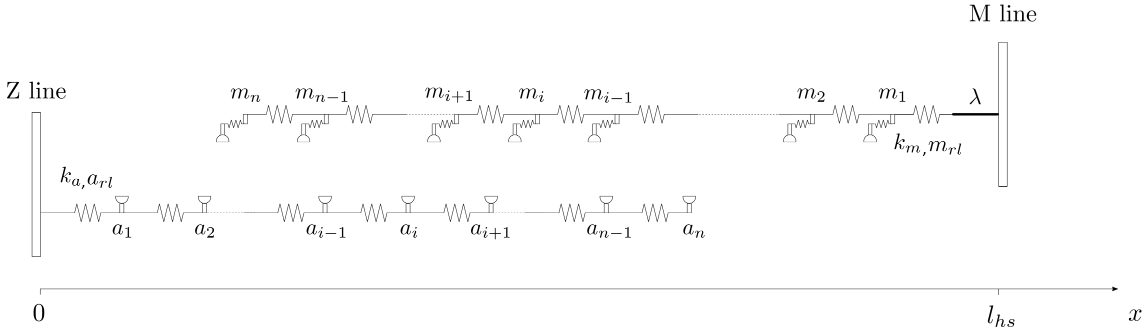
Force-balance in FiberSim

1 Compliant myofilaments

FiberSim is a spatially-explicit model of a half-sarcomere. It describes a compliant network of protein filaments, which is detailed below.



Geometry and mechanical arrangement of the thick and thin filament in FiberSim.

1.1 Actin filaments

Thin filaments are composed of nodes joined by linear springs of stiffness k_a with a resting length a_{rl}. If the position of the i^{th} node along an x-axis is noted a_i, then the force-balance equations for the thin filament can be written as:

$$\begin{aligned} 2\,k_a\,a_1 - k_a\,a_2 &= 0 \\ -k_a\,a_{i-1} + 2\,k_a\,a_i - k_a\,a_{i+1} &= 0 \ \text{(for } 1 < i < n) \\ -k_a\,a_{n-1} + k_a\,a_n &= k_a\,a_{rl} \end{aligned} \tag{1}$$

1.2 Myosin filaments

Thick filaments are composed of nodes joined by linear springs of stiffness k_m with a resting length m_{rl}. A rigid link of length λ connects the thick filament to the M-line. If the position of the i^{th} node along an x-axis is noted m_i, and the half-sarcomere length is noted l_{hs}, then the force-balance equations for the thick filament can be written as:

$$\begin{gathered} 2\,k_m\,m_1 - k_m\,m_2 = k_m\,(l_{hs} - \lambda) \\ -k_m\,m_{i-1} + 2\,k_m\,m_i - k_m\,m_{i+1} = 0 \;\; (\text{for } 1 < i < n) \\ k_m\,m_{n-1} - k_m\,m_n = k_m\,m_{rl} \end{gathered} \tag{2}$$

The system of equations (1)-(2) can be written in matrix form:

$$Kx = F \tag{3}$$

where K is a matrix containing the springs stiffness, x is a vector containing the positions of the actin and myosin nodes (a_i and m_i, respectively) and F is a vector containing the constant terms (independent of nodes positions). K is a tridiagonal matrix:

$$K = \begin{bmatrix} \# & \# & & 0 \\ \# & \ddots & \ddots & \\ & \ddots & \ddots & \# \\ 0 & & \# & \# \end{bmatrix}$$

and numerical methods exist to solve Eq.(3) for x.

Crossbridges can bind during contraction, and thus induce crossbridge links between actin and myosin filaments. This will bring new force contributions to Eqs.(1)-(2), and "non-tridiagonal" elements inside K. This is described in the next section.

2 Crossbridge links

Myosin heads located at the thick filament nodes can attach to neighboring binding sites at the thin filament nodes, thus affecting the filament lattice framework.
A crossbridge located at the j^{th} thick filament node which attaches to the i^{th} node of the thin filament generates a force f_{cb} given by:

$$f_{cb} = k_{cb}\,(m_j - a_i + x_{ps}) \tag{4}$$

where k_{cb} is the crossbridge spring stiffness and x_{ps} is the crossbridge extension when deploying the power stroke.

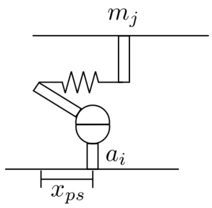


Figure 2: Schematic representation of a crossbridge link.

This additional force on the filaments should be added to the force-balance equations (1)-(2):

$$\begin{aligned} -k_a\, a_{i-1} + 2\, k_a\, a_i - k_a\, a_{i+1} - k_{cb}\, a_i + k_{cb}\, m_j &= f_{cb}\, x_{ps} && (5) \\ -k_m\, m_{j-1} + 2\, k_m\, m_j - k_m\, m_{j+1} + k_{cb}\, a_i - k_{cb}\, m_j &= -f_{cb}\, x_{ps} && (6) \end{aligned}$$

The terms in red will add non-tridiagonal, opposite elements to the K matrix:

$$K = \begin{bmatrix} \# & \# & \# & 0 \\ \# & \ddots & \ddots & \\ -\# & \ddots & \ddots & \# \\ 0 & & \# & \# \end{bmatrix}$$

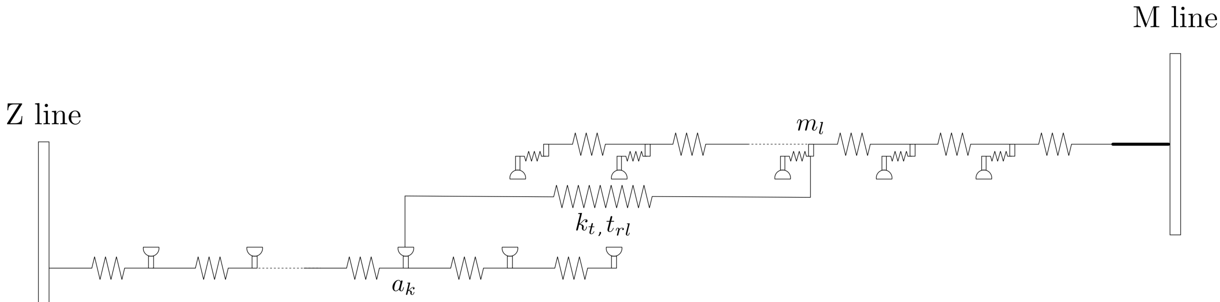
while the blue terms will contribute to the F vector.
Crossbridge linking toughen the numerical solving of Eq.(3), which notably requires an iterative procedure to find a solution x that satisfies a certain precision.

3 Titin

Titin is responsible for the passive force developing within the half-sarcomere when it is streched, and for the recoil force when it is shortened. In the model, it is assumed that titin is a linear spring of stiffness k_t and rest length t_{rl}. This spring is attached at both ends, on a particular thick and thin filament node respectively. Similar to crossbridge links, titin adds some force contribution f_t to Eqs.(1)-(2):

$$f_t = k_t\,(m_l - a_k + t_{rl}) \tag{7}$$

4 Myosin-binding Protein C



Z line
M line
m_l
k_t, t_{rl}
a_k