Energy System Modelling
Karlsruhe Institute of Technology
Institute for Automation and Applied Informatics
Summer Term 2020

Tutorial IV: Electricity Markets

Will be worked on in the exercise session on Thursday, 25 June 2020.

Problem IV.1 (analytical) – shadow prices

Suppose that the utility for the electricity consumption of an industrial company is given by

$$U(d) = 70d - 3d^2 \ [€/\text{MW h}] \quad , \quad d \in [d_{min}, d_{max}] = [2, 10],$$

where d is the demand in MW and d_{min}, d_{max} are the minimum and maximum demand.

Assume that the company is maximising its net surplus for a given electricity price π, i.e. it maximises $\max_d [U(d) - \pi d]$.

(a) If the price is $\pi = 5$ €/MWh, what is the optimal demand d^*? What is the value of the KKT multiplier μ_{max} for the constraint $d \leq d_{max} = 10$ at this optimal solution? What is the value of μ_{min} for $d \geq d_{min} = 2$?

(b) Suppose now the electricity price is $\pi = 60$ €/MWh. What are the optimal demand d^*, μ_{max} and μ_{min} now?

Problem IV.2 (analytical) – Economic dispatch in a single bidding zone

Consider an electricity market with two generator types, one with the cost function $C_1(g_1) = c_1 g_1$ with variable cost $c_1 = 20$ €/MWh, capacity $G_1 = 300$ MW and a dispatch rate of g_1 [MW], and another with the cost function $C_2(g_2) = c_2 g_2$ with variable cost $c_2 = 50$ €/MWh, capacity $G_2 = 400$ MW and a dispatch rate of g_2 [MW]. The demand-side has utility function $U(d) = 8000d - 5d^2$ [€/h] for a consumption rate of d [MW].

(a) What are the objective function and constraints required for an optimisation problem to maximise short-run social welfare in this market?

(b) Write down the Karush-Kuhn-Tucker (KKT) conditions for this problem.

(c) Determine the optimal rate of production of the generators and the value of all KKT multipliers. What is the interpretation of the respective KKT multipliers?

PROBLEM IV.3 (ANALYTICAL) – EFFICIENT DISPATCH IN A TWO-BUS POWER SYSTEM

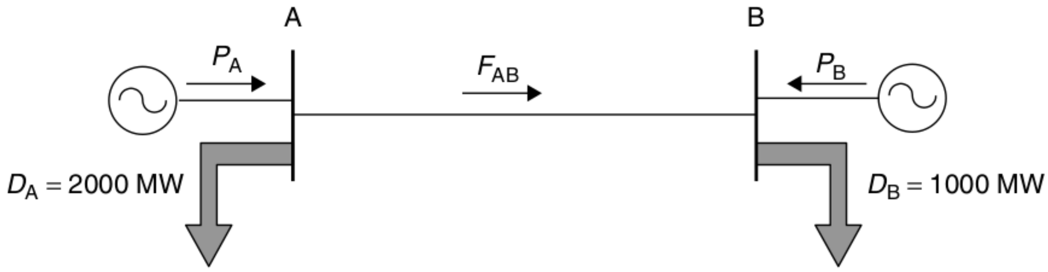

Figure 1: A simple two-bus power system.

Consider the two-bus power system shown in Figure 1, where the two nodes represent two markets, each with different total demand D_i, and one generator at each node producing P_i. At node A the demand is $D_A = 2000$MW, whereas at node B the demand is $D_B = 1000$MW. Furthermore, there is a transmission line with a capacity denoted by F_{AB}. The marginal cost of production of the generators connected to buses A and B are given respectively by the following expressions:

$$
\begin{aligned}
MC_A &= 20 + 0.03P_A \qquad \text{€ /MW h} \\
MC_B &= 15 + 0.02P_B \qquad \text{€ /MW h}
\end{aligned}
$$

Assume that the demands D_A and D_B are constant and insensitive to price, that energy is sold at its marginal cost of production and that there are no limits on the output of the generators.

(a) Calculate the price of electricity at each bus, the production of each generator, and the flow on the line for the following cases. You may also calculate the values of any KKT multiplier as a bonus.

   (i) The line between buses A and B is disconnected.

   (ii) The line between buses A and B is in service and has an unlimited capacity.

   (iii) The line between buses A and B is in service and has an unlimited capacity, but the maximum output of Generator B is 1500 MW.

   (iv) The line between buses A and B is in service and has an unlimited capacity, but the maximum output of Generator A is 900 MW. The output of Generator B is unlimited.

   (v) The line between buses A and B is in service but its capacity is limited to 600 MW. The output of the generators is unlimited.

(b) Calculate the generator revenues, generator profits, consumer payments and consumer net surplus for all the cases considered in the above problem. Who benefits from the line connecting these two buses?

(c) Calculate the congestion surplus for case (v). For what values of the flow on the line between buses A and B is the congestion surplus equal to zero?