

This homework is worth 10 points and will be graded partially on completion (5), partially on accuracy (4), and partially on formatting (1) - see the Homework rubric in Brightspace under the Assignments tab. Carefully write out fully detailed solutions as modeled in class. Please upload a single file containing pictures/scans of your written work into the appropriate homework assignment in Brightspace.

1. In each of the augmented matrices below, circle the leading entries and identify the echelon form.

$$A = \left[\begin{array}{ccc|c} 4 & 0 & -2 & 0 \\ 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

~~Row echelon~~
neither

$$B = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

row echelon
by values

$$C = \left[\begin{array}{cc|c} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{array} \right]$$

neither

$$D = \left[\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

~~reduced~~
row echelon

$$E = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

~~reduced~~
row echelon

$$F = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & 6 \\ 0 & 1 & -3 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

reduced row echelon

2. Use row operations to convert augmented matrix E above to reduced row-echelon form.

SWAP

$$\begin{array}{l} R_3 \rightarrow R_4 \\ R_4 \rightarrow R_3 \end{array}$$

$$E = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

3. Use row operations to convert augmented matrix C above to reduced row-echelon form.

$$C = \left[\begin{array}{cc|c} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{array} \right]$$

$$3. \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 2 & 3 & 1 \end{array} \right]$$

5. $-3R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & -\frac{3}{2} \end{array} \right]$$

$$1. \frac{1}{2}R_2 + R_1 \rightarrow R_1$$

$$\frac{1}{2}R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|c} 0 & 1 & \frac{1}{2} \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{array} \right]$$

$$4. -2R_1 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 3 & -1 \end{array} \right]$$

5. $3R_2 + R_3 \rightarrow R_3$

$$C = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{array} \right]$$

$$2. \text{SWAP } R_2 \rightarrow R_1$$

$$R_1 \rightarrow R_2$$

4. Solve the system of equations represented by augmented matrix D above (do you see a quick way to solve it without row operations?).

$$D = \left[\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\begin{array}{l} x - 2(8) - 3(1) = 4 \\ x - 16 - 3 = 4 \end{array}$$

$$z = 1$$

$$y - 3(1) = 5$$

$$y = 8$$

$$x - 19 = 4$$

$$x = 23$$

5. Show off your row operation skills to convert $\begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 5 & 3 & 4 & | & 8 \\ 2 & -1 & 5 & | & 0 \end{bmatrix}$ into $\begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$ (I recommend beginning with $-R_3 + R_1$, eliminating x , eliminating z , and then eliminating y).

$$\begin{aligned} &\begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 5 & 3 & 4 & | & 8 \\ 2 & -1 & 5 & | & 0 \end{bmatrix} \xrightarrow{-R_3 + R_1} \begin{bmatrix} 1 & 2 & 2 & | & 7 \\ 5 & 3 & 4 & | & 8 \\ 2 & -1 & 5 & | & 0 \end{bmatrix} \xrightarrow{-5R_1 + R_2} \begin{bmatrix} 1 & 2 & 2 & | & 7 \\ 0 & -7 & -6 & | & -27 \\ 2 & -1 & 5 & | & 0 \end{bmatrix} \\ &\xrightarrow{-2R_1 + R_3} \begin{bmatrix} 1 & 2 & 2 & | & 7 \\ 0 & -7 & -6 & | & -27 \\ 0 & -5 & 1 & | & -14 \end{bmatrix} \xrightarrow{-2R_3 + R_1} \begin{bmatrix} 1 & 2 & 0 & | & 35 \\ 0 & -7 & -6 & | & -27 \\ 0 & -5 & 1 & | & -14 \end{bmatrix} \xrightarrow{6R_3 + R_2} \begin{bmatrix} 1 & 2 & 0 & | & 35 \\ 0 & -37 & 0 & | & -111 \\ 0 & -5 & 1 & | & -14 \end{bmatrix} \\ &\xrightarrow{-2R_2 + R_1} \begin{bmatrix} 1 & 2 & 0 & | & 35 \\ 0 & -37 & 0 & | & -111 \\ 0 & -5 & 1 & | & -14 \end{bmatrix} \xrightarrow{5R_2 + R_3} \begin{bmatrix} 1 & 2 & 0 & | & 35 \\ 0 & -37 & 0 & | & -111 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \xrightarrow{R_3 \div 37} \begin{bmatrix} 1 & 2 & 0 & | & 35 \\ 0 & -1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \\ &\xrightarrow{-R_3 + R_1} \begin{bmatrix} 1 & 2 & 0 & | & 35 \\ 0 & -1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \xrightarrow{-5R_2 + R_1} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & -1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & -3 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \end{aligned}$$

