15.095 Homework 1

due September 19, 2018

1. **Modeling: Furniture Ordering**

   Jack is an office manager of a large New York City based accounting firm. He needs to replace the aging and out-of-style furniture in the firm's offices. The firm has decided to purchase new furniture sets for all 2,000 offices in New York City. Jack has received bids from 4 different companies who are willing to supply the furniture sets. The bids from these 4 companies are shown in Table 1.

   (a) Help Jack build a mixed-integer linear optimization model to minimize the total purchasing cost. What are the decision variables? Which variables are integer? (You do not need to solve this optimization problem, just provide the formulation.)

   (b) What extra constraints should we add if Jack needs to purchase under the following conditions:

      i. Jack must order from at least 3 companies.

      ii. Jack can order from Carolina Woodworks or Nashawtuc Millworks, but not both.

      iii. If Jack orders from Carolina Woodworks, he must also order from Nashawtuc Millworks. However, if Jack orders from Nashawtuc Millworks, he may or may not order from Carolina Woodworks.

      iv. Jack can either order from both Carolina Woodworks and Nashawtuc Millworks or from neither.

      v. If Jack does not order from Carolina Woodworks, then he must order from Nashawtuc Millworks.

   (c) Suppose that a fifth company, Delaware Mills, has submitted a bid to supply up to 1,500 furniture sets. In the bid, the first 1,000 sets will cost $2,530 per set with a one-time delivery charge of $9,000. However, if the firm purchases more than 1,000 sets, then it will be charged $2,430 for any and all additional furniture sets beyond the 1,000, with

	Carolina Woodworks	Nashawtuc Millworks	Adirondack Furnishing Designs	Lancaster Artisan Company
Quantity	Up to 1,000	Up to 1,200	Up to 800	Up to 1,100
Price per set	\$2,500	\$2,450	\$2,510	\$2,470
Onetime delivery charge	\$10,000	\$20,000	\$0	\$13,000

Table 1: Bids from the Furniture Companies

an additional delivery charge of \$7,000. Incorporate this new bid into the optimization model from part (a).

2. **Robust Linear Regression**

   (a) Using Julia/JuMP, write functions to solve the following optimization problems:

$$\min_{\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2 + \rho\|\boldsymbol{\beta}\|_1, \tag{1}$$

$$\min_{\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2 + \rho\|\boldsymbol{\beta}\|_2. \tag{2}$$

   Given data $\mathbf{X} \in \mathbb{R}^{n \times p}, \mathbf{y} \in \mathbb{R}^n$, and a constant $\rho \in \mathbb{R}$, these functions should return the optimal β which solves the specified regression problem.

   (b) Using these functions, fit ℓ_1 and ℓ_2-regularized linear regression models on the data in the files `housing.csv` and `communities-and-crime.csv`. The last column in each file is the target $\mathbf{y}$, and the remaining columns make up the matrix of independent variables $\mathbf{X}$. For each data set, split into 50% training, 25% validation, and 25% testing. Use the validation set to select an optimal value for the parameter ρ in the range $[0.001, 0.01, 0.1, 1, 2]$ for both methods. Compute $\|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2$ on the testing set using the β's from ℓ_1-regularized linear regression, ℓ_2-regularized linear regression, and standard linear regression ($\rho = 0$). Which method performs best? Also compare with a baseline model, which predicts using the mean $\mathbf{y}$ on the training and validation sets.

   (c) Based on the performance on these two datasets, what are the benefits of regularized linear regression? When you should use this method over standard linear regression?

3. **Best Subset Selection**

Write down a first-order method to solve(find warm starts for MIO):

$$\min_{\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \Gamma\|\boldsymbol{\beta}\|_1 \quad s.t. \quad \|\boldsymbol{\beta}\|_0 \leq k, \tag{3}$$

where $\Gamma > 0$. You do not need to implement this in Julia, just derive and write out the steps of the algorithm.

Hint: Follow the method presented in Lecture 2. Majorize the quadratic term by a quadratic separable in β_i's, and leave other terms unchanged.