaires :

$$\boxed{\theta_4 = \text{atan2}\left(-s_1n_y - c_1n_x, -s_1o_y - c_1o_x\right)}$$

$$\boxed{d_3 = a_z(p_z+300) - 690}$$