Instructions

- The homework is due on **Friday 2/18 at 5pm ET**.
- There are 3 problems in total.
- No extension will be provided, unless for serious documented reasons.
- **Start early!**
- Study the material taught in class, and feel free to do so in small groups, but the solutions should be a product of your own work.
- This is not a multiple choice homework; reasoning, and mathematical proofs are required before giving your final answer.

1 Short exercises [15 points]

(a) [5 pts] Prove that $1 + x \leq e^x$ for all real x. For what range of x is $1 + x \approx e^x$ within 0.01.

(b) [2.5pts] Compute for $n = 1, \ldots, 10$, $n!$ exactly, and using Stirling's approximation formula $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n$. What do you observe? Present your findings as a 2-column table.

(c) [7.5pts] Give an example of a random variable for which Chebyshev's inequality is tight, namely the inequality holds as equality.

2 To Handshake or Not?[25 points]

Suppose n people walk into a party. Due to covid-19, each pair $\{i, j\}$ shakes hands with probability only $\frac{1}{10}$. Let $1 > \epsilon > 0$ be a positive number. Prove that *almost surely* **every** person from that party shook hands in the range $[(1-\epsilon)\frac{n}{10}, (1+\epsilon)\frac{n}{10}]$.

3 Bayes hits again [60 points]

This exercise is given in the form of Jupyter notebook in our Github page `https://github.com/tsourolampis/cs365-spring22` under the HW directory.