6. Solve the system of equations

$$3x + y + 7z = 7$$

$$5x + 3y + 4z = 8$$

$$2x - y + 5z = 0$$

$$\begin{aligned} &\begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 5 & 3 & 4 & | & 8 \\ 2 & -1 & 5 & | & 0 \end{bmatrix} \xrightarrow{-R_3 + R_1} \begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 5 & 3 & 4 & | & 8 \\ 1 & 2 & 2 & | & 7 \end{bmatrix} \xrightarrow{6R_3 + R_2} \begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 1 & 2 & 2 & | & 7 \\ 5 & 3 & 4 & | & 8 \end{bmatrix} \xrightarrow{-R_3 + R_1} \begin{bmatrix} 1 & 2 & 2 & | & 7 \\ 1 & 2 & 2 & | & 7 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \\ &\xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 1 & 2 & 2 & | & 7 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 2 & 2 & | & 8 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 1 & | & 4 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \\ &\xrightarrow{-R_2 + R_3} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 1 & | & 4 \\ 0 & 0 & -7 & | & -22 \end{bmatrix} \xrightarrow{R_3 \div -7} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 1 & | & 4 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \end{aligned}$$

7. Solve the matrix equation

$$\begin{bmatrix} 3 & 1 & 7 \\ 5 & 3 & 4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 0 \end{bmatrix}$$

$$\begin{aligned} &\begin{cases} 3x + y + 7z = 7 \\ 5x + 3y + 4z = 8 \\ 2x - y + 5z = 0 \end{cases} \xrightarrow{-R_3 + R_1} \begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 5 & 3 & 4 & | & 8 \\ 1 & 2 & 2 & | & 7 \end{bmatrix} \xrightarrow{6R_3 + R_2} \begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 1 & 2 & 2 & | & 7 \\ 5 & 3 & 4 & | & 8 \end{bmatrix} \xrightarrow{-R_3 + R_1} \begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 1 & 2 & 2 & | & 7 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \\ &\xrightarrow{-R_2} \begin{bmatrix} 3 & 1 & 7 & | & 7 \\ 1 & 0 & 0 & | & -1 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 2 & 1 & 7 & | & 8 \\ 1 & 0 & 0 & | & -1 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 2 & 1 & 7 & | & 8 \\ 1 & 0 & 0 & | & -1 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 1 & 0 & 0 & | & -1 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \\ &\xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ 0 & -3 & -6 & | & -14 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 3 & 6 & | & 14 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \end{aligned}$$

8. Write $\begin{bmatrix} 7 \\ 8 \\ 0 \end{bmatrix}$ as a linear combination of the columns of $\begin{bmatrix} 3 & 1 & 7 \\ 5 & 3 & 4 \\ 2 & -1 & 5 \end{bmatrix}$.

$$\begin{aligned} &4 \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} + 1 \begin{bmatrix} 7 \\ 4 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 12 \\ 20 \\ 8 \end{bmatrix} + \begin{bmatrix} 3 \\ 9 \\ -3 \end{bmatrix} + \begin{bmatrix} 7 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 22 \\ 32 \\ 10 \end{bmatrix} \end{aligned}$$