

# BigProject in Cybersecurity

## Cryptanalysis of ForkAES-\*<sup>-2-2</sup>

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### 1 Attack on ForkAES-\*<sup>-2-2</sup> with Reflection Trails

The attack can work for an arbitrary number of rounds before the state is forked. Our analysis and diagrams refer to 5 rounds before the fork and 2 rounds after that, then the keys are numbered from 0 to 9.

The analysis will make use of the diagram in Figure 1 because only the two rounds after the fork will be considered<sup>1</sup>.

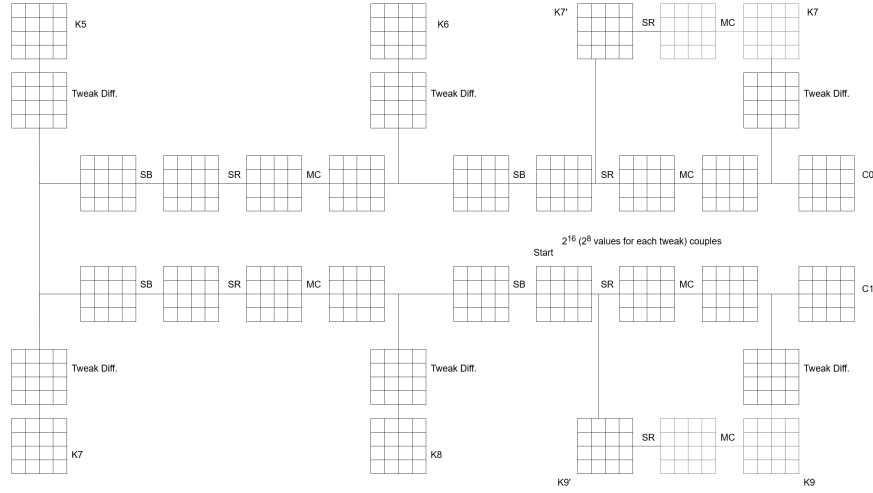


Figure 1: Considered part of the cipher

**Reflection Trails** The analysis will make use of a modified version of the Reflection property which considers the differences between the states of two different encryptions. In particular, we are searching for a differential characteristic which makes possible to find one of the keys involved in the tag-reconstruction operation.

**Proposition: Differential Reflection Trail** If there exists a differential  $F_r$  for the  $r$ -round transformation that propagates a difference  $\Delta I$  to  $\Delta O$  with probability  $p$ , there exists a differential for the  $2r$ -round transformation  $(F^{-1})^r \circ F^r$  that propagates a difference  $\Delta I$  to  $\Delta O$  with probability at least  $p^2$ . This property holds for any choice of round keys and constants in the  $2r$  rounds<sup>2</sup>.

We can apply Reflection trails technique on the tag-reconstruction procedure because it is a back and forward computation of the same round function with different round-keys and constants. The addition of constants and round-keys does not interfere with the property we are going to use.

<sup>1</sup>Notice that the values indicated with the apex in the diagrams are shown with a  $\sim$  in the text

<sup>2</sup>Cryptanalysis of ForkAES, page 6

**The characteristic** Thanks to the mix column operation, a single active byte propagates to 4 in the next round and to 16 in the second one ( $1 \xrightarrow{F} 4 \xrightarrow{F} 16 \xrightarrow{F} 16$ ). Tweak injection helps us to get better trails. Notice that, if we decide a specific tweak difference, we can find a couple of states whos difference from the fork is  $1 \xrightarrow{F} 0 \xrightarrow{F} 1 \xrightarrow{F} 4$ . That means that whilst the common diffusion is reduced, the probability of guessing the key increases.

**The reflection trail** From the above differential characteristic, we can build a reflection trail which enables the attacker to break the cipher efficiently. The trail is:  $4 \xrightarrow{F^{-1}} 1 \xrightarrow{F^{-1}} 0 \xrightarrow{F^{-1}} 1 \xrightarrow{F} 0 \xrightarrow{F} 1 \xrightarrow{F} 4$ .

**Attack** The probability that the one byte difference in  $\tilde{C}_1$  becomes equal to the tweak difference after the inverse sub bytes operation in order to make it disappear is  $2^8$ . This means that for each byte of the key we can obtain a very little number of possibilities (1-2) by only considering  $2^8 \times 2^8$  couples of ciphertexts  $C_0$ .

Such trail does not work with 3 or 4 rounds after the fork because it will imply that the states at the fifth round should be equal, which is slightly impossible.

