MACHINE LEARNING: LINEAR REGRESSION

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Dataset split

Size of the house (m^2)	Price (millions)
20	60
30	100
40	120
50	180
60	200
70	250
80	300
90	350
100	450
110	480

Training set
70%

Test set
30%

Training/cross validation/test set

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Dataset split

Size of the house (m^2)	Price (millions)
20	60
30	100
40	120
50	180
60	200
70	250
80	300
90	350
100	450
110	480

Training set
60%

Cross validation set (dev set)
20%

Test set
20%

Cross-validation

https://www.youtube.com/watch?v=TIgfjmp-4BA

Background-What is Machine Learning?

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Applications

- Website search (group website pages)
- Streaming movies (similar movies recommendation)
- OK Google, Siri, Alexa
- Email classification (spam, no spam)
- Healthcare
- Agriculture
- ……..

MACHINE LEARNING

"Field of study that gives computers the ability to learn without being explicitly programmed" Arthur Samuel (1959).

Machine Learning Algorithms

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- Supervised Learning
- Unsupervised Learning

Supervised Learning

- The algorithm learns from x to y mapping.

$$\begin{array}{ccc} x & \rightarrow & y \\ \text{Input} & & \text{Output} \end{array}$$

- The algorithm takes examples to learn from. It has to know the “correct labels”.

Supervised Learning

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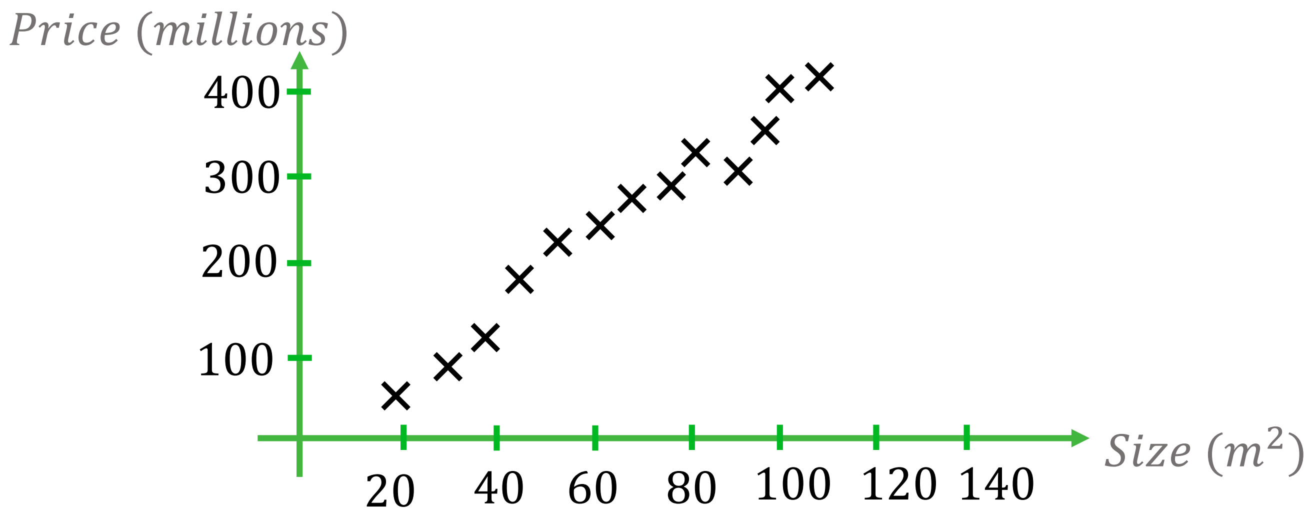
- Regression: house price prediction



Supervised Learning

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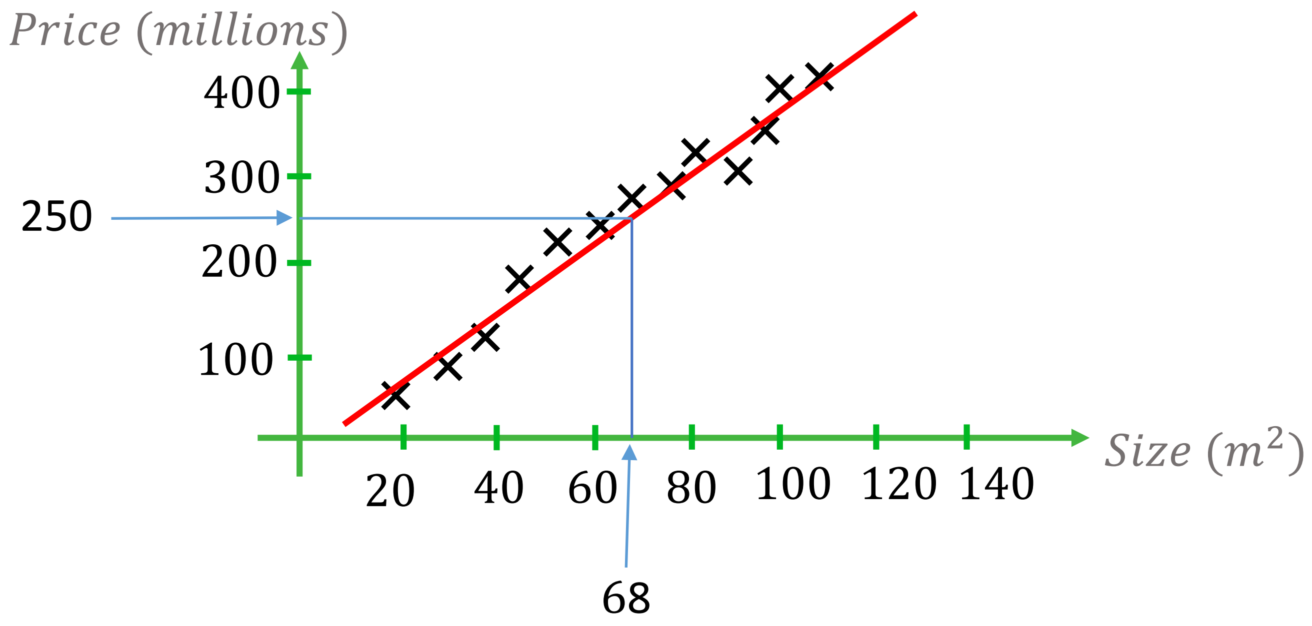
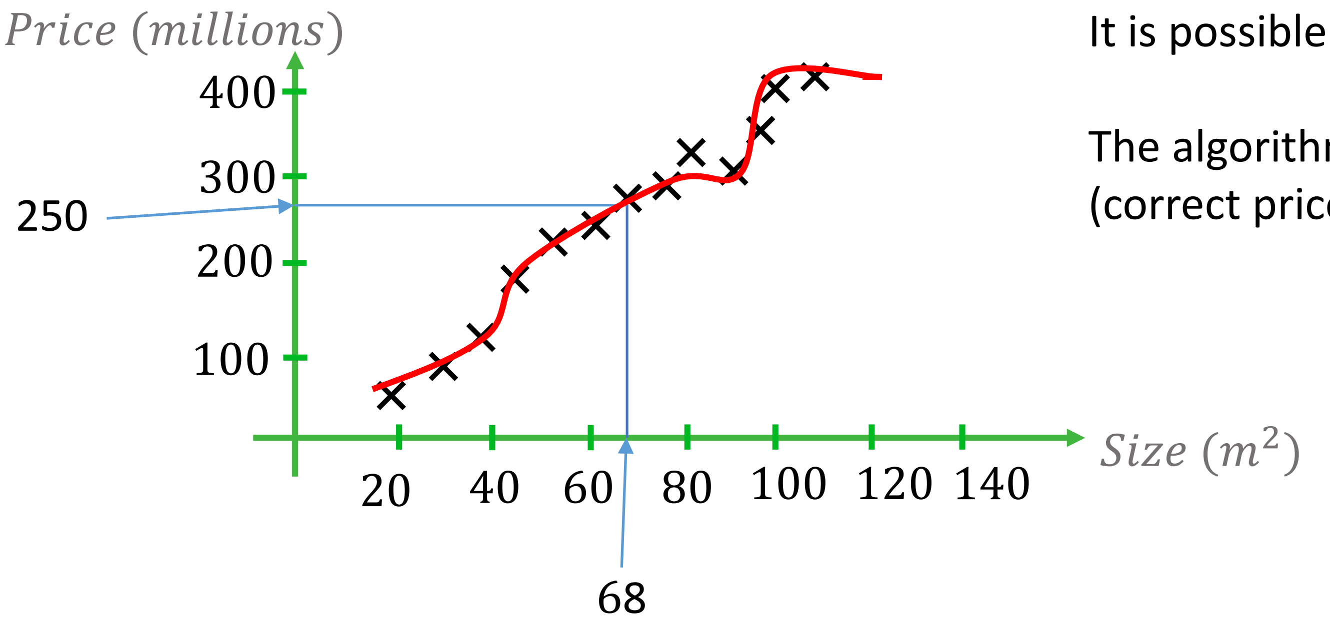
- Regression

It is possible to fit a straight line or a curve



Supervised Learning

- Regression



It is possible to fit a straight line or a curve

The algorithm has to know the “right answer” (correct price)

