

$$\nabla \phi \psi \rightarrow$$

$$\frac{\partial \phi \psi}{\partial x} \hat{i} + \frac{\partial \phi \psi}{\partial y} \hat{j} + \frac{\partial \phi \psi}{\partial z} \hat{k}$$

$$\left(\psi \frac{\partial \phi}{\partial x} + \phi \frac{\partial \psi}{\partial x} \right) \hat{i} + \left[\psi \frac{\partial \phi}{\partial y} + \phi \frac{\partial \psi}{\partial y} \right] \hat{j} + \left[\psi \frac{\partial \phi}{\partial z} + \phi \frac{\partial \psi}{\partial z} \right] \hat{k}$$

$$\psi \left[\frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k} \right] + \phi \left[\frac{\partial \psi}{\partial x} \hat{i} + \frac{\partial \psi}{\partial y} \hat{j} + \frac{\partial \psi}{\partial z} \hat{k} \right]$$

$$\psi \nabla \phi + \phi \nabla \psi = \nabla \phi \psi$$

$$\nabla \cdot (\nabla \times \mathbf{a})$$

$$(\nabla \times \mathbf{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_1 & a_2 & a_3 \end{vmatrix} = \left[\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z} \right] \hat{i} - \left[\frac{\partial a_3}{\partial x} - \frac{\partial a_1}{\partial z} \right] \hat{j} + \dots$$

$$\dots \left[\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y} \right] \hat{k}$$

$$\nabla \cdot (\nabla \times \mathbf{a})$$

$$\frac{\partial}{\partial x} \left[\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z} \right] - \frac{\partial}{\partial y} \left[\frac{\partial a_3}{\partial x} - \frac{\partial a_1}{\partial z} \right] + \frac{\partial}{\partial z} \left[\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y} \right]$$

$$\underbrace{\frac{\partial^2 a_3}{\partial y \partial x}}_{\text{purple}} - \underbrace{\frac{\partial^2 a_2}{\partial z \partial x}}_{\text{green}} - \underbrace{\frac{\partial^2 a_3}{\partial y \partial x}}_{\text{purple}} + \underbrace{\frac{\partial^2 a_1}{\partial y \partial z}}_{\text{red}} + \underbrace{\frac{\partial^2 a_2}{\partial z \partial x}}_{\text{green}} - \underbrace{\frac{\partial^2 a_1}{\partial z \partial y}}_{\text{red}} = 0$$