Anomaly detection

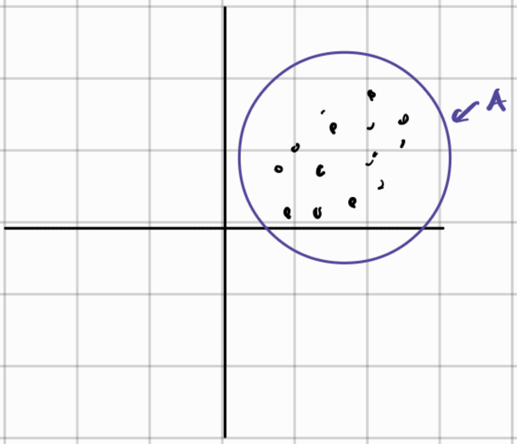
The goal is given a trainning sample be able to detect anomalies in new images.

assumption: if s is normal then $s \approx Dx$ where x is sparse.

to do this we have the following phases:

training phase

1. extract S of normal patches
2. learn D (K-SVD, MOD)
3. compute for a patch s $a(s)$ indicator vector
4. extract V normal patches, compute $A = [a(s)$ for s in $V]$



5. Build confidence region

Test phase

1. compute $a(s)$
2. if $a(s) \in$ confidence region
   - s is normal

   else
   - s is anormal

How to compute $a(s)$

first, we compute x

$$\hat{x} = \arg\min \underbrace{\frac{1}{2}\|s - Dx\|_2^2 + \lambda\|x\|_1}_{\varphi(x)}$$

if $\varphi(x)$ is small then
$\varphi(x)$ is small
else
$\varphi(x)$ is large

so, a good indicator would be

$$a(s) = \begin{bmatrix} \|s - Dx\|_2^2 \\ \|x\|_1 \end{bmatrix} \in \mathbb{R}^2$$

now we build A:

$$A = \begin{bmatrix} \underset{a(s)}{|} & & \end{bmatrix} \in \mathbb{R}^{2 \times w}$$

then, we build our confidence region

$u = \text{mean}(A) \in \mathbb{R}^2$ $\quad$ $\Sigma = \text{cov}(A) \in \mathbb{R}^2$

↳ row-wise

so, our region is:

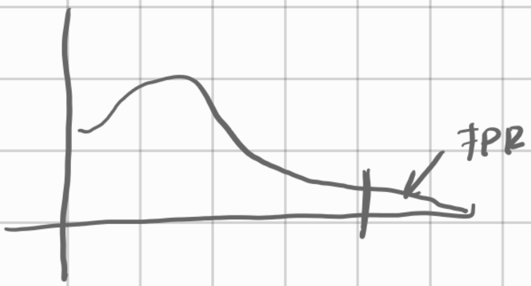
$$Q = \{a \in \mathbb{R}^2 : \underbrace{(a-\mu)^T \Sigma^{-1} (a-\mu)}_{\text{defines the shape of the region}} < \underbrace{\tau}_{\text{size of the region}}\}$$

and, how do we choose τ, τ actually controls the probability of false positive rate.

so, we fix FPR and choose τ accordingly

$$\underbrace{J(s)}_{\text{anomaly score}} = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

so we select an area of the distribution equal to 0.1



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so,

$$\tau = \text{quantile}([J(s) \text{ for } s \text{ in } V], 1 - FPR)$$

now, moving on to the test phase for the test image we will agregate the estimations of $a(s)$ for each pixel.

so,

Algorithm (test phase)

$I \in \mathbb{R}^{R \times C}$

```
for i in range(R-P, STEP)
    for j in range(C-P, STEP)
        extract s
        compute x
        compute α
        compute J(s) = (α-μ)^T Σ^{-1} (α-μ)

        H[i:i+P, j:j+P] += J(s)
        W[i:i+P, j:j+P] += 1
```

$H = H/W$

$mask = H > \tau$

Remark (practical) :

- $P = 15$ for building D
- do sparse coding with FISTA using $\lambda = 0.18$
- STEP = P
- be careful with black patches (set $J(s) = 0$ since they are exception)