- **Regression**

It predicts a number

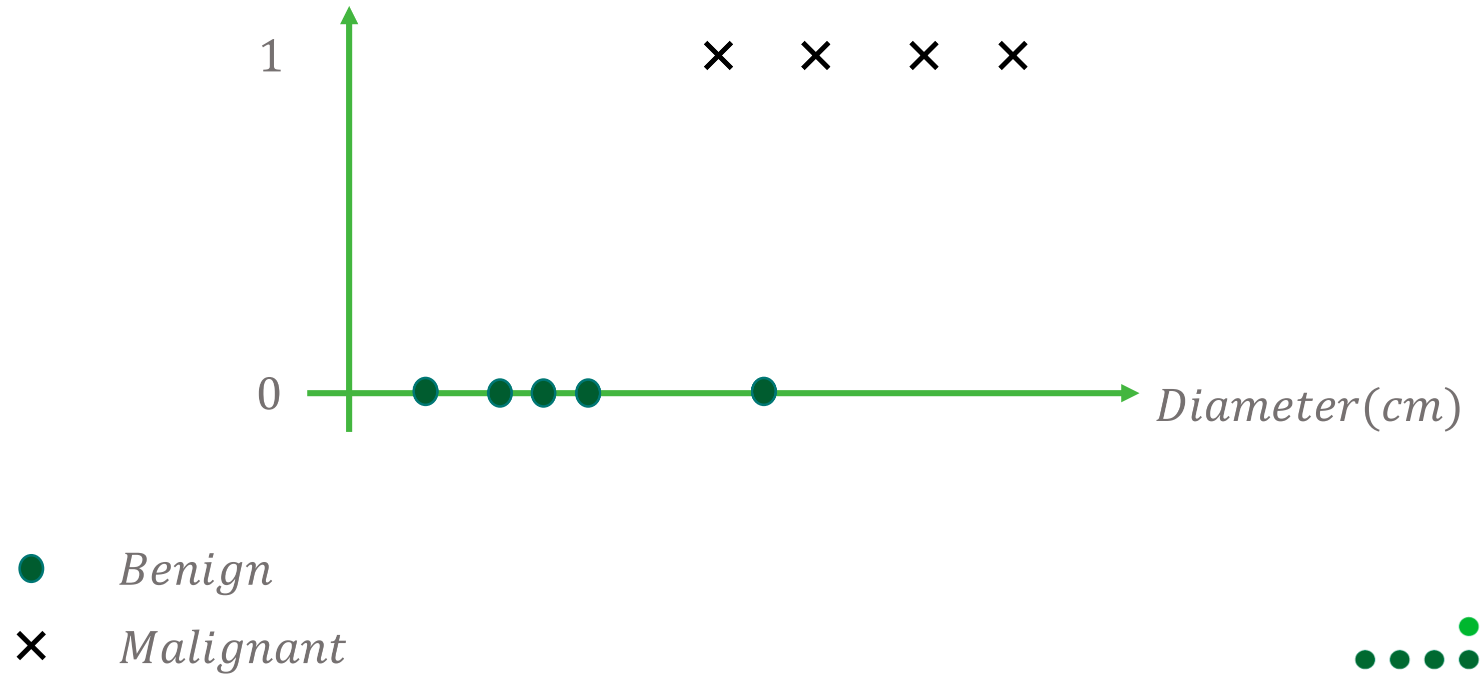
There are many possible outputs

- **Classification**

It predicts a small number of possible outputs or categories (classes)

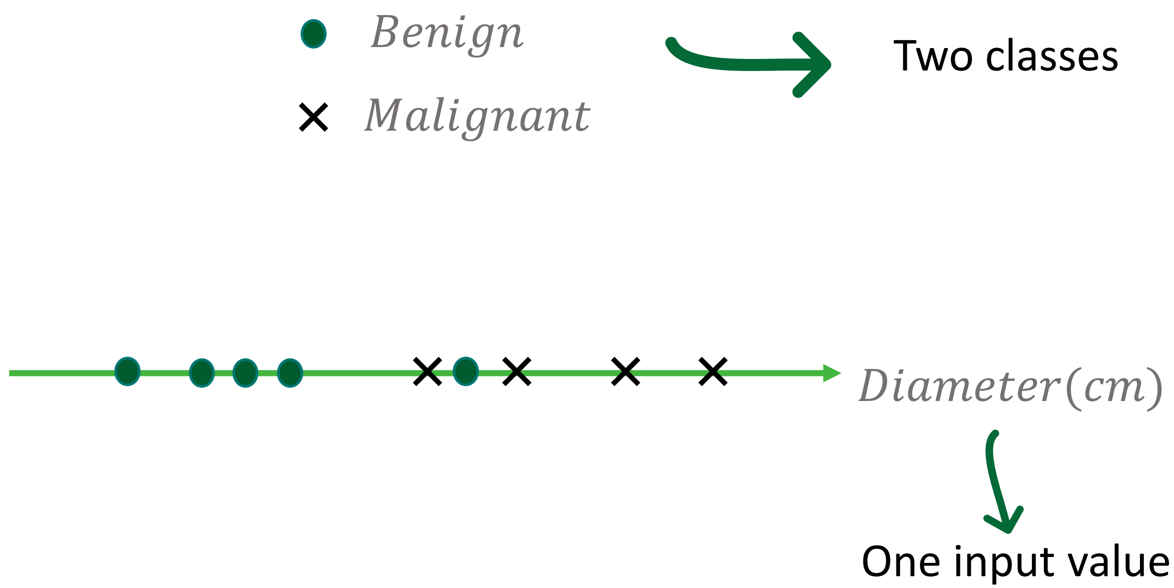
Categories or classes do not have to be numbers. For example, classify between cats and dogs, classify between malignat or benign for a tumor

- Classification: melanoma detection



Supervised Learning

- Classification: melanoma detection



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- **Classification: melanoma detection**

