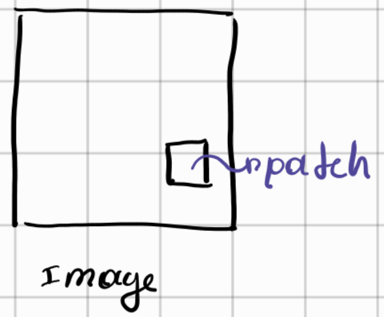
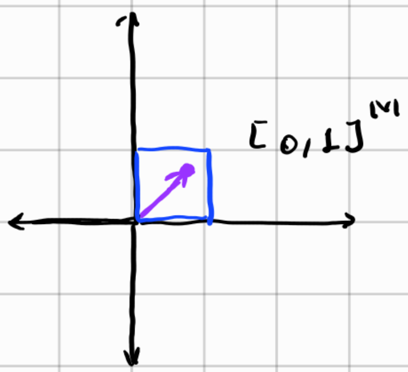
Signals and sparsity

we will consider signals and images as vectors



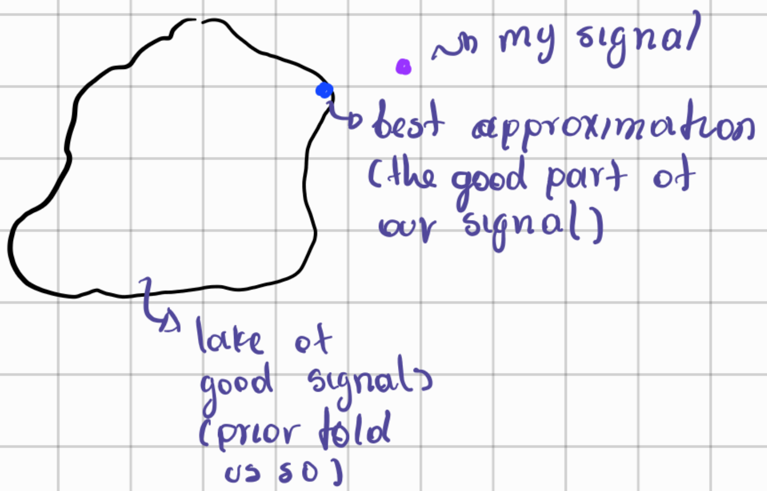
to see images as vectors we need to unfold them so we get a signal wich can represent a vector.

so to normalize a signal means.



Priors

what we need is a prior to restrict our search in the space and find the good signal.



we assume that if s is our measured signal:

$$s = y + \eta$$

↳ noise

we also assume that $\|\eta\| < \delta$ → maximum noise

↳ "the effect of the noise"

Conclusions

Priors guide us to evaluate what a good signal is, it's a function $f: \mathbb{R}^M \to [0,1]$, where the closest to 1 the most likely to be a "true signal".

Sparsity

we assume that every signal can be approximated as a sparse linear combination of some basis elements.

→ go to linear algebra recap

so, assume:

$$\underline{s} = D\underline{x}$$ ⤳ sparsity tell us that x should have few non-zero entries

so, with this in mind we make the followings algorithms.

Enforce sparcity

Steps:

1. change basis (get x)
2. threshold x (set to 0 small coefficients
3. come back to $\hat{s}$

For this algorithm to "work", we assume

$s = y + n$

y ↓ true signal

n ↳ noise

- $D^T y$ is sparse
- $D^T n$ is not sparse
- (with low coefficients in the suitable basis)?

$\longrightarrow x = \underbrace{D^T y}_{\text{recover } y} + \underbrace{D^T n}_{\text{remove this part (thresholding)}}$

Footnote: Sparcity is not relevant in canonical basis that's why we need to change basis, one where we think sparcity is more relevant.