This homework is worth 10 points and will be graded partially on completion (5), partially on accuracy (4), and partially on formatting (1) - see the Homework rubric in Brightspace under the Assignments tab. Carefully write out fully detailed solutions as modeled in class. Please upload a single file containing pictures/scans of your written work into the appropriate homework assignment in Brightspace.

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1. In each of the augmented matrices below, circle the leading entries and identify the echelon form.

$$A = \left[\begin{array}{ccc|c} 4 & 0 & -2 & 0 \\ 0 & 3 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \qquad B = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad C = \left[\begin{array}{cc|c} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{array}\right]$$

neither — row-echelon by vacuity — neither

$$D = \left[\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad E = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \qquad F = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & 6 \\ 0 & 1 & -3 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

row-echelon — neither — reduced row-echelon

2. Use row operations to convert augmented matrix E above to reduced row-echelon form.

Swap $R3 \to R4$, $R4 \to R3$

$$E = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

3. Use row operations to convert augmented matrix C above to reduced row-echelon form.

$$C = \left[\begin{array}{cc|c} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{array}\right]$$

1. ~~$-\frac{1}{2}R2 + R1 \to$~~ $\left[\begin{array}{cc|c} 0 & 1 & \frac{1}{2} \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{array}\right]$ $\frac{1}{2}R1 \to R1$

2. Swap $R2 \to R1$, $R1 \to R2$

3. $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 2 & 3 & 1 \end{array}\right]$

4. $-2R1 + R3 \to R3$ $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 3 & -1 \end{array}\right]$

5. $-3R2 + R3 \to R3$ $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & -\frac{3}{2} \end{array}\right]$

6. $3R2 + R3 \to R3$ $C = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{array}\right]$

4. Solve the system of equations represented by augmented matrix D above (*do you see a quick way to solve it without row operations?*).

