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MONASH University

Semester One 2019
Examination Period

Faculty of Business and Economics

EXAM CODES: **ETF2700-ETF5970**

TITLE OF PAPER: Mathematics for Business – SAMPLE EXAM

EXAM DURATION: 2 hours writing time

READING TIME: 10 minutes

THIS PAPER IS FOR STUDENTS STUDYING AT: (tick where applicable)

☑ Caulfield ☐ Clayton ☐ Parkville ☐ Peninsula
☐ Monash Extension ☐ Off Campus Learning ☐ Malaysia ☐ Sth Africa
☐ Other (specify)

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SAMPLE EXAM

AUTHORISED MATERIALS

OPEN BOOK ☐ **YES** ☑ **NO**

CALCULATORS ☑ **YES** ☐ **NO**
Only HP 10bII+ or Casio FX82 (any suffix) calculator permitted
SPECIFICALLY PERMITTED ITEMS ☐ **YES** ☑ **NO**
if yes, items permitted are:

Candidates must complete this section if required to write answers within this paper

STUDENT ID: __ __ __ __ __ __ __ __ DESK NUMBER: __ __ __ __ __

Important: There are eights questions. Please attempt all the questions, show all the steps of your calculations, and provide explanations to justify your answers. To obtain full marks, it is important to provide complete answers supported by logically sound explanations, unless the question explicitly states that no explanation is needed. It is not sufficient to simply provide calculator instructions.

Total marks: 60

[This is a sample exam, which is NOT part of the unit assessment. The final examination is worth 60% towards the final mark of this unit].

Formulae are provided at the end of this paper.

Question 1: 3+2+5=10 Marks

The demand and supply functions for two goods X and Y are given below.

$$\begin{aligned} \text{Demand:} \quad & Q_X = 120 - 2P_X + 3P_Y, \quad Q_Y = 150 + 6P_X - 4P_Y \\ \text{Supply:} \quad & Q_X = -240 + 6P_X, \quad Q_Y = -150 + 6P_Y \end{aligned}$$

(a) Determine the matrix $\mathbf{A}$ in the equation of the form

$$\mathbf{A}\begin{bmatrix} P_X \\ P_Y \end{bmatrix} = \begin{bmatrix} 360 \\ 300 \end{bmatrix}$$

which determine market equilibrium.

(b) Show that $\mathbf{A}$ is invertible, by verifying that $\det(A) \neq 0$.

(c) Using $\mathbf{A}^{-1}$, obtain the prices of the two goods at market equilibrium. Show your working and provide answers correct to 2 decimal places.

Question 2: 2+3+3+2=10 Marks

A monopolist faces a demand function $P = 173 - 2Q$, with P denoting the market price and Q denoting the quantity demanded. There is a fixed cost of 200, the total variable cost function is $TVC(Q) = \frac{1}{3}Q^3 - 10Q^2 + 188Q$.

(a) Obtain an expression for the profit function $f(Q)$ defined for $Q > 0$.

(b) Determine the stationary point(s) of the profit function.

(c) Assume the maximal point of the profit function exist. What is the maximal value of the profit function?

(d) Compute the elasticity $El_Q f(Q)$ at point $Q = 10$, to 2 decimal places. Provide a brief interpretation of the computed value of the price elasticity, in the context of this example.

Question 3: 5 Marks

Suppose the market price P can be written as a function of the market demand quantity $Q \in [0, 5)$

$$P(Q) = 100 - 10Q.$$

The current demand quantity is $Q_0 = 2$. Showing all steps of your working, evaluate the consumer surplus in this market

$$\text{CS} = \int_0^{Q_0} P(Q)dQ - P_0 Q_0$$

where Q_0 is the current market demand quantity.

Question 4: 3+1+3+1=8 Marks

Energy company GreenVolt (GV) owns a property at the wind-swept, sunny location of Ocean Heads. GV is evaluating two projects: a wind farm and a solar energy plant. The wind farm requires an initial investment of \$10m, and a \$5m loss is expected for the first year. For the following 3 years, GV expects annual returns of \$8m from electricity sales. The solar plant requires an initial investment of \$15m. GV expects a loss of \$2m for the first year and annual returns of \$9m for the following 3 years.

Assume a discount (interest) rate of 8% compounded annually.

(a) Calculate, in \$m to 3 decimal places, the present value of the two projects.

(b) Based on the present values that you have carried out in (a), explain which one you think is preferable.

(c) Calculate, in percentage to 3 decimal places, the internal rate of return of the two projects.

(d) On the basis of the internal rate of return of the two projects, which project do you think is preferable?

Question 5: 2+2+3+1=8 Marks

Given the Cobb-Douglas production function

$$Q = 100L^{0.3}K, \quad L, K > 0.$$

(a) Write down the equation of the isoquant for $Q = 800$ in the form $K = f(L)$

(b) Show by differentiation that $f(L)$ is convex.