**Attack Procedure for one key byte** With the following procedure we are going to find one byte of  $\tilde{K}_7$ .

- 1) Choose tweaks  $T_0$  and  $T_1$  with the chosen fixed difference  $\Delta T$ . This means that you will end up with  $2^8$  couples  $(T_0, T_1)$ .
- 2) For each tweaks couple:
  - For each tweak  $T$  in the couple:
    - \* take the  $2^8$  distinct values of  $\tilde{C}_1$  for the same byte number in which the tweak difference is considered. Meaning that, if the tweak difference is at byte 0, we will take all the possible values of the byte 0 of  $\tilde{C}_1$ . Fix the other 15 bytes to constant values and compute the corresponding  $C_1$
    - \* You will obtain  $2^8$  values of  $C_1$ . Query each of those values to the tag-reconstruction procedure in order to obtain the corresponding  $C_0$  values. For each of those  $2^8$   $C_0$  values, compute the corresponding  $\tilde{C}_0$ .
  - We obtain  $2^8$  values computed with  $T_0$  and  $2^8$  values computed with  $T_1$ .
  - From the  $2^{16}$  couples between different tweaks, pick the one with 15 inactive bytes in the byte positions different from the one of the tweak difference. If such couple is not found, skip to the next tweak couple, otherwise pick such couple and go to step 3.
- 3) We expect at least one right pair. Initialize a counter for each possible byte value of the key we are trying to find. For each possible byte value of  $\tilde{K}_7$ :
  - For each value in the couple:
    - \* add that key byte, add the corresponding tweak ( $T_0$  for the first value in the couple and  $T_1$  for the second one), apply inverse mix-column and inverse shift-row.

- If there is no difference between the two computed states increase the counter of the tried key byte value.
- 4) Consider only the values whos counter value is equal to the maximum. We expect at most 2 values with a probability of the 95,4% (computed empirically).

**Full Attack** Iterate the procedure above for each key byte (the figures in Appedix show the trails for the key bytes of  $\tilde{K}_7$  column 0). After having computed the possibilities for each key byte, compute all the possible keys  $\tilde{K}_7$  and test them progressively. Once the possibilities are reduced to one key candidate, compute the corresponding  $K_7$ . From the computed  $K_7$ , generate all the other round keys by applying the key-scheduling and the inverse-key-scheduling algorithm.

### Complexity

- **Time:**
  - Encryptions: we need to distinguish two cases
    - \* If we validate the keys by checking differential characteristic (as done in the provided code), the number of encryptions is 1.
    - \* If we validate the keys by exhaustive searching the right key from all the found possibilities, the number of encryptions is  $\simeq 2^{16}$ . Rarely this number could be slightly greater.
  - Memory Accesses:  $2^{13}$  if the validation is done with exhaustive search,  $2^{14}$  with the other method.
- **Data:** the attack requires  $16 \times (2^8 + 2^8) = 2^{13}$  tag-reconstruction queries if the validation is performed with the exhaustive search technique. Otherwise, the attack requires  $2^{14}$  queries.
- **Memory:** we need to store  $2^9$  AES states, meaning the  $2^9$  values of  $\tilde{C}_0$ .

Here a brief summary:

Validation Technique Applied	Encr	Mem. Acc.	Data	Store (Memory)
Exhaustive Search	$2^{16}$	$2^{13}$	$2^{13}$	$2^9$
Checking with differential characteristic	1	$2^{14}$	$2^{14}$	$2^9$

**Possible Countermeasures** What it makes the attack easier to apply is the injection of the tweak, which is never modified during the process. That characteristic helps us to reduced the complexity of the differential trail. Then, a possible countermeasure could be to modify the tweak during the process in such a way that the tweaks applied in the two branches are always different. In particular, a permutation of the bytes could be applied. In our example, 5+2 rounds ForkAES, we would need to generate tweaks from  $T_0$  to  $T_9$  as done with the round keys. The application of the round tweaks should be done by adding the round tweak  $i$  to the round  $i$  state and so on.

A possible simple implementation can be found on the repository. In particular, the file `utilities.c` contains a procedure called `TweakExpansion` which permutes the value within the initial tweak. The countermeasure should work also for 3, 4 and 5 rounds after the fork. What is important is to implement a scheduling function for the tweak which is able to avoid cycles after the fork in order to prevent the exploitation of similarities between the two branches.

