Instructions

- The homework is due on **Friday 2/22 at 5pm ET**.
- There are 2 problems in total.
- No extension will be provided, unless for serious documented reasons.
- **Start early!**
- Study the material taught in class, and feel free to do so in small groups, but the solutions should be a product of your own work.
- This is not a multiple choice homework; reasoning, and mathematical proofs are required before giving your final answer.

1 Short proofs [50 points]

(a) [10 pts] Given a square matrix $A^{n\times n}$ and a constant κ we define the shifted matrix $A-\kappa I$ where $I^{n\times n}$ is the identity matrix. Prove that if λ is an eigenvalue of A, then $\lambda-\kappa$ is an eigenvalue of $A-\kappa I$.

(b) [10 pts] Consider the (undirected) clique graph K_n on n nodes, where every two distinct vertices in the clique are adjacent. Compute analytically the eigenvalues of the adjacency matrix.

Hint: Shift the adjacency matrix with an appropriate value κ to obtain a matrix who eigenvalues are easy to calculate, and then shift back to the original matrix using (a).

(c) [10 pts] Consider a random walk on a connected undirected graph with n nodes and m edges. Prove that the stationary distribution $\pi=(\pi_1,\ldots,\pi_n)$ satisfies $\pi_j=\frac{deg(j)}{2m}$ for all nodes $j\in[n]$.

(d) [10 pts] Can a Markov chain have infinite stationary distributions? Explain your answer.

(e) [10 pts] What is the expected number of coin tosses to obtain heads-tails-heads (HTH) consecutively using a fair coin?

2 HITS algorithm [50 points]

For the programming assignment see the Jupyter notebook in our Github page `https://github.com/tsourolampis/cs365-spring22` under the HW directory.