6. Demuestre que:

a) $Log(ie) = 1 - \frac{\pi}{2} i$ $\quad * Log(z) = \ln \|z\| + i(\theta + 2\pi n)$

$ie = 0 + ei = e \cdot e^{\frac{\pi}{2} i}$

$Log(e \cdot e^{\pi/2 i}) = \ln(e) + (\pi/2 \, i)$ ✓

b) $Log(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4} i$

$1 - i = 2e^{\frac{\pi}{4} i}$ $\quad \|z\| = \sqrt{1+1} = \sqrt{2}$

$Log(2e^{\pi/4 i}) = \ln(\sqrt{2}) + \pi/4 \, i$

$= \ln(2)^{1/2}$

$= \frac{1}{2}\ln(2) + \frac{\pi}{4} i$ ✓

c) $Log(e) = 1 + 2n\pi i$

$e = e + 0i = e \cdot e^{2\pi i}$

$Log(e \cdot e^{2\pi i}) = \ln(e) + 2\pi n i$

$= 1 + 2\pi n i$ ✓

D) $Log(i) = (2n + \frac{1}{2})\pi i$

$i = 0 + 1i = e^{\pi/2 \, i}$

$Log(e^{\pi/2 \, i}) = \ln(1) + i(\frac{\pi}{2} + 2\pi n)$

$= 0 + \pi i(\frac{1}{2} + 2n)$

$= (\frac{1}{2} + 2n)\pi i$

10. $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$ $\quad$ $Senh(x) = \dfrac{e^x - e^{-x}}{2}$

Demuestre

a) $\cosh(x) = \cos(ix)$ $\quad * \; z + \frac{1}{z} = 2\cos(\theta)$

• $\cosh(x) = \frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}(2\cos(\theta))$ $\quad * \; x = i\theta$

$\theta = -ix$

$\cosh(x) = \cos(-ix) = \cos(ix)$

$\hookrightarrow \cos(-\theta) = \cos(\theta)$

$\cosh(x) = \cos(ix)$ ✓

• $i\,Senh(x) = Sen(ix)$ $\quad z - \frac{1}{z} = 2i\,Sen\,\theta$

$Senh(x) = \frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}(2i\,Sen\,\theta)$ $\quad * \theta = -ix$

$Senh(x) = i\,Sen(-ix) = -i\,Sen(ix)$

$i\,Senh(x) = Sen(ix)$ ✓

- $\cos(x) = \cosh(ix)$

$\cosh(ix) = \frac{1}{2}(e^{ix} + e^{-ix}) = \frac{1}{2}(2\cos\theta)$

$\hookrightarrow \theta = x$

$\cosh(ix) = \cos(x)$ ✓

- $\operatorname{Senh}(ix) = i\operatorname{sen}(x)$

$\operatorname{Senh}(ix) = \frac{1}{2}(e^{ix} - e^{-ix}) = \frac{1}{2}(2i\operatorname{sen}\theta)$; $\theta = x$

$\operatorname{Senh}(ix) = i\operatorname{sen}(x)$ ✓

b) Mostrar

- $\cosh^2(x) - \operatorname{Senh}^2(x) = 1$

$\cosh^2(x) - \operatorname{Senh}^2(x) = \frac{1}{2}(e^x + e^{-x})^2 - \frac{1}{2}(e^x - e^{-x})^2$

$\frac{1}{4}(e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x})$

$\frac{1}{4}(4) = 1$ ✓

- $\operatorname{Sech}^2(x) = 1 - \tanh^2(x)$

$(\operatorname{Sech}^2(x) = 1 - \tanh^2(x)) * \cosh^2(x)$

$\frac{1}{\cosh^2(x)} = 1 - \frac{\operatorname{Senh}^2(x)}{\cosh^2(x)}$

$1 = \cosh^2(x) - \operatorname{Senh}^2(x)$ (Demostrado arriba) ✓

- $\cosh(2x) = \cosh^2(x) + \operatorname{Senh}^2(x)$

$\cosh(2x) = \cos(2ix) = \cos(ix)\cos(ix) - \dots$

$\dots \operatorname{Sen}(xi)\operatorname{Sen}(ix)$

$= \cos^2(ix) - \operatorname{Sen}^2(ix) = \cosh^2(x) - \operatorname{Senh}^2(x)$ ✓

c) Resolver

- $\cosh(x) - 5\operatorname{Senh}(x) - 5 = 0$

$\frac{1}{2} - \frac{10}{2} = \frac{9}{2}$

$\cosh(x) - 5\operatorname{Senh}(x) - 5 = 0$

$\frac{1}{2}(e^x - e^{-x}) - 5(e^x - e^{-x}) - 5 = 0$

$\frac{1}{2}e^x - 5e^x - \frac{1}{2}e^{-x} + 5e^{-x} - 5 = 0$

$e^x(\frac{1}{2} - 5) - e^{-x}(\frac{1}{2} - 5) - 5 = 0$

$\frac{9}{2}e^x - \frac{9}{2}e^x - 5 = 0$

$\frac{9}{2}(e^x - e^{-x}) = 5$

$\operatorname{Senh}(x) = 5/9$

$\operatorname{Senh}^{-1}(5/9) = x$ ✓

• $2\cosh(4x) - 8\cosh(2x) + 5 = 0$

$2(\text{Senh}^2(2x) + \cosh^2(2x) - 4\cosh(2x)) = 5$

$2(1 + \cosh^2(2x) + \cosh^2(2x) - 4\cosh(2x) = 5$

$2(1 + 2\cosh^2(2x) - 4\cosh(2x)) = 5$

$2(2\cosh^2(2x) - 4\cosh(2x) + 1)$

$(\sqrt{2}\cosh(2x) - 1)^2 = 5/2$

$\sqrt{2}\cosh(2x) - 1 = \dfrac{\sqrt{5}}{\sqrt{2}}$

$\cosh(2x) = \dfrac{\sqrt{5}}{2} - \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{5}}{2} - \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{5}-\sqrt{2}}{2}$