$Benign$

$\times$ $Malignant: type\ I$

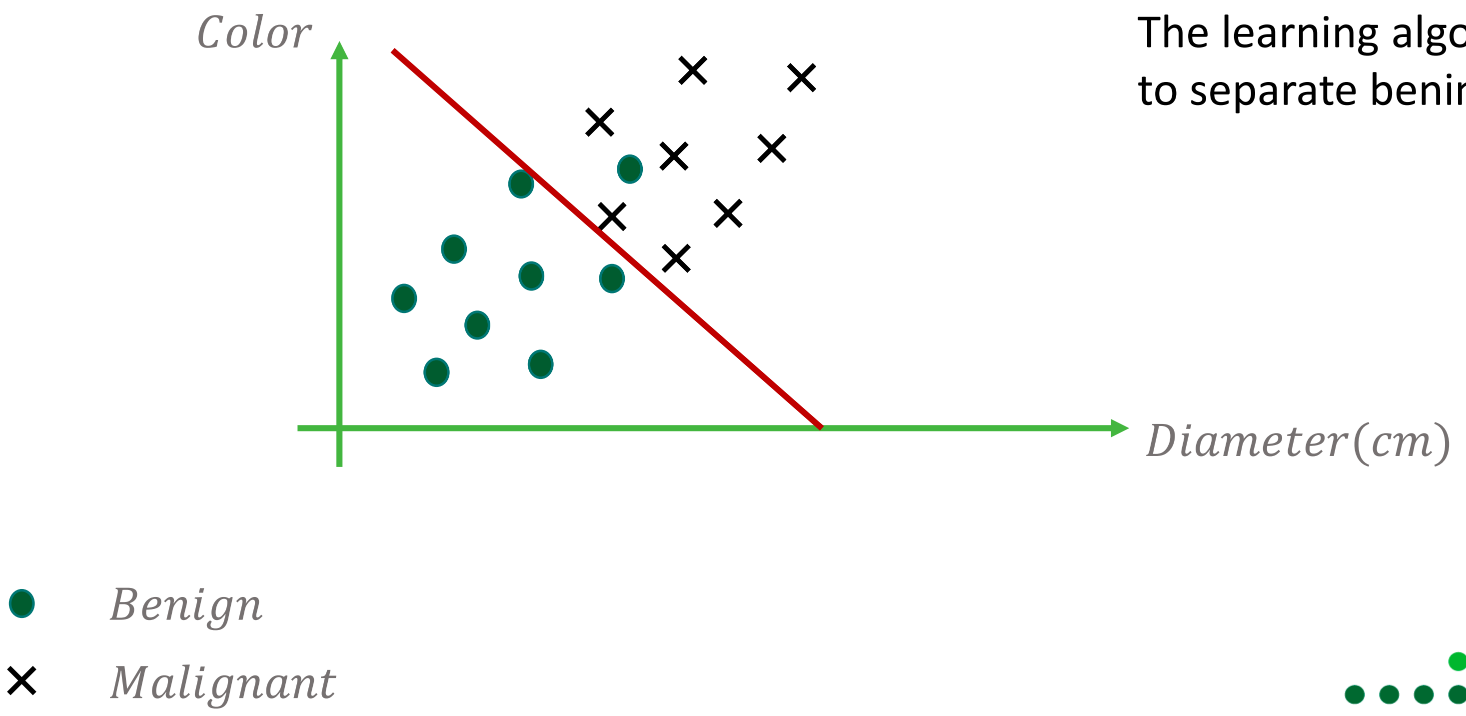
$Malignant: type\ II$

Three classes

$Diameter(cm)$

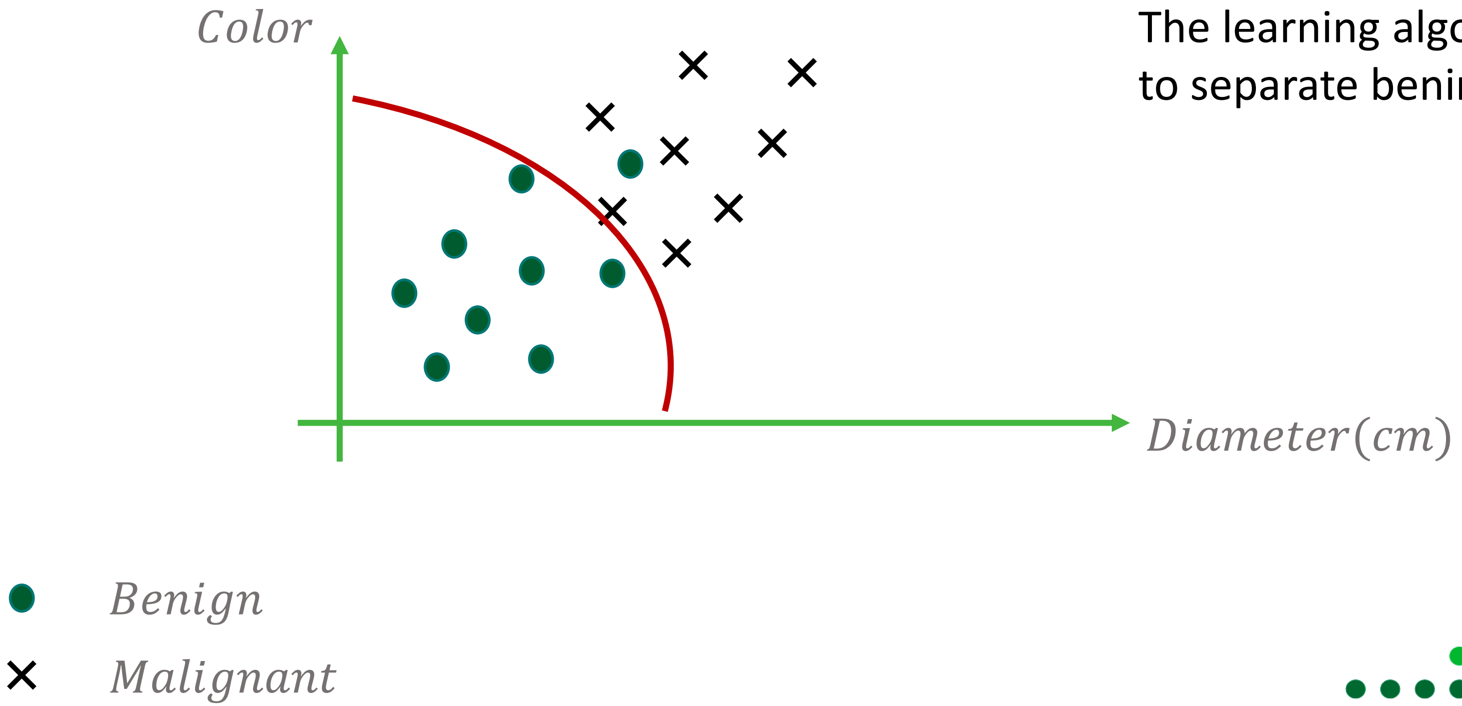
One input value

▪Classification: melanoma detection



The learning algorithm finds a **boundary** to separate bening and malignant tumors

▪Classification: melanoma detection



The learning algorithm finds a **boundary** to separate bening and malignant tumors

Supervised Learning

Regression

Classification

Unsupervised Learning

- Data is not associated with any labels or classes or categories.

- The objective is to find some structure in the data, something interesting.

Unsupervised Learning

Find some structure un unlabeled data

Unsupervised Learning

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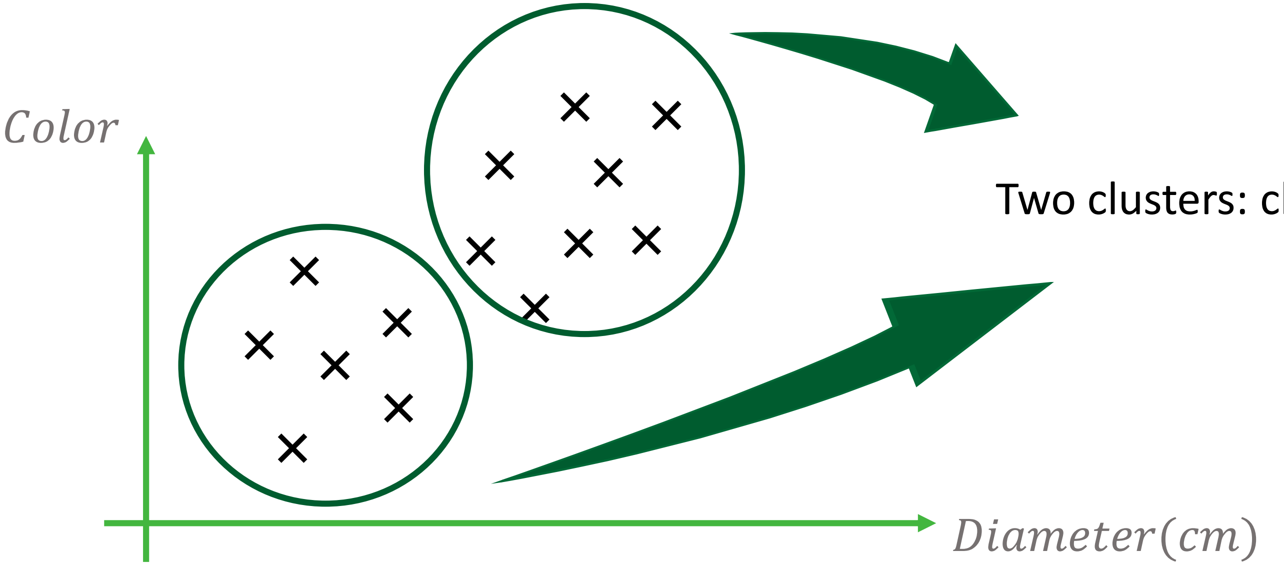
Find some structure un unlabeled data



Two clusters: clustering (group similar data)

Example: Google search about a topic

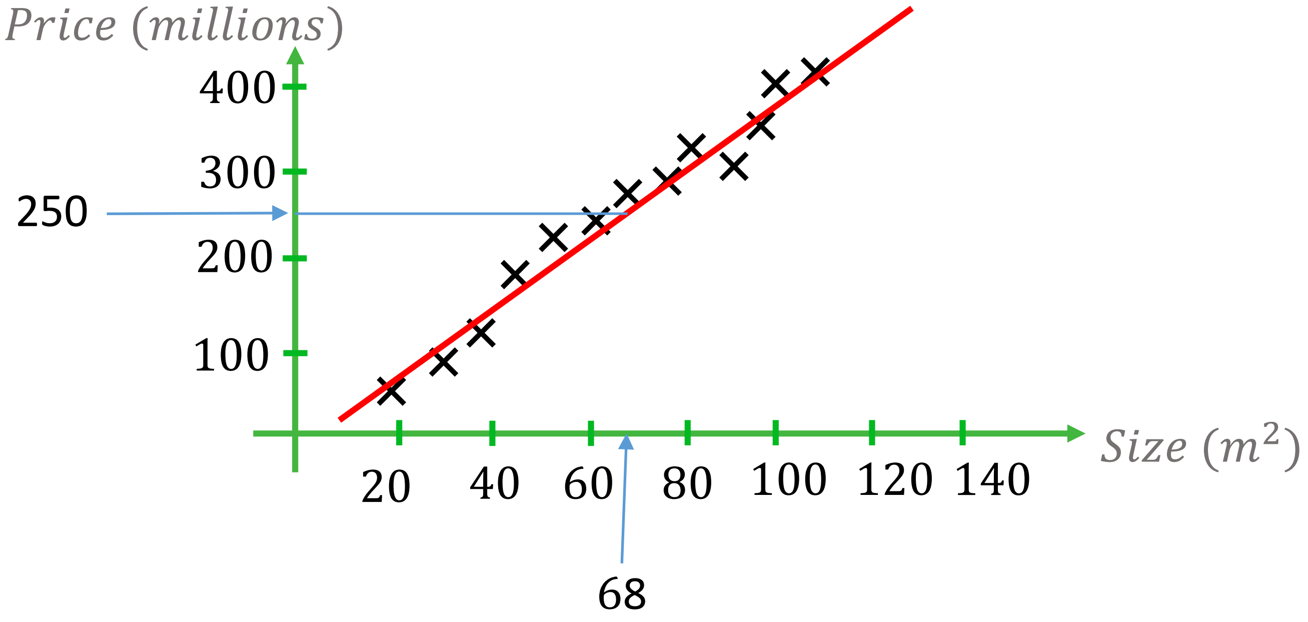
Unsupervised Learning

- Clustering
- Anomaly detection: it finds unusual data
- Dimensionality reduction: PCA