## 2 Appendix

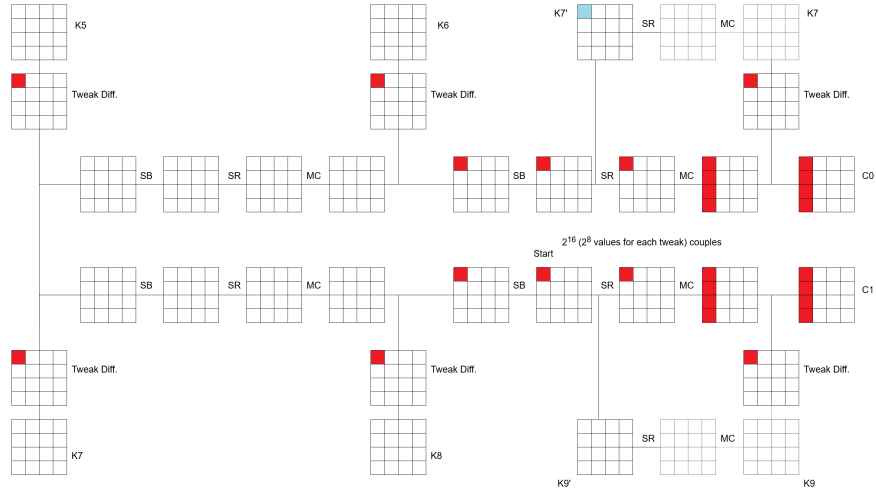


Figure 2: Differential Reflection Trail: Byte 0

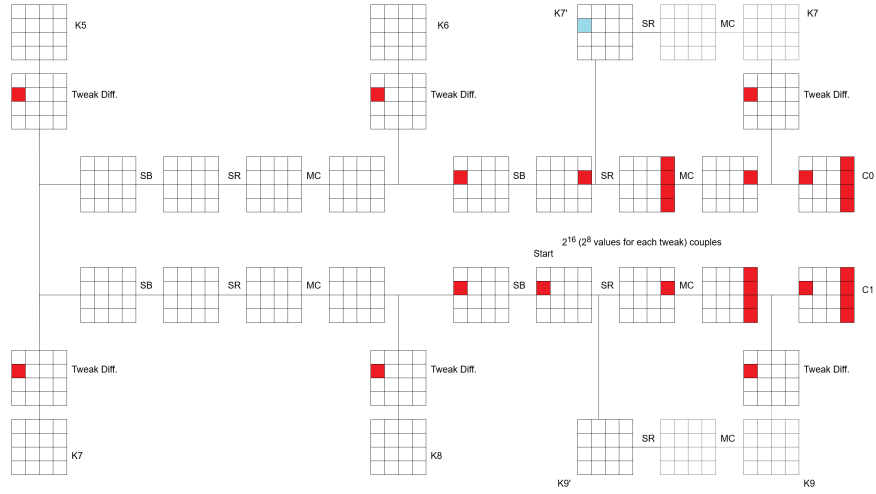


Figure 3: Differential Reflection Trail: Byte 1

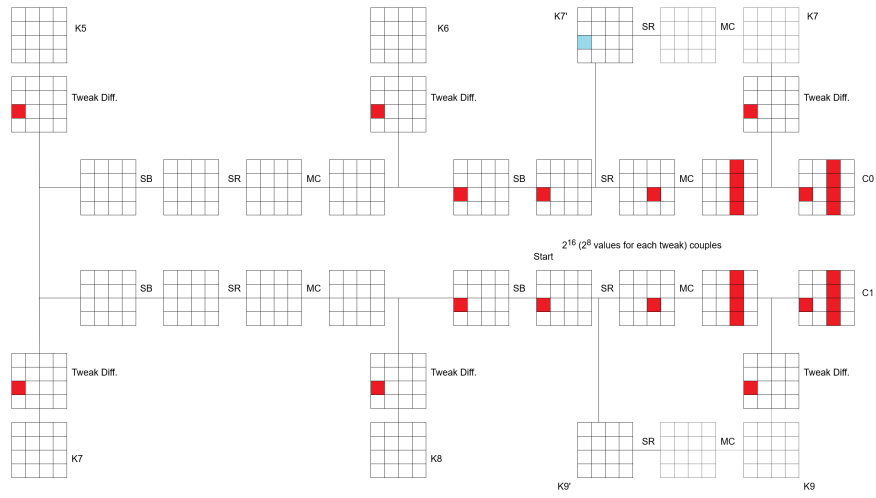


Figure 4: Differential Reflection Trail: Byte 2

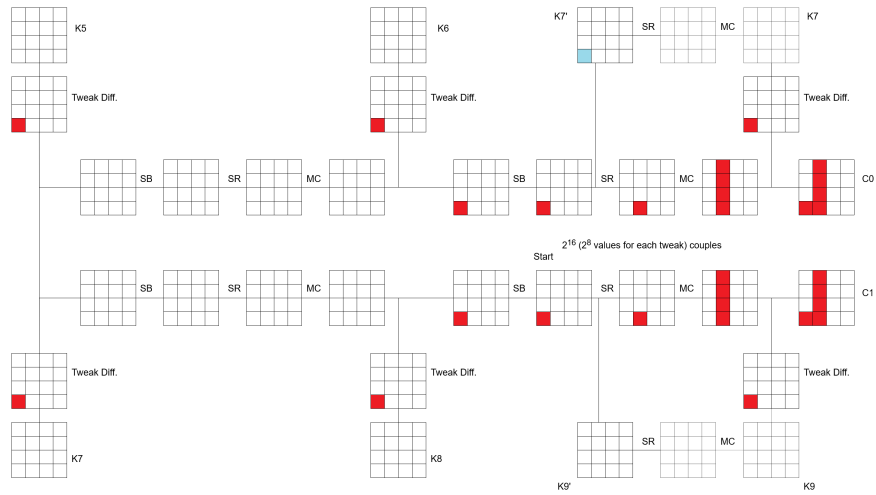


Figure 5: Differential Reflection Trail: Byte 3