$2x = \cosh^{-1}\left(\dfrac{\sqrt{5}-\sqrt{2}}{2}\right)$

$x = \dfrac{1}{2}\cosh^{-1}\left(\dfrac{\sqrt{5}-\sqrt{2}}{5}\right)$ ✓

• $\cosh(x) = \text{Senh}(x) + 2\,\text{Sech}(x)$

$1 = \tanh(x) + 2\,\text{Sech}^2(x)$

$\text{Sech}^2(x) + \tanh^2(x) = \tanh(x) + 2\,\text{Sech}^2(x)$

$\tanh^2(x) = \tanh(x) + \text{Sech}^2(x)$

$\text{Senh}^2(x) = \text{Senh}(x)\cosh(x) + 1$

$\text{Senh}^2(x) - 1 = \text{Senh}(x)\cosh(x)$

Vol 2.

4. Derivar.

a) $\dfrac{z+i}{z-i}\Big|_i$ * derivar Como cociente.

$$\frac{(z-i)-(z+i)}{(z-i)^2} = \frac{-2i}{(z-i)^2}\Big|_i \quad \nexists$$

No es analítica

b) $(z-4i)^8\Big|_{3+4i}$ * cadena

$u = z - 4i$ $\quad \dfrac{du}{dz}\dfrac{dy}{du}$

$y = u^8$

$= 8(z-4i)^7\Big|_{3+4i} = 8(3+4i-4i)^7 = 8(3^7)$

c) $\left.\dfrac{(1{,}5z+2i)}{(3iz-4)}\right]_z$

$$\frac{(1.5)(3iz-4)-(3i)(1.5z+2i)}{(3iz-4)^2}$$

$= 4.5iz - \cancel{6} - 4.5iz + \cancel{6} = 4.5z(i-1)$

$$= \frac{4.5.z(i-1)}{(3iz-4)^2}$$

d) $i(1-z)^n \Big|_0$

$u = 1-z$ $\qquad \dfrac{du}{dz}\,\dfrac{dw}{du}$

$w = iu^n$

$-1(inu^{n-1}) = -(in(1-z)^{n-1}\Big|_0 = -in$

e) $(iz^3+3z^2)^3\Big|_{2i}$

$(3iz^2+6z)(3(iz^3+3z^2)^2\Big|_{2i}$

$(3i(2i)^2+6(2i))\ 3(i(2i)^3+3(2i)^2)^2$

$(3i(-4)+12i)(3i(-8i)+9(-4))^2$

$(-12i+12i)(24-36)^2 = 12^2 = 144$

f) $\dfrac{z^3}{(z+i)^3}\Big|_i$

$$\left.\frac{3z^2(z+i)^3-3(z+i)z^3}{(z+i)^5}\right|_i$$

$$\frac{3(i)^2(i+i)^3-3(i+i)^2i^3}{(i+i)^5}$$

$$\frac{(-3)(-8i)-3(-4)(-i)}{32i}$$

$$\frac{12i}{32i} = \frac{3}{8}i$$

Encuentre si las funciones son analíticas

$$\frac{\partial u(x,y)}{\partial x} = \frac{\partial v(x,y)}{\partial y}$$

$$\frac{\partial u(x,y)}{\partial y} = -\frac{\partial v(x,y)}{\partial x}$$

a) $f(z) = i z z^*$

$f(x,y) = i(x^2+y^2)$

$u(x,y) = 0$

$v(x,y) = (x^2+y^2)$

$\frac{\partial u}{\partial x} = 0 \neq \frac{\partial v}{\partial y} = 2y$ No es analítica

b) $f(z) = e^{-2x}(\cos(2y) - i\,\mathrm{sen}(2y))$

$u(x,y) = e^{-2x}\cos(2y)$; $v(x,y) = -e^{-2x}\,\mathrm{sen}(2y)$

$\frac{\partial u}{\partial x} = -2e^{-2x}\cos(2y)$

Es analítica

$\frac{\partial v}{\partial y} = -2e^{-2x}\cos(2y)$

c) $f(z) = \mathrm{Re}(z^2) - i\,\mathrm{Im}(z^2)$

$z^2 = x^2 + 2xyi - y^2$

$f(x,y) = x^2 - y^2 - i\,2xy$

$u(x,y) = x^2 - y^2$; $v(x,y) = 2xy$

$\frac{\partial u}{\partial x} = 2x$ $\qquad \frac{\partial u}{\partial y} = -2y$ Es analítica.

$\frac{\partial v}{\partial y} = 2x$ $\qquad \frac{\partial v}{\partial x} = 2y$

d) $f(z) = e^x(\cos(y) - i\,\mathrm{sen}(y))$

$u(x,y) = e^x\cos(y)$; $v(x,y) = -e^x\,\mathrm{sen}(y)$

$\frac{\partial u}{\partial x} = e^x\cos(y)$ $\qquad \frac{\partial u}{\partial y} = -e^x\,\mathrm{sen}(y)$ No es analítica.

$\frac{\partial v}{\partial y} = e^x\cos(y)$ $\qquad \frac{\partial v}{\partial x} = -e^x\cos(y)$