Linear Regression

- It fits a straight line to the data

- Linear Regression: house price prediction



Size (m^2)	Price (millions)
20	60
30	100
40	120
50	180
…	…

Supervised Algorithm: Linear Regression

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Terminology

- **Training set:** data that is used to train the model
  - $x \longrightarrow$ input variable (feature)
  - $y \longrightarrow$ output variable
  - $m \longrightarrow$ number of training examples
  - $(x, y) \longrightarrow$ a single training example

Size (m^2)	Price (millions)
x	y
20	60
30	100
40	120
50	180
…	…

Supervised Algorithm: Linear Regression

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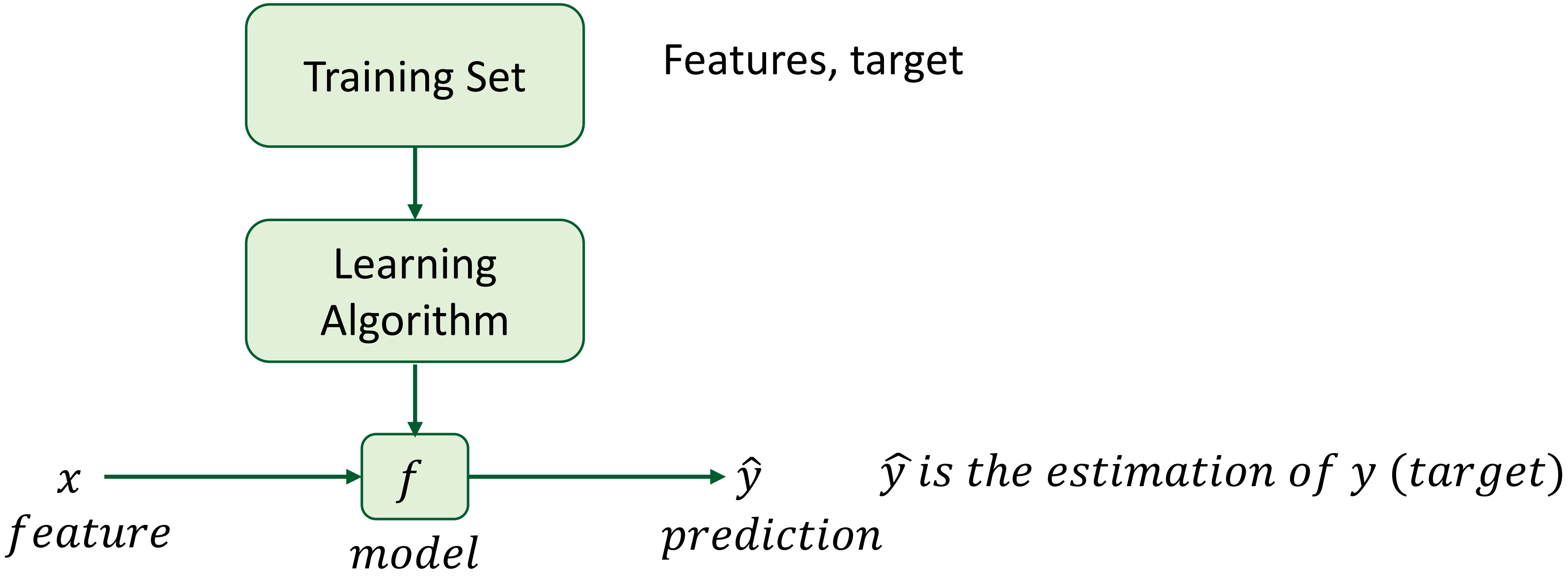
Terminology

- **Training set:** data that is used to train the model
  - $(x^{(i)}, y^{(i)}) \longrightarrow i^{th}$ training example
  - i refers to the rows in the table
  - $(x^{(3)}, y^{(3)}) = (40, 120)$

Size (m^2)	Price (millions)
x	y
20	60
30	100
40	120
50	180
…	…

Supervised Algorithm: Linear Regression



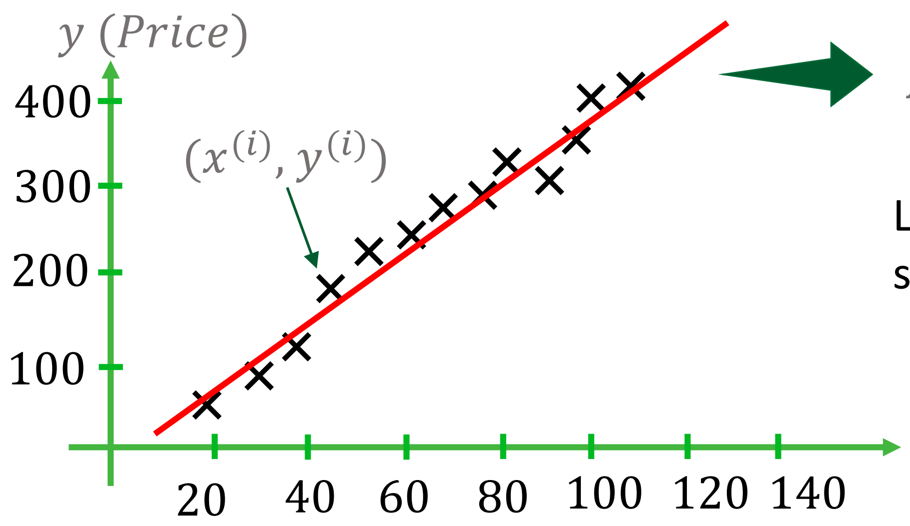
House Price prediction

f model representation:

$$f_{w,b}(x) = wx + b \longrightarrow f(x) = wx + b$$

The values of w and b are going to determine the prediction $\hat{y}$

- Linear Regression: house price prediction



$\boldsymbol{f(x) = wx + b}$

Linear regression with one variable
single feature (x)

Cost Function:

It tells how well the model is working, so the idea is to get better results based on it.

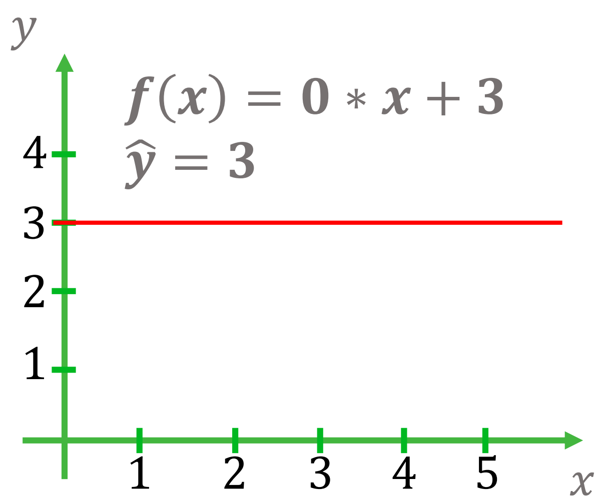
For the model:

$$\boldsymbol{f(x) = wx + b}$$

$\boldsymbol{w}, \boldsymbol{b}$ are called parameters, coefficients or weights

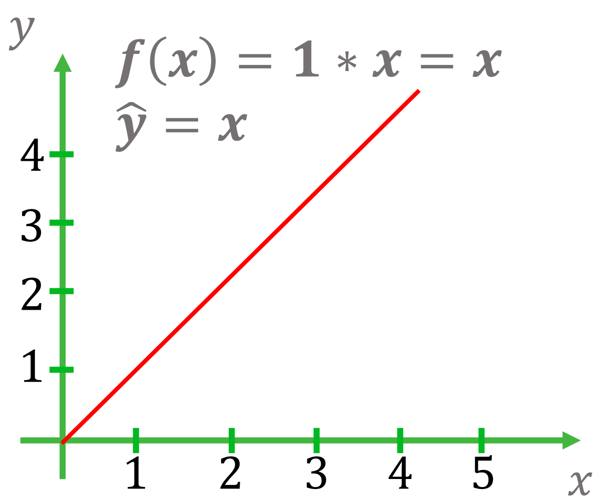
Supervised Algorithm: Linear Regression with one variable

