

## Instructions

- The homework is due on Friday 3/18 at 5pm ET.
- There are 3 problems in total.
- No extension will be provided, unless for serious documented reasons.
- **Start early!**
- Study the material taught in class, and feel free to do so in small groups, but the solutions should be a product of your own work.
- This is not a multiple choice homework; reasoning, and mathematical proofs are required before giving your final answer.

### 1 Reservoir Sampling [30 Points]

Design an algorithm that samples  $k \geq 1$  elements uniformly at random from an insert-only stream, whose length is unknown. Prove the correctness of your claim.

### 2 Variance of Morris Counter [35 points]

Prove equation  $\text{Var}(Z) = \frac{m(m-1)}{2}$  on slide 27.

### 3 A useful technique [35 points]

Prove the claim on slide 16. Be specific about the values of the constants  $C_1, C_2$  you use in your proof, where  $t = C_1 \log \frac{1}{\delta}$ ,  $k = C_2 \frac{\text{Var}[X]}{\epsilon^2 \mathbb{E}[X]^2}$ .