$$H(s) = \frac{s - 1/2}{s^2 - s + 1/3} = \frac{s - 1/2}{(s - P_1)(s - P_2)}$$

simplificamos la ecuacion

$$H(s) = \frac{(s - 1/2)}{(s - P_1)(s - P_2)}$$

donde identificamos que

los ceros aquellos que hacen el numerador cero es cuando

$$s = 1/2$$

mientras que los polos aquellos que hacen el denominador cero

$$s \to \begin{array}{l} P_1 = 1/2 + \sqrt{3}/6\, j \\ P_2 = 1/2 - \sqrt{3}/6\, j \end{array}$$

por lo que procedemos a realizar el analisis

$$H(s) = \frac{s - 1/2}{(s - P_1)(s - P_2)} = \frac{A}{s - P_1} + \frac{B}{s - P_2}$$

$$H(s) = \frac{A}{s - 1/2 - \frac{\sqrt{3}}{6}j} + \frac{B}{s - 1/2 + \frac{\sqrt{3}}{6}j}$$

$$H(s)(s - P_1) = \frac{A\cancel{(s - P_1)}}{\cancel{(s - P_1)}} + \frac{B(s - P_1)}{(s - P_2)}$$

$$H(s)(s - P_1)\Big|_{s = P_1} = A$$

$$A = \frac{(s - 1/2)\cancel{(s - P_1)}}{\cancel{(s - P_1)}(s - P_2)}\Bigg|_{s = P_1}$$

$$A = \frac{(s - 1/2)}{(s - P_2)} \Bigg|_{s = P_1}$$

$$A = \frac{P_1 - 1/2}{P_1 - P_2} = \frac{1}{2}$$

Luego hallamos B

$$H(s)(s-P_2) = \frac{A(s-P_2)}{s-P_1} + \frac{B\cancel{(s-P_2)}}{\cancel{(s-P_2)}}$$

$$H(s)(s-P_2)\Big|_{s=P_2} = B$$

$$B = \frac{(s-1/2)\cancel{(s-P_2)}}{(s-P_1)\cancel{(s-P_2)}}\Bigg|_{s=P_2} = \frac{s-1/2}{s-P_1}\Bigg|_{s=P_2}$$

$$B = \frac{P_2 - 1/2}{P_2 - P_1} = \frac{1}{2}$$

$$H(s) = \frac{1/2}{s-P_1} + \frac{1/2}{s-P_2} = \frac{1/2}{s - \left(\frac{1}{2} + \frac{\sqrt{3}}{6}j\right)} + \frac{1/2}{s - \left(\frac{1}{2} - \frac{\sqrt{3}}{6}j\right)}$$

Luego procedemos a implementar invarianza impulsional cambiando a dom z

$$H(s) = \sum_{k=1}^{N} \frac{C_k}{s - P_k} \xrightarrow{z} H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$$

$$H(s) = \frac{1/2}{s - \frac{1}{2} - \frac{\sqrt{3}}{6}j} + \frac{1/2}{s - \frac{1}{2} + \frac{\sqrt{3}}{6}j}$$

$$H(z) = \frac{1/2}{1 - e^{-1/2T} e^{-j\frac{\sqrt{3}}{6}T} z^{-1}} + \frac{1/2}{1 - e^{-1/2T} e^{j\frac{\sqrt{3}}{6}T} z^{-1}}$$

$$H(z) = \frac{1\quad \frac{1}{2}\left(1 - e^{-1/2T} e^{j\frac{\sqrt{3}}{6}T} z^{-1}\right) + \frac{1}{2}\left(1 - e^{-1/2T} e^{-j\frac{\sqrt{3}}{6}T} z^{-1}\right)}{2\quad \left(1 - e^{-1/2T} e^{-j\frac{\sqrt{3}}{6}T} z^{-1}\right)\left(1 - e^{-1/2T} e^{j\frac{\sqrt{3}}{6}T} z^{-1}\right)}$$

SIMPLIFICAMOS LA EXPRESION

$$② = 1 - e^{-\frac{1}{2}T} e^{j\frac{\sqrt{3}}{6}T} z^{-1} - e^{-\frac{1}{2}T} e^{-j\frac{\sqrt{3}}{6}T} z^{-1} + \left(e^{-1/2T} \cancel{e^{j\frac{\sqrt{3}}{6}T}} z^{-1}\right)\left(e^{-\frac{1}{2}T} \cancel{e^{-j\frac{\sqrt{3}}{6}T}} z^{-1}\right)$$

$$= 1 - e^{-1/2T} z^{-1}\left(e^{j\frac{\sqrt{3}}{6}T} + e^{-j\frac{\sqrt{3}}{6}T}\right) + e^{-T} z^{-2}$$

$$= 1 - 2e^{-1/2T} z^{-1} \cos\left(\frac{\sqrt{3}}{6}T\right) + e^{-T} z^{-2}$$

$$① = \frac{1}{2} - \frac{1}{2} e^{-1/2T} e^{j\frac{\sqrt{3}}{6}T} z^{-1} + \frac{1}{2} - \frac{1}{2} e^{-1/2T} e^{-j\frac{\sqrt{3}}{6}T} z^{-1}$$

$$= 1 - \frac{1}{2} e^{-1/2T} z^{-1}\left(e^{j\frac{\sqrt{3}}{6}T} + e^{-j\frac{\sqrt{3}}{6}T}\right)$$

$$= 1 - \frac{1}{\cancel{2}} e^{-1/2T} z^{-1}\left(\cancel{2}\cos\left(\frac{\sqrt{3}}{6}T\right)\right)$$

$$= 1 - e^{-1/2T} z^{-1} \cos\left(\frac{\sqrt{3}}{6}T\right)$$

LUEGO TENEMOS QUE

$$H(z) = \frac{1 - e^{-\frac{1}{2}T}\cos\left(\frac{\sqrt{3}}{6}T\right) z^{-1}}{1 - 2e^{-\frac{1}{2}T}\cos\left(\frac{\sqrt{3}}{6}T\right) z^{-1} + e^{-T} z^{-2}} \left(\frac{z^2}{z^2}\right)$$

$$= \frac{z^2 - e^{-\frac{1}{2}T}\cos\left(\frac{\sqrt{3}}{6}T\right) z}{z^2 - 2e^{-1/2T}\cos\left(\frac{\sqrt{3}}{6}T\right) z + e^{-T}}$$