Instructions

- The homework is due on **Friday 4/1 at 5pm ET**.
- There are 3 problems. The last problem is on Git, and it is a programming assignment.
- No extension will be provided, unless for serious documented reasons.
- **Start early!**
- Study the material taught in class, and feel free to do so in small groups, but the solutions should be a product of your own work.
- This is not a multiple choice homework; reasoning, and mathematical proofs are required before giving your final answer.

1 Projections [30 points]

Let $\pi_b(x)$ be the orthogonal projection of $x \in \mathbb{R}^n$ on the subspace $U = span(b)$ where $b \in \mathbb{R}^n$.

(a) Prove that $\pi_b(x)$ is the closest vector to x on U.

(b) Prove that the Euclidean length of $\pi_b(x)$ is less than or equal to that of x.

(c) Can two orthogonal vectors be linearly dependent? Give an answer with a proof.

(d) Transform the basis $B = \{v_1 = (4,2), v_2 = (1,2)\}$ of $\mathbb{R}^2$ into an orthonormal basis whose first basis vector is in the span of v_1.

2 A special matrix-vector multiplication [20 points]

Let $u \in \mathbb{R}^n$ be a fixed vector. Let $U = uu^T$. Show that maximizing $x^T U(\vec{1} - x)$ over all binary vectors $x \in \{0,1\}^n$ is equivalent to partitioning the coordinates of u into two subsets wheret he sum of the elements in both subsets are as equal as possible. Here 1 represent the all-ones vectors $\vec{1} = \underbrace{[1, 1, ..., 1]^T}_{n \text{ coordinates}}$.

3 F_0, F_1 estimation : Programming Assignment [50 points]

Please see our Github repo for this problem.