

## TUTORIAL I: TIME SERIES ANALYSIS

Will be worked on in the exercise session on Thursday, 4 June 2020.

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### PROBLEM I.1 (PROGRAMMING) – DATA ANALYSIS

The following data are made available to you on the course home page<sup>1</sup> and the github repository<sup>2</sup>:

`de_data.csv, gb_data.csv, eu_data.csv, (wind.csv, solar.csv, load.csv).`

They describe (quasi-real) time series for wind power generation  $W(t)$ , solar power generation  $S(t)$  and load  $L(t)$  in Great Britain (GB), Germany (DE) and Europe (EU). The time step is 1 h and the time series are several years long.

- (a) Check that the wind and solar time series are normalized to 'per-unit of installed capacity', and that the load time series is normalized to MW.
- (b) For all three regions, calculate the maximum, mean, and variance of the time series.
- (c) For all three regions, plot the time series  $W(t)$ ,  $S(t)$ ,  $L(t)$  for a winter month (January) and a summer month (July).
- (d) Resample the time series to daily, weekly and monthly data points and visualise them in plots. Can you identify some recurring patterns?
- (e) For all three regions, plot the duration curve for  $W(t)$ ,  $S(t)$ ,  $L(t)$ .
- (f) For all three regions, plot the probability density function of  $W(t)$ ,  $S(t)$ ,  $L(t)$ .
- (g) Recurring patterns of time series can also be visualised more rigorously by applying a Fourier Transform. Apply a (Fast) Fourier Transform to the three time series  $X \in W(t), S(t), L(t)$ :

$$\tilde{X}(\omega) = \int_0^T X(t) e^{i\omega t} dt .$$

For all three regions, plot the energy spectrum  $|\tilde{X}(\omega)|^2$  as a function of  $\omega$ . Discuss the relationship of these results with the findings obtained in (b)-(f).

- (h) Normalize the time series to one, so that  $\langle W \rangle = \langle S \rangle = \langle L \rangle = 1$ . Now, for all three regions, plot the mismatch time series

$$\Delta(t) = \gamma \alpha W(t) + \gamma(1 - \alpha) S(t) - L(t)$$

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<sup>1</sup><https://nworbmot.org/courses/esm-2020/>

<sup>2</sup>[https://github.com/lisazeyen/ESM\\_tutorial/tree/master/tutorial/01-tutorial-04.06.2020/notebooks/data](https://github.com/lisazeyen/ESM_tutorial/tree/master/tutorial/01-tutorial-04.06.2020/notebooks/data)

for the same winter and summer months as in (c). Choose  $\alpha \in \{0.0, 0.5, 0.75, 1.0\}$  with  $\gamma = 1$ , and  $\gamma \in \{0.5, 0.75, 1.0, 1.25, 1.5\}$  with  $\alpha = 0.75$ .

Which configuration entails the lowest mismatch on average and in extremes?

- (i) For all three regions, repeat (b)-(g) for the mismatch time series.

## PROBLEM I.2 (ANALYTICAL) – EFFECT OF SEASONALITY

Figure 1 shows