$$\boldsymbol{f(x) = wx + b}$$



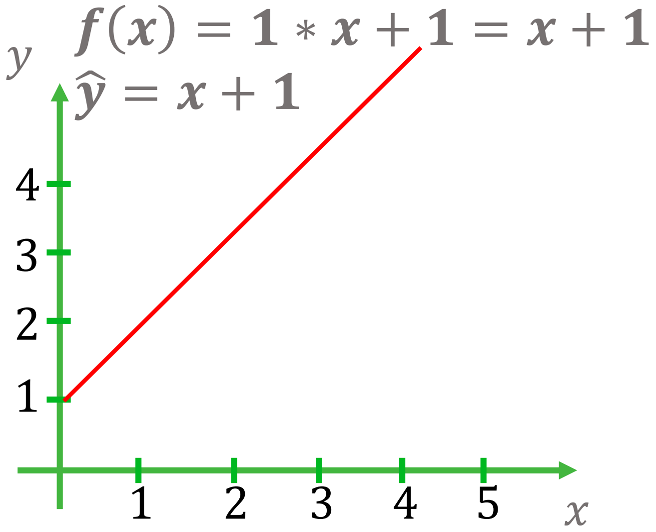
$w = 0, slope$

$b = 3, intercept$



$w = 1, slope$

$b = 0, intercept$



$w = 1, slope$

$b = 1, intercept$

Supervised Algorithm: Linear Regression

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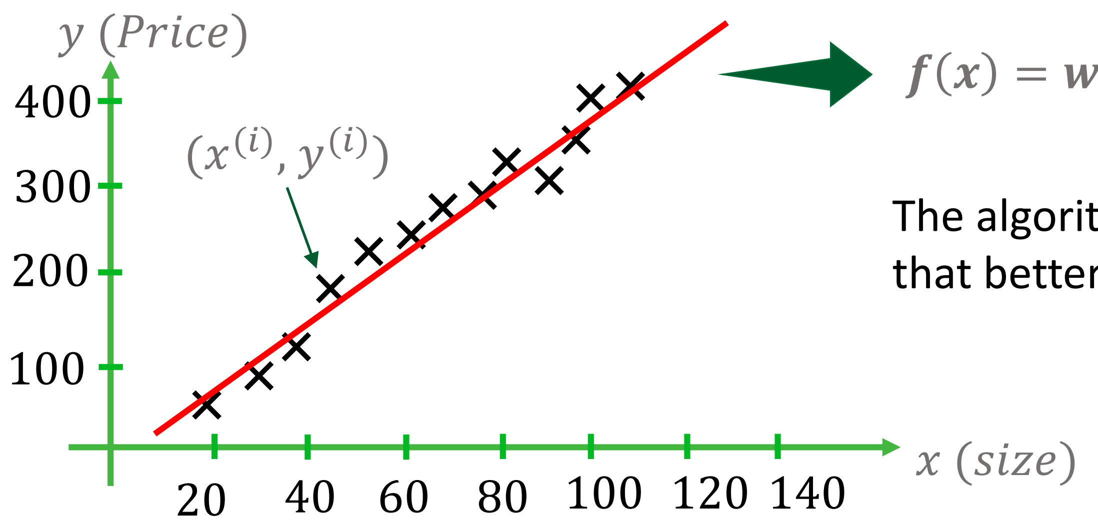
- Linear Regression: house price prediction



$\boldsymbol{f(x) = wx + b}$

The algorithm finds the $\boldsymbol{w}$ and $\boldsymbol{b}$ (parameters) that better fits the data

$\hat{y}^{(i)}=f_{w,b}\left(x^{(i)}\right)$

$f_{w,b}\left(x^{(i)}\right) = w\, x^{(i)} + \mathrm{b}$

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How to find w and b values so,

$\hat{y}^{(i)}$ is close to $y^{(i)}$ for all $(x^{(i)}, y^{(i)})$

?

Cost Function: squared error cost function

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2$$

$m: number\ of\ training\ examples$

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Cost Function: squared error cost function

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}\underbrace{\left(\hat{y}^{(i)} - y^{(i)}\right)^2}_{\text{Error}}$$

m: *number of training examples*

$\hat{y}^{(i)}$: prediction

$y^{(i)}$: target

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Cost Function: squared error cost function

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)^2$$

$m: number\ of\ training\ examples$

$f_{w,b}\left(x^{(i)}\right)$: prediction

$y^{(i)}$: target

Supervised Learning: Linear Regression

Model: $f_{w,b}(x) = wx + b$

Parameters: w, b

Cost function: $J(w, b) = \frac{1}{2m} \sum_{i=1}^{m} \left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)^2$

Goal: $\underset{w,b}{minimize}\, J(w, b)$

Supervised Learning: Linear Regression

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If b=0,

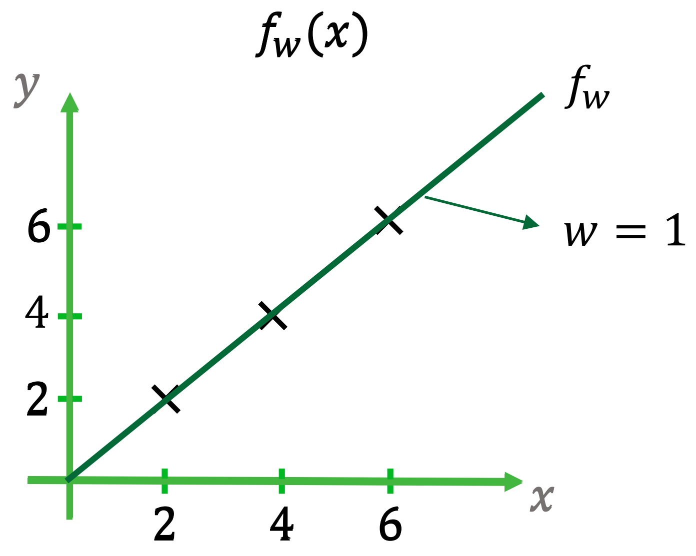
Model: $f_w(x) = wx$

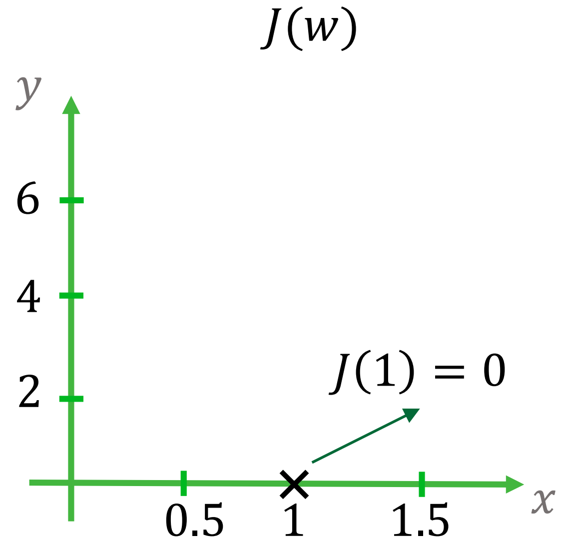
Parameters: w

Cost function: $J(w) = \frac{1}{2m}\sum_{i=1}^{m}(f_w\left(x^{(i)}\right) - y^{(i)})^2$