(c) Find the values of L and K, to 2 decimal places, for which the production is maximised under the budget restriction $L + 2K = 30$ using Lagrange method.

(d) If the budget increases by 1 (that is, increases from 30 to 31), compute the resulting change (rounded off to 3 decimal places) in the maximal level of production, using the Lagrange multiplier method.

Question 6: 1+4+1=6 Marks

The following difference equation models the salary scale for part-time staff

$$Y_t = 20 + 1.2Y_{t-1}$$

where Y_t denotes the salary (in dollars) in year $t = 0, 1, 2, \ldots$.

(a) If $Y_0 = 2300$, deduce Y_1, Y_2 and Y_3 directly from the difference equation.

(b) Solve the difference equation. In other words, determine Y_t for all $t = 0, 1, \ldots$.

(c) Calculate the number of year until the salary first exceeds \$15,000.

Question 7: 4+2=6 Marks

Suppose that a firm's capital stock $K(t)$ satisfies the differential equation

$$\frac{dK}{dt} = I - \delta K(t)$$

where investment I is constant, and $\delta K(t)$ denotes depreciation, with δ a positive constant

(a) Find the solution of the equation if the capital stock at time $t = 0$ is K_0.

(b) Let $\delta = 0.05$ and $I = 10$. Explain what happens as $t \to \infty$ when: (i) $K_0 = 150$; (ii) $K_0 = 250$.

Question 8: 2+5=7 Marks

A daily diet mix requires a minimum of: 160 mg of Vitamin K and 1000 mg of Vitamin D. Two foods A and B contain these vitamins:

	Vitamin K	Vitamin D	Cost per 1 kg
Food A	10 mg	100 mg	40
Food B	8 mg	40 mg	20

(a) Write down the inequality constraints for each vitamin. Denote the consumption (in kg) of food A and food B as x and y.

(b) Using extreme point theorem, determine the number of units of food A and B which fulfil the daily requirements at a minimum cost. You may assume the optimal point exist.

END OF EXAMINATION

Formulae are provided in the next two pages.

Formulae

- 'abc' formula for quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \text{ if } b^2 - 4ac \geq 0$$

- Inverse of a 2×2 matrix:

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ (if } ad - bc \neq 0)$$

- Arithmetic sequences $(a, a + d, a + 2d, \ldots)$:

$$\text{sum of the first } n (n \geq 1) \text{ terms is } S_n = \sum_{i=1}^{n} T_i = \frac{n}{2} [2a + (n-1)d]$$

- Geometric sequences $(a, ar, ar^2, \ldots)$:

$$\text{sum of the first } n (n \geq 1) \text{ terms is } S_n = \sum_{i=1}^{n} T_i = \frac{a(1 - r^n)}{1 - r}$$

- Annuities, Loans (m-payments per year)

$$\text{Value at End of Year } n: \quad V_n = P_0 \left(1 + \frac{r}{m}\right)^n + \frac{A_0}{r/m} \left[\left(1 + \frac{r}{m}\right)^n - 1\right]$$

$$\text{Debt Repayments:} \quad A_0 = L \cdot \frac{r/m}{1 - (1 + r/m)^{-n}},$$

$$\text{Net Present Value:} \quad V_0 = A_0 \frac{1 - (1 + r/m)^{-n}}{r/m}$$

- Differentiation Rules

Rule	$f(x)$	$f'(x)$
Power Rule	$x^p, p \neq 0$	px^{p-1}
Constant	K	0
Natural Exponential	e^x	e^x
Exponential	a^x	$a^x \ln(a)$
Logarithm	$\ln(x)$	$1/x$
Product Rule	$u(x) \cdot v(x)$	$u'(x) \cdot v(x) + u(x) \cdot v'(x)$
Quotient Rule	$\frac{u(x)}{v(x)}$	$\frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{(v(x))^2}$
Chain Rule	$u(v(x))$	$u'(v(x)) \cdot v'(x)$

- Integration Rules

Rule	$f(x)$	$\int f(x)dx$
Power Rule	x^p, $p \neq -1$	$\frac{x^{p+1}}{p+1} + C$
One exception to power rule	x^{-1}	$\ln(x) + C$
Integral of a constant	K	$Kx + C$
Natural Exponential	e^x	$e^x + C$

- Integration by substitution

$$\int_a^b f(\varphi(t))\varphi'(t)dt = \int_{\varphi(a)}^{\varphi(b)} f(x)dx$$

- Integration by parts

$$\int u(x) \cdot v'(x)dx = u(x)v(x) - \int v(x) \cdot u'(x)dx$$

- First-order differential equation: for any constant k,

$$\frac{dy}{dx} = ky, \qquad y = Ae^{kx}, A \text{ is real}$$