

Anomaly detection

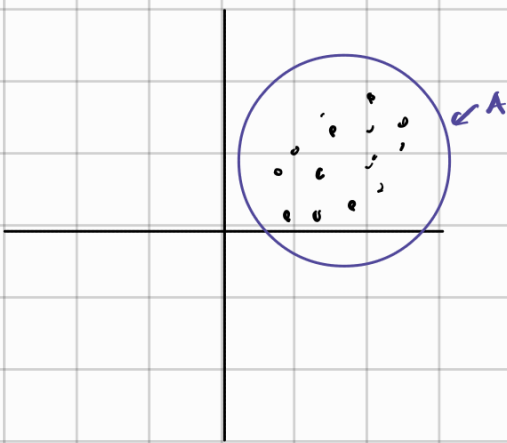
The goal is given a training sample be able to detect anomalies in new images.

assumption: if I is normal then $S \approx Dx$ where x is sparse.

to do this we have the following phases:

training phase

1. extract S of normal patches
2. learn D (K-SVD, MOD)
3. compute for a patch s $a(s)$ indicator vector
4. extract V normal patches, compute $A = [a(s) \text{ for } s \text{ in } V]$



5. Build confidence region

Test phase

1. compute $a(s)$
2. if $a(s) \in$ confidence region
 S is normal
else
 S is abnormal

How to compute $\alpha(s)$

first, we compute x

$$\hat{x} = \arg \min \underbrace{\frac{1}{2} \|s - Dx\|_2^2 + \lambda \|x\|_1}_{\psi(x)}$$

if $\psi(x)$ is small then

$\psi(x)$ is small

else

$\psi(x)$ is large

so, a good indicator would be

$$\alpha(s) = \begin{bmatrix} \|s - Dx\|_2^2 \\ \|x\|_1 \end{bmatrix} \in \mathbb{R}^2$$

now we build A :

$$A = \begin{bmatrix} | \\ | \\ \vdots \\ | \end{bmatrix} \in \mathbb{R}^{2 \times w}$$

\downarrow
 $\alpha(s)$

then, we build our confidence region

$$\mu = \text{mean}(A) \in \mathbb{R}^2$$

\hookrightarrow row-wise

$$\Sigma = \text{cov}(A) \in \mathbb{R}^2$$

so, our region is:

$$\mathcal{Q} = \{a \in \mathbb{R}^2 : (a-u)^T \Sigma^{-1} (a-u) < \tau\}$$

\downarrow defines the shape of the region \downarrow size of the region

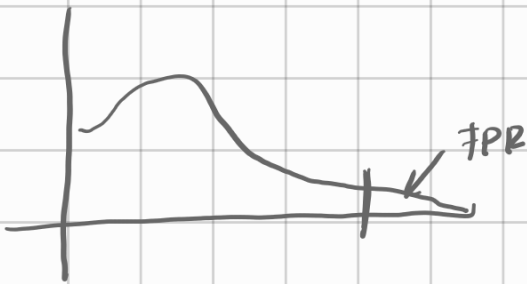
and, how do we choose τ , τ actually controls the probability of false positive rate.

so, we fix FPR and choose τ accordingly

$$J(s) = (a-u)^T \Sigma^{-1} (a-u)$$

\downarrow anomaly score

so we select an area of the distribution equal to 0.1



so,

$$\tau = \text{quantile}([J(s) \text{ for } s \text{ in } V], 1 - \text{FPR})$$

now, moving on to the test phase for the test image we will aggregate the estimations of $a(s)$ for each pixel.

so,

Algorithm (test phase)

$$I \in \mathbb{R}^{R \times C}$$

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for i in range(R-P, STEP)
  for j in range(C-P, STEP)
    extract  $s$ 
    compute  $x$ 
    compute  $a$ 
    compute  $J(s) = (a - \mu)^T \tilde{\Sigma}^{-1} (a - \mu)$ 
```

$$H[i:i+P, j:j+P] \leftarrow J(s)$$

$$W[i:i+P, j:j+P] \leftarrow I$$

$$H = H / W$$

$$\text{mask} = H > \gamma$$

Remark (practical):

- $P = 15$ for building D
- do sparse coding with FISTA using $\lambda = 0.18$
- $\text{STEP} = P$
- be careful with black patches (set $J(s) = 0$ since they are exception)