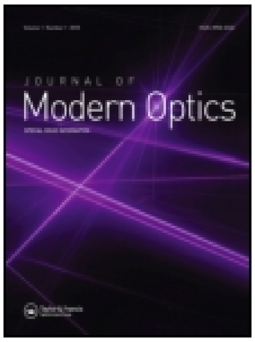

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Depolarization and Polarization Indices of an Optical System

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Depolarization and polarization indices of an optical system

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Abstract. The depolarization index and the direct and reciprocal polarization indices may be derived from the Mueller matrix of any optical system. These indices give direct information about the depolarizing and polarizing power of the optical system.

1. Introduction

The use of the Stokes–Mueller formalism allows a complete study of the polarization properties of any optical system [1]. From the Mueller matrix of an optical system, we derive parameters such as the depolarization index and the direct and reciprocal polarization indices.

The depolarization index P_D gives an average measure of the depolarizing power of the optical system. $P_D = 1$ corresponds to a non-depolarizing optical system, while $P_D = 0$ corresponds to a total depolarizer. The direct polarization index P_A gives a measure of the polarizing power of an optical system for incident unpolarized light in a given direction, hereafter called 'direction A'; conversely, the reciprocal polarization index P_B gives a measure of the polarizing power of that optical system for incident unpolarized light in the opposite direction, hereafter called 'direction B'.

Let us consider a Stokes vector $\mathbf{S}$ with elements S_i $(i = 0, 1, 2, 3)$. We can define a positive-semidefinite quadratic form F associated with $\mathbf{S}$ as

$$F = S_0^2 - S_1^2 - S_2^2 - S_3^2. \tag{1}$$

So, the degree of polarization P given by

$$P = \left(\frac{S_1^2 + S_2^2 + S_3^2}{S_0^2}\right)^{1/2}, \tag{2}$$

can also be written as

$$P = \left(1 - \frac{F}{S_0^2}\right)^{1/2}. \tag{3}$$

In the same way, the indices P_D, P_A and P_B will be defined from corresponding quadratic forms.

2. Depolarization index

All the polarization properties of a given optical system $\mathcal{O}$ can be described by a Mueller matrix $M_{\rm A}$ with elements m_{ij} $(i, j = 0, 1, 2, 3)$. So, for an incident light beam of Stokes vector $\mathbf{S}$, the Stokes vector $\mathbf{S}'$ of the outgoing light beam is given by

$$\mathbf{S}' = M_{\rm A}\mathbf{S}. \tag{4}$$

Then, if the directions of the incident and outgoing light beams are exchanged or reversed, in the absence of magnetic fields (the Faraday effect) the Mueller matrix $M_{\rm B}$ which describes $\mathcal{O}$ in this case is related to $M_{\rm A}$ as [2]

$$M_{\rm B} = QM_{\rm A}^{\rm T}Q, \tag{5}$$

where $M_{\rm A}^{\rm T}$ is the transpose of $M_{\rm A}$, and Q is a diagonal matrix with elements $(1, 1, 1, -1)$.

Hereafter we will refer to $M_{\rm A}$ and $M_{\rm B}$ respectively as the forward and reverse Mueller matrices of $\mathcal{O}$ (these names are totally arbitrary and have no significance).

Let us consider the set of Stokes vectors $\mathbf{S}_{{\rm p}i}$ and $\mathbf{S}_{{\rm n}i}$ $(i = 1, 2, 3)$ given by

$$\mathbf{S}_{\rm p1} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \mathbf{S}_{\rm p2} = \begin{pmatrix}1\\0\\1\\0\end{pmatrix}, \mathbf{S}_{\rm p3} = \begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \mathbf{S}_{\rm n1} = \begin{pmatrix}1\\-1\\0\\0\end{pmatrix}, \mathbf{S}_{\rm n2} = \begin{pmatrix}1\\0\\-1\\0\end{pmatrix}, \mathbf{S}_{\rm n3} = \begin{pmatrix}1\\0\\0\\-1\end{pmatrix}. \tag{6}$$

The Stokes vectors $\mathbf{S}_{{\rm p}i}$ and $\mathbf{S}_{{\rm n}i}$ correspond to different totally polarized light beams, because their corresponding quadratic forms are

$$F_{{\rm p}i} = 0, \qquad F_{{\rm n}i} = 0, \qquad i = 1, 2, 3. \tag{7}$$

First we consider the direction A for the incident light beams. To the Stokes vectors given by

$$\mathbf{S}'_{{\rm p}i} = M_{\rm A}\mathbf{S}_{{\rm p}i}, \tag{8 a}$$

$$\mathbf{S}'_{{\rm n}i} = M_{\rm A}\mathbf{S}_{{\rm n}i}, \tag{8 b}$$

there correspond the positive-semidefinite quadratic forms

$$F'_{{\rm p}i} = m_{00}^2 + m_{0i}^2 + 2m_{00}m_{0i} - \sum_{j=0}^{3} (m_{j0}^2 + m_{ji}^2 + 2m_{j0}m_{ji}), \tag{9 a}$$