Goal: $\underset{w}{minimize}\, J(w)$

Supervised Learning: Linear Regression





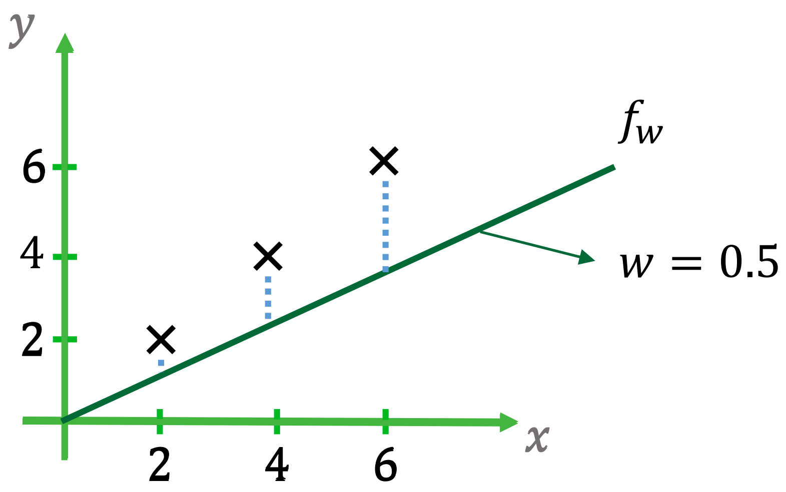
$$w = 1$$

$$J(w) = \frac{1}{2m}(0^2 + 0^2 + 0^2) = 0$$

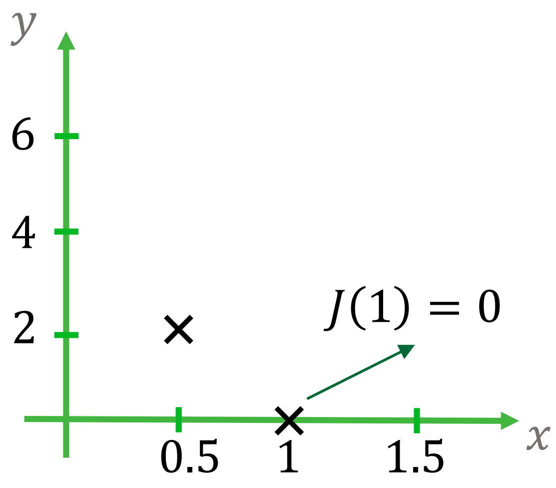
$$J(1) = 0$$

Supervised Learning: Linear Regression

$f_w(x)$



$J(w)$



$w = 0.5$

$$J(0.5) = \frac{1}{2 * 3}\left((1-2)^2 + (2-4)^2 + (3-6)^2\right) = 2.3$$

$$J(0.5) = 2.3$$

Supervised Learning: Linear Regression

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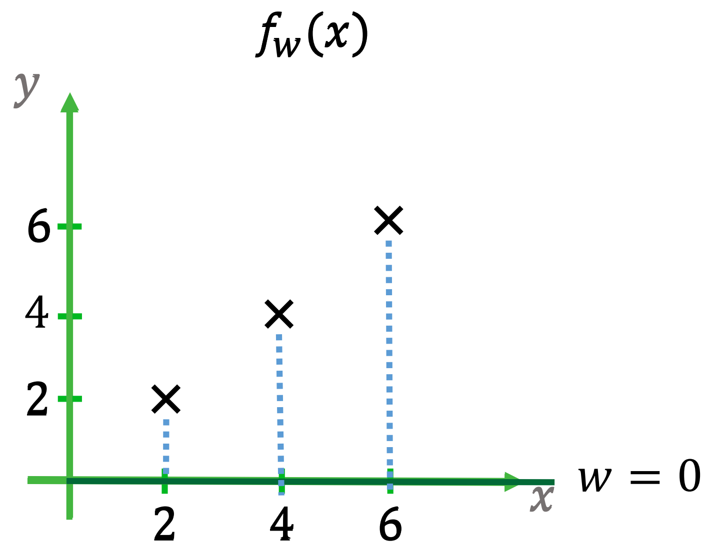
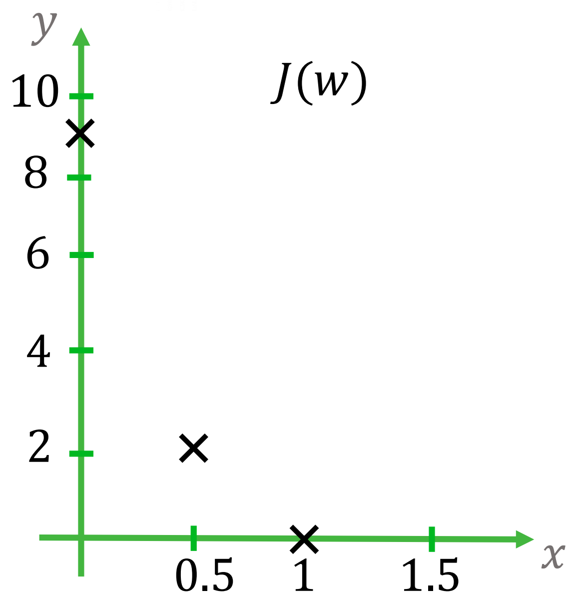




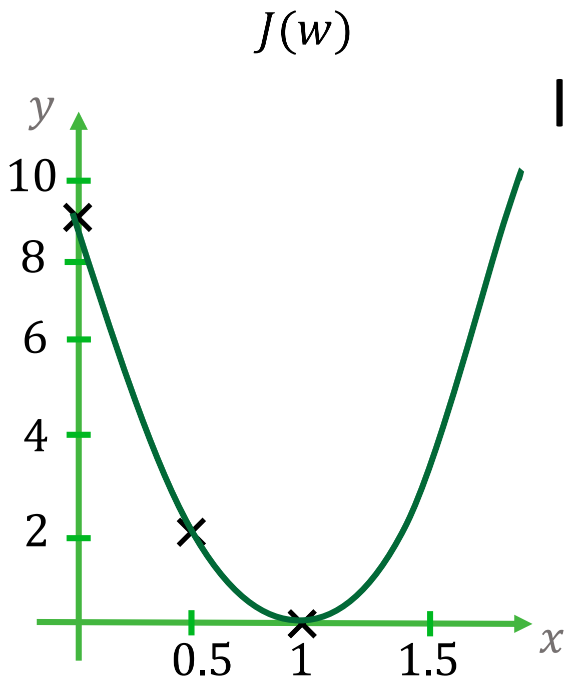
$$w = 0$$

$$J(0) = \frac{1}{2 * 3}\left((0-2)^2 + (0-4)^2 + (0-6)^2\right) = 9.3$$

$$J(0) = 9.3$$

Supervised Learning: Linear Regression

How to choose *w* to minimize *J(w)*?



In this case is when *w=1*

Supervised Learning: Linear Regression

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Model: $f_{w,b}(x) = wx + b$

Parameters: w, b

Cost function: $J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}(f_{w,b}(x^{(i)}) - y^{(i)})^2$

Goal: $\underset{w,b}{minimize}\, J(w,b)$

Supervised Learning: Linear Regression

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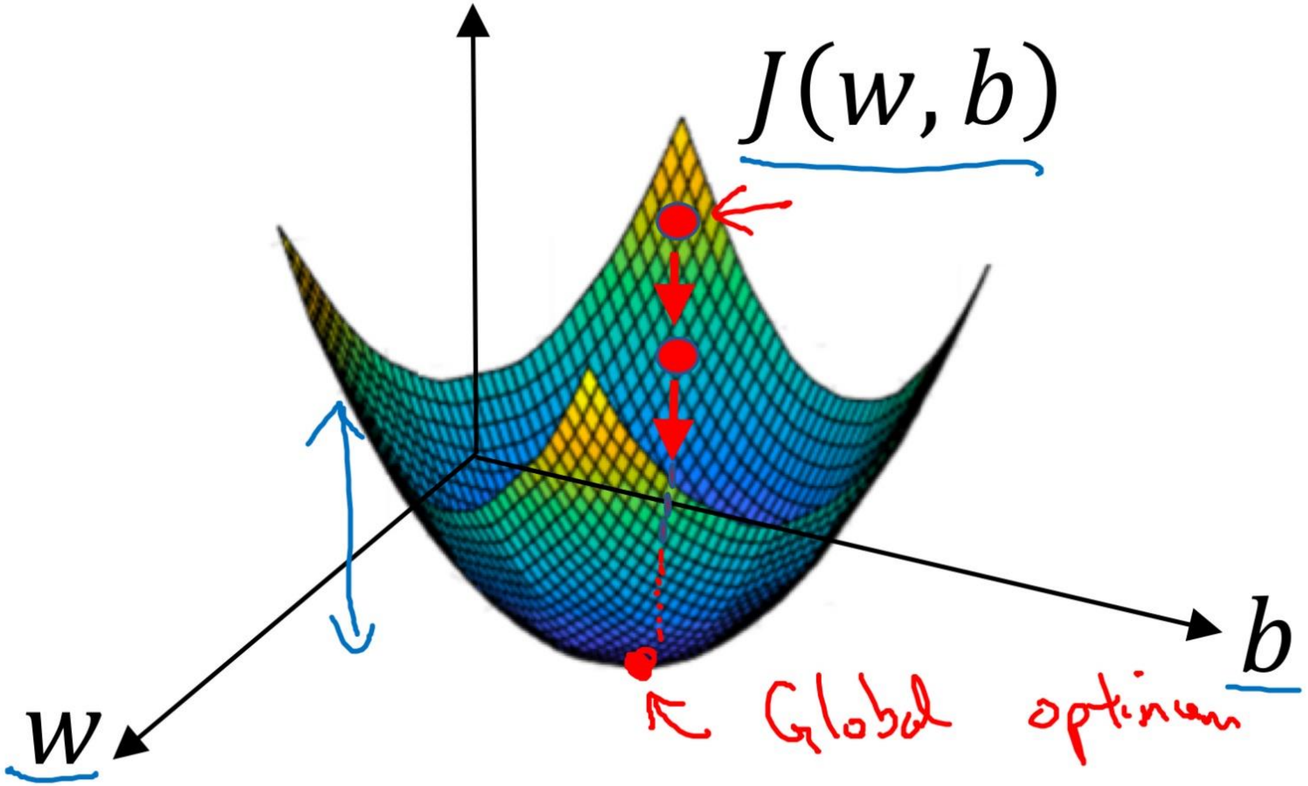
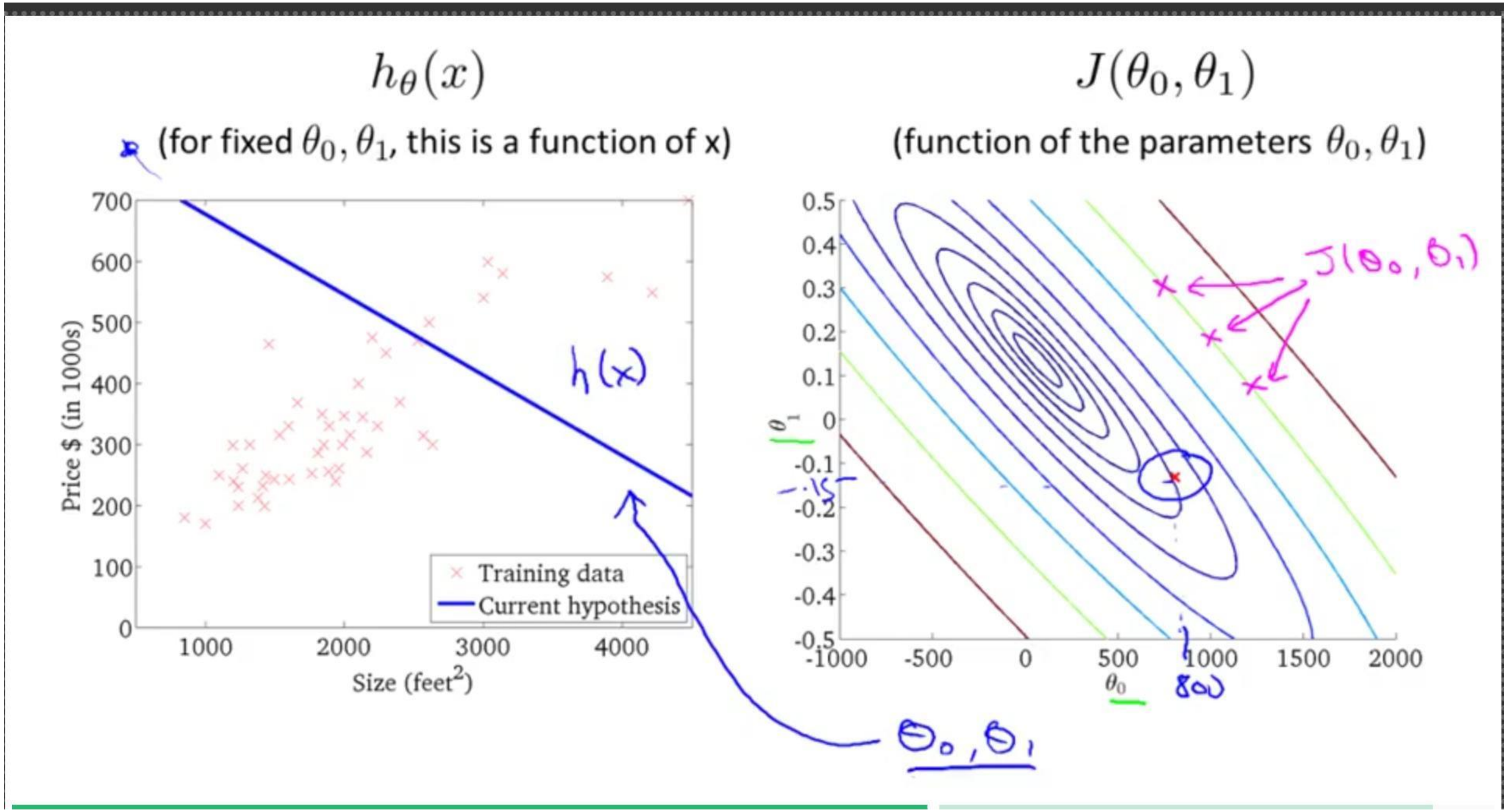