$$D = \left[\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z = 1$

$y - 3 = 5$, $y = 8$

$y - 3(1) = 5$

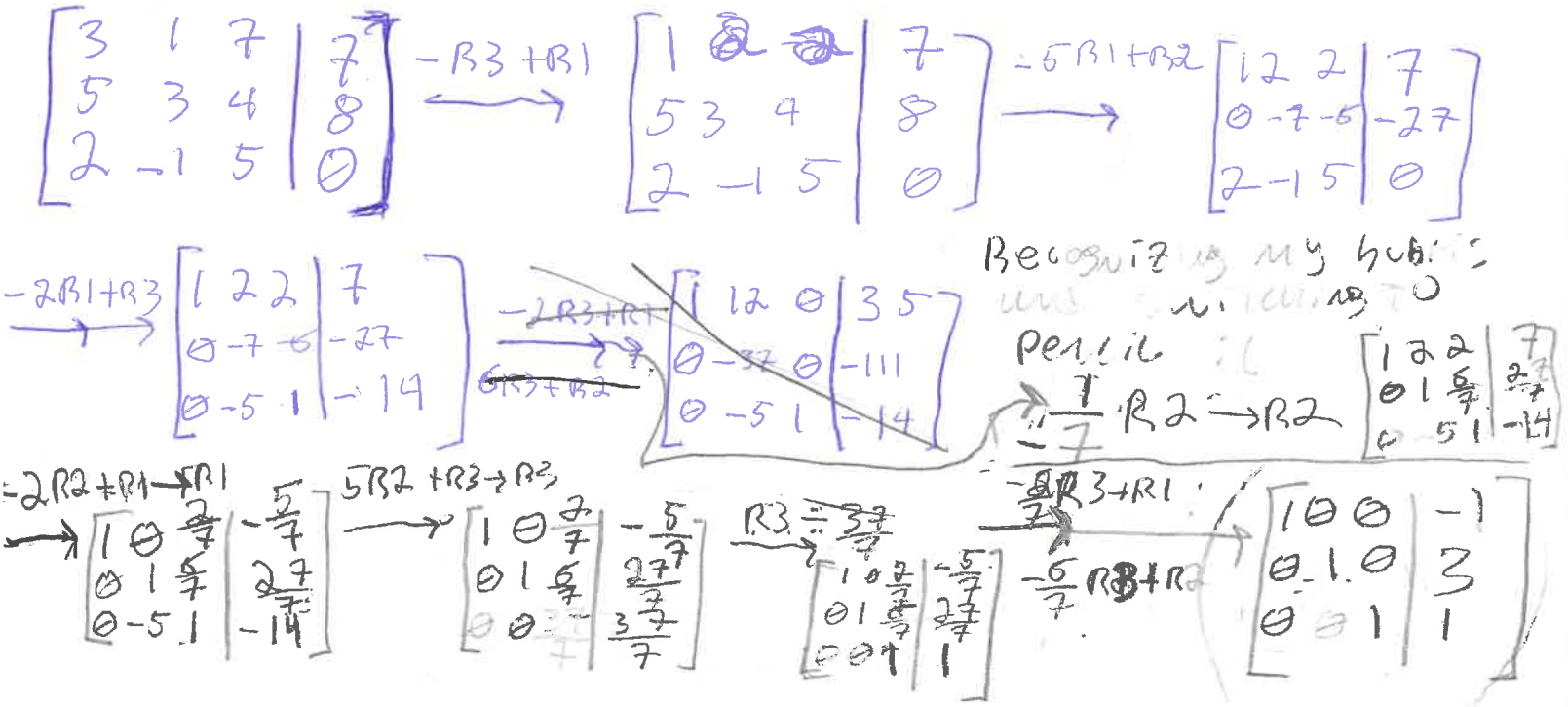
$x - 2(8) - 3(1) = 4$

$x - 16 - 3 = 4$

$x - 19 = 4$

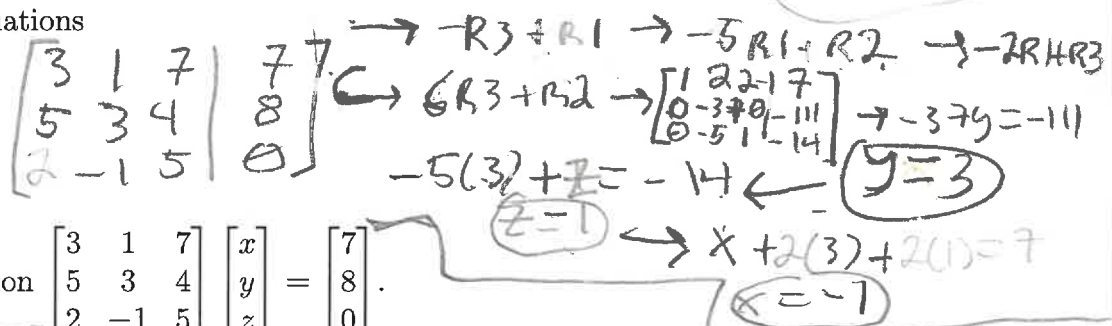
$x = 23$

5. Show off your row operation skills to convert $\left[\begin{array}{ccc|c} 3 & 1 & 7 & 7 \\ 5 & 3 & 4 & 8 \\ 2 & -1 & 5 & 0 \end{array}\right]$ into $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]$ (*I recommend beginning with* $-R3+R1$, *eliminating* x, *eliminating* z, *and then eliminating* y).

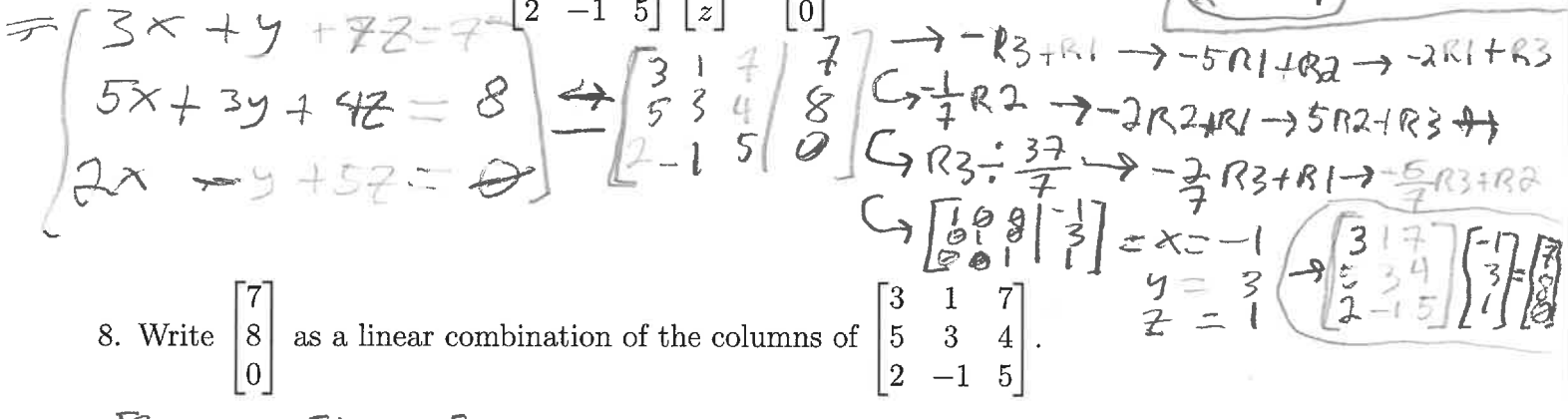


6. Solve the system of equations

$$\begin{aligned} 3x+y+7z &= 7 \\ 5x+3y+4z &= 8 \\ 2x-y+5z &= 0 \end{aligned}$$



7. Solve the matrix equation $\begin{bmatrix} 3 & 1 & 7 \\ 5 & 3 & 4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 0 \end{bmatrix}$.



8. Write $\begin{bmatrix} 7 \\ 8 \\ 0 \end{bmatrix}$ as a linear combination of the columns of $\begin{bmatrix} 3 & 1 & 7 \\ 5 & 3 & 4 \\ 2 & -1 & 5 \end{bmatrix}$.