$$F'_{{\rm n}i} = m_{00}^2 + m_{0i}^2 - 2m_{00}m_{0i} - \sum_{j=0}^{3} (m_{j0}^2 + m_{ji}^2 - 2m_{j0}m_{ji}), \tag{9 b}$$

for $i = 1, 2, 3$. The quadratic form $F'_{\rm D}$ given by the average of the above quadratic forms is

$$F'_{\rm D} = \frac{1}{6}\sum_{i=1}^{3} (F'_{{\rm p}i} + F'_{{\rm n}i}). \tag{10}$$

Then, from equation (9)

$$F'_{\rm D} = m_{00}^2 + \frac{1}{3}\sum_{i=1}^{3} m_{0i}^2 - \sum_{i=1}^{3} m_{i0}^2 - \frac{1}{3}\sum_{i,j=1}^{3} m_{ij}^2. \tag{11}$$

This parameter $F'_{\rm D}$ represents the mean value of the differences between the square of the total intensity of the output light, and the square of the intensity of that part of the outgoing light which is totally polarized for incident light of Stokes vectors $\mathbf{S}_{pi}$ and $\mathbf{S}_{ni}$.

Taking into account the inequalities

$$F'_{pi} \geq 0, \quad F'_{ni} \geq 0, \quad i = 1, 2, 3, \tag{12 a}$$

$$F'_{D} \geq 0, \tag{12 b}$$

we can write

$$F'_{D} = 0 \Leftrightarrow F'_{pi} = F'_{ni} = 0, \qquad i = 1, 2, 3. \tag{13}$$

This result shows that $F'_D = 0$ is a necessary and sufficient condition, so that $\mathcal{O}$ does not depolarize incident light beams of Stokes vectors $\mathbf{S}_{pi}$ and $\mathbf{S}_{ni}$. However, this statement does not mean that totally polarized light beams of Stokes vectors other than $\mathbf{S}_{pi}$ and $\mathbf{S}_{ni}$ may not be depolarized by $\mathcal{O}$.

Now, considering the direction B, we can see that the quadratic forms associated with the Stokes vectors

$$\mathbf{S}''_{pi} = M_B \mathbf{S}_{pi}, \tag{14 a}$$

$$\mathbf{S}''_{ni} = M_B \mathbf{S}_{ni}, \tag{14 b}$$

are respectively

$$F'_{pi} = m_{00}^2 + m_{i0}^2 + 2m_{00}m_{i0} - \sum_{j=0}^{3} (m_{0j}^2 + m_{ij}^2 + 2m_{0j}m_{ij}), \tag{15 a}$$

$$F''_{ni} = m_{00}^2 + m_{i0}^2 - 2m_{00}m_{i0} - \sum_{j=0}^{3} (m_{0j}^2 + m_{ij}^2 - 2m_{0j}m_{ij}), \tag{15 b}$$

for $i = 1, 2, 3$. In the same way that we defined F'_D, we can define the quadratic form F''_D as

$$F''_D = \frac{1}{6} \sum_{i=i}^{3} (F''_{pi} + F''_{ni}). \tag{16}$$

Then, from equations (15) and (16)

$$F''_D = m_{00}^2 + \frac{1}{3} \sum_{i=1}^{3} m_{i0}^2 - \sum_{i=1}^{3} m_{0i}^2 - \frac{1}{3} \sum_{i,j=1}^{3} m_{ij}^2. \tag{17}$$

From the averages F'_D and F''_D, we can derive a new positive-semidefinite quadratic form F_D:

$$F_D = \frac{1}{2}(F'_D + F''_D). \tag{18}$$

Then, from equations (11) and (17)

$$F_D = \frac{1}{3}\left[4m_{00}^2 - \sum_{i,j=0}^{3} m_{ij}^2\right] \tag{19}$$

or

$$F_D = \frac{1}{3}[4m_{00}^2 - \Gamma^2(M_A)] = \frac{1}{3}[4m_{00}^2 - \Gamma^2(M_B)], \tag{20}$$

where

$$\Gamma(M_{\mathrm{A}}) = \Gamma(M_{\mathrm{B}}) = [\mathrm{tr}\,(M_{\mathrm{A}}^{\mathrm{T}}M_{\mathrm{A}})]^{1/2} = \left[\sum_{i,j=0}^{3} m_{ij}^2\right]^{1/2}, \tag{21}$$

is a positive-semidefinite norm associated with M_{A} (or M_{B}) [3].