One global minimum in Squared Error Function

Supervised Learning: Linear Regression



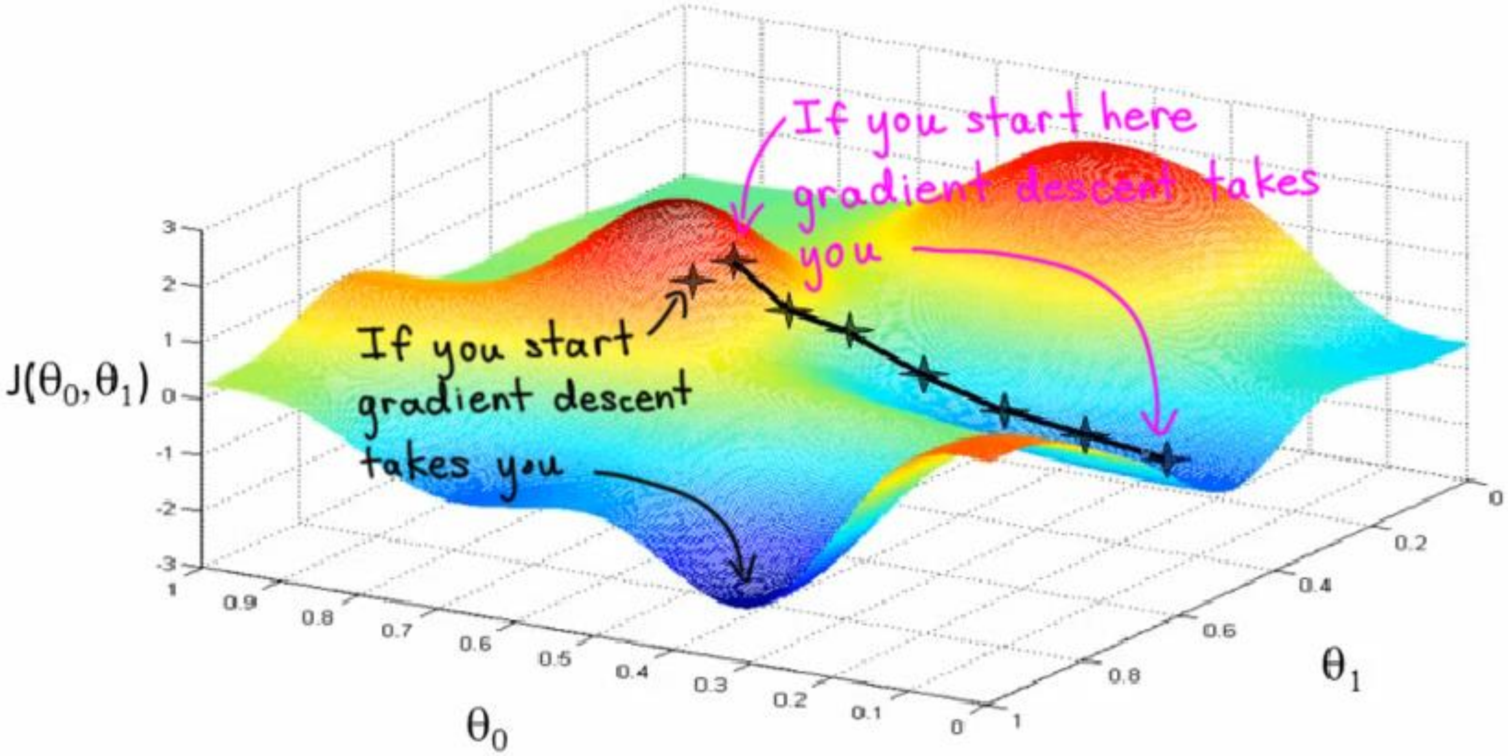
Contour plots

Gradient Descent: minimize any function

Outline:

- Start with some *w, b*
- Change *w, b* to reduce *J(w,b)*
- Keep changing until at or near a minimum

Gradient Descent: minimize any function



More tan one local Minimum: Neural Networks

Gradient Descent algorithm:

$$Repeat\ until\ convergence$$

$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

α: Learning rate

Control the steps of the algorithm to update the parameters

Gradient Descent algorithm:

$$w\_temp = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

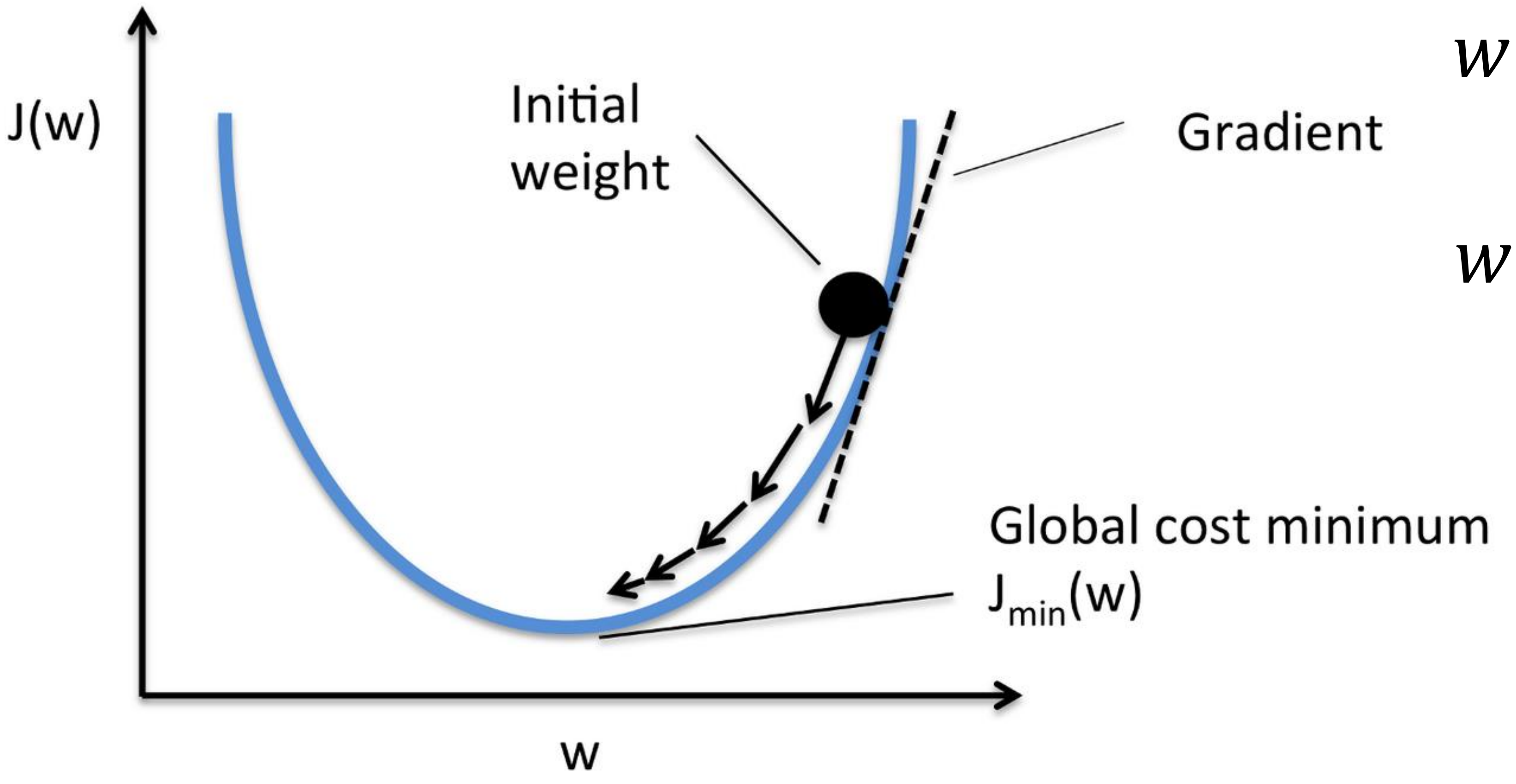
Simultaneous update

$$b\_temp = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

$$w = w\_temp$$

$$b = b\_temp$$

Gradient Descent algorithm:

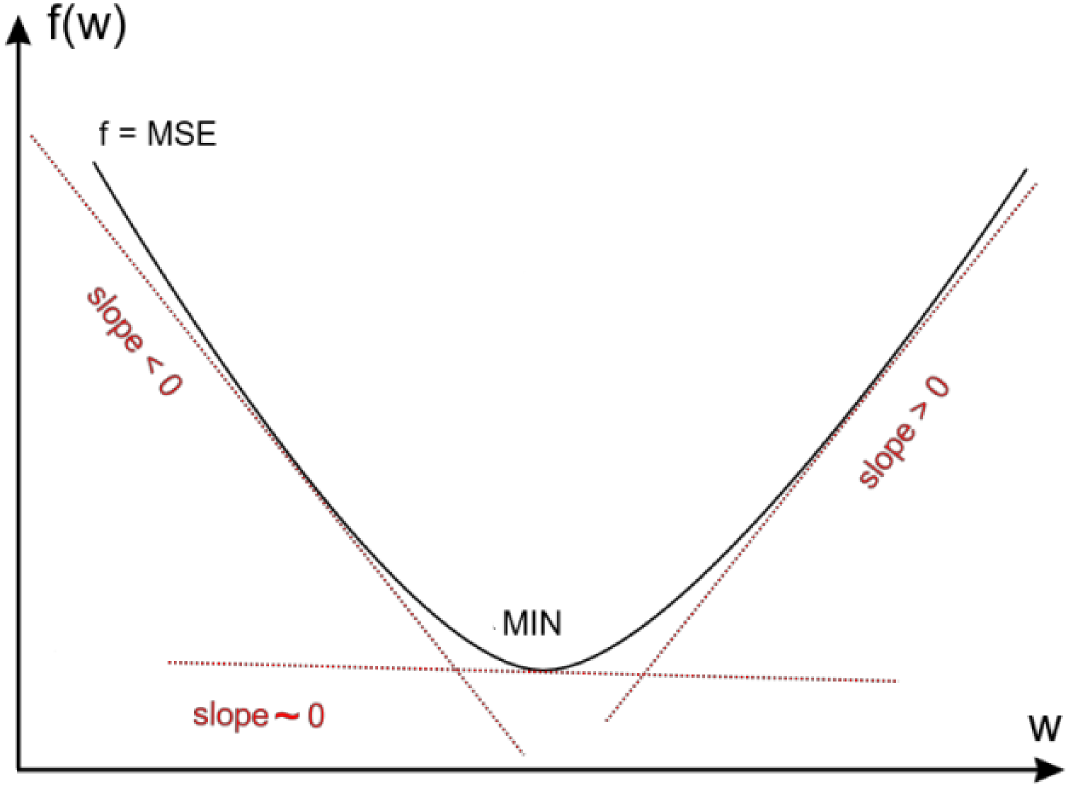


$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$$w = w - \alpha * positive$$

Supervised Learning: Linear Regression

Gradient Descent algorithm:



$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$$w = w - \alpha * positive$$

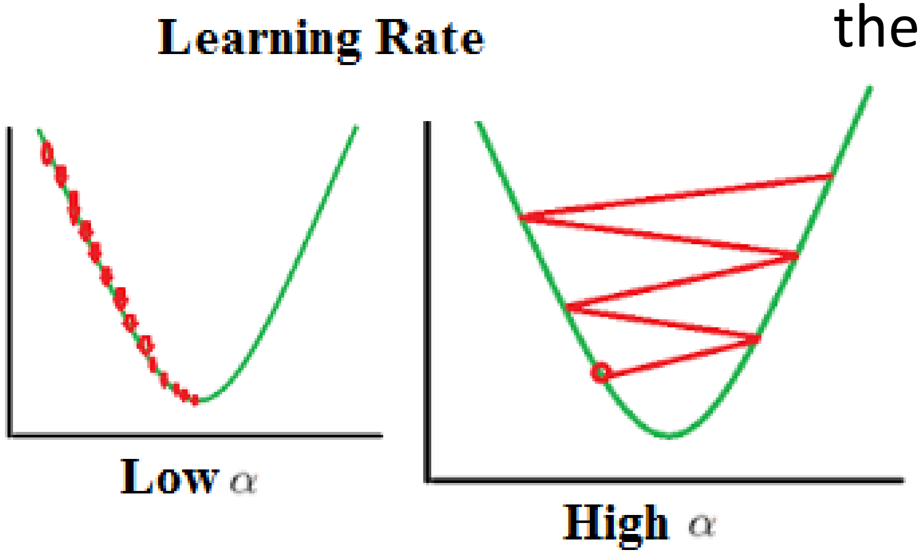
$$w = w - \alpha * negative$$

Supervised Learning: Linear Regression

Learning Rate:

If ∝ is too small, gradient descent can be very slow

If ∝ is too large, gradient descent can never reach the minimum (diverge)



When near a local minimum, gradient descent takes smaller steps (derivative smaller)

Gradient Descent for Linear Regression: $f_{w,b}(x) = wx + b$

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}(f_{w,b}(x^{(i)}) - y^{(i)})^2$$

$$\frac{\partial}{\partial w}J(w,b) = \frac{1}{m}\sum_{i=1}^{m}(f_{w,b}(x^{(i)}) - y^{(i)})x^{(i)}$$

$$\frac{\partial}{\partial b}J(w,b) = \frac{1}{m}\sum_{i=1}^{m}(f_{w,b}(x^{(i)}) - y^{(i)})$$

Supervised Learning: Linear Regression

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Gradient Descent Algorithm:

$$f_{w,b}(x) = wx + b$$

Repeat until convergence {

$$w = w - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{w,b}(x^{(i)}) - y^{(i)}) x^{(i)}$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{w,b}(x^{(i)}) - y^{(i)})$$

}

“Batch” Gradient Descent:

All the training examples are used in each step of the algorithm.

Linear Regression with Multiple features (variables)

Size in m^2 (x_1)	Number of bedrooms (x_2)	Number of bathrooms (x_3)	Price in millions (y)
20	1	1	60
30	1	1	100
40	1	1	120
50	1	2	180
…	…	…	…

$x_j = j^{th}\ feature$

$n = number\ of\ features$

$\vec{x}^{(i)} = features\ of\ i^{th}\ training\ example$

Linear Regression with Multiple features (variables)

Size in m^2 (x_1)	Number of bedrooms (x_2)	Number of bathrooms (x_3)	Price in millions (y)
20	1	1	60
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…	…	…	…

$x_j = j^{th}\ feature$

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Linear Regression with Multiple features (variables)

Size in m^2 (x_1)	Number of bedrooms (x_2)	Number of bathrooms (x_3)	Price in millions (y)
20	1	1	60
30	1	1	100
40	1	1	120
50	1	2	180
…	…	…	…

$x_j = j^{th}\ feature$

$n = number\ of\ features$

$x^{(i)} = features\ of\ i^{th}\ training\ example$

$x_j^{(i)} = feature\ j\ in\ the\ i^{th}\ training\ example$

Linear Regression with one variable:

$$f_{w,b}(x) = wx + b$$

Linear Regression with multiple variables:

$$f_{w,b}(x) = w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_n x_n + b$$

Linear Regression with multiple variables:

$$w = [w_1\ w_2\ w_3\ w_4 \ldots w_n]$$

$$x = [x_1\ x_2\ x_3\ x_4 \ldots x_n]$$

$$f_{w,b}(x) = w \cdot x + b$$

Dot product

Multiple Linear Regression

Gradient Descent for Multiple Linear Regression:

Repeat until convergence {

$$w_1 = w_1 - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)x^{(i)}} \rightarrow \frac{\partial}{\partial w_1}J(w,b)$$

....

$$w_n = w_n - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)x^{(i)}} \rightarrow \frac{\partial}{\partial w_n}J(w,b)$$

$$b = b - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)$$

}

Gradient Descent for Multiple Linear Regression:

Repeat until convergence {

$$w_1 = w_1 - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)x^{(i)}} \rightarrow \frac{\partial}{\partial w_1}J(w,b)$$

....

$$w_n = w_n - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)x^{(i)}} \rightarrow \frac{\partial}{\partial w_n}J(w,b)$$

$$b = b - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)$$

}

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THANK YOU ANDREW NG

References

- Christopher M Bishop et al. Pattern recognition and machine learning. Springer New York, 2006.
- Ng, A. (2023). Machine Learning Specialization Coursera. Standford University, DeepLearning.AI
- Samuel, A. L. (1959). Some studies in machine learning using the game of checkers. IBM Journal of research and development, 3(3), 210-229.

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