Equation (20) can also be written as

$$\Gamma^2(M_{\mathrm{A}}) = 4m_{00}^2 - 3F_{\mathrm{D}}. \tag{22}$$

Since m_{00} is the transmittance of $\mathcal{O}$ for unpolarized light, and F_{D} is the average given by equation (18), equation (22) provides an interpretation of the norm $\Gamma(M_{\mathrm{A}})$.

Taking into account that $\Gamma(M_{\mathrm{A}}) = 2m_{00}$ is a necessary and sufficient condition for M_{A} (or M_{B}) to be non-depolarizing [3], from equation (22) we deduce that this condition is equivalent to $F_{\mathrm{D}}(M_{\mathrm{A}}) = 0$. The parameter $F_{\mathrm{D}}(M_{\mathrm{A}})$ will be called the 'depolarization factor' of $\mathcal{O}$, because it gives an average measure of the depolarization produced by $\mathcal{O}$.

In the same way that equation (3) gives a relation between P and F for a light beam, we can define the depolarization index P_{D} of $\mathcal{O}$ as

$$P_{\mathrm{D}} = \left(\frac{\Gamma^2(M_{\mathrm{A}}) - m_{00}^2}{3m_{00}^2}\right)^{1/2}, \tag{23}$$

or

$$P_{\mathrm{D}} = \left(1 - \frac{F_{\mathrm{D}}}{m_{00}^2}\right)^{1/2}. \tag{24}$$

From equations (19) and (20) we can see that the values of F_{D} and P_{D} are restricted by the limits

$$0 \leqslant F_{\mathrm{D}} \leqslant m_{00}^2, \tag{25}$$

$$0 \leqslant P_{\mathrm{D}} \leqslant 1, \tag{26}$$

where the values $F_{\mathrm{D}} = 0$ and $P_{\mathrm{D}} = 1$ correspond to non-depolarizing optical media, and $F_{\mathrm{D}} = m_{00}^2$ and $P_{\mathrm{D}} = 0$ correspond to a total depolarizer.

3. Polarization indices

Let us consider an optical system $\mathcal{O}$, and an incident unpolarized light beam in direction A. The Stokes vector $\mathbf{S}'$ of the output light beam is given by

$$\mathbf{S}' = M_{\mathrm{A}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}m_{00}\\m_{10}\\m_{20}\\m_{30}\end{pmatrix}. \tag{27}$$

Now, we can define the 'direct polarization factor' of $\mathcal{O}$ as the quadratic form F_{A} associated with $\mathbf{S}'$. In the same way, we can define the 'reciprocal polarization factor' of $\mathcal{O}$ as the quadratic form F_{B} associated with the Stokes vector

$$\mathbf{S}'' = M_{\mathrm{B}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}m_{00}\\m_{01}\\m_{02}\\-m_{03}\end{pmatrix}. \tag{28}$$

From $\mathbf{S}'$ and $\mathbf{S}''$ we can also derive the 'direct polarization index' P_{A} and the 'reciprocal polarization index' P_{B} as

$$P_{\mathrm{A}} = \left(\frac{m_{10}^2+m_{20}^2+m_{30}^2}{m_{00}^2}\right)^{1/2} = \left(1-\frac{F_{\mathrm{A}}}{m_{00}^2}\right)^{1/2}, \tag{29}$$

$$P_{\mathrm{B}} = \left(\frac{m_{01}^2+m_{02}^2+m_{03}^2}{m_{00}^2}\right)^{1/2} = \left(1-\frac{F_{\mathrm{B}}}{m_{00}^2}\right)^{1/2}. \tag{30}$$

The parameters F_{A} and F_{B} (or P_{A} and P_{B}) give information on the polarizing power of $\mathcal{O}$ for incident light in each direction.

The possible values of these parameters are restricted by the limits

$$0 \leqslant F_{\mathrm{A}} \leqslant m_{00}^2, \quad 0 \leqslant F_{\mathrm{B}} \leqslant m_{00}^2, \tag{31}$$

$$0 \leqslant P_{\mathrm{A}} \leqslant 1, \quad 0 \leqslant P_{\mathrm{B}} \leqslant 1. \tag{32}$$

Note that $P_{\mathrm{A}} = 1$ (or $F_{\mathrm{A}} = 0$) is not a sufficient condition for $\mathcal{O}$ to produce the total polarization of any incident light beam in direction A; the same applies to direction B.

Any non-depolarizing optical system satisfies the equalities

$$F_{\mathrm{A}} = F_{\mathrm{B}}, \tag{33 a}$$

$$P_{\mathrm{A}} = P_{\mathrm{B}}. \tag{33 b}$$

Therefore, in this case, a necessary and sufficient condition for $\mathcal{O}$ to be a total polarizer is

$$F_{\mathrm{A}} = F_{\mathrm{B}} = 0, \tag{34 a}$$

$$P_{\mathrm{A}} = P_{\mathrm{B}} = 1. \tag{34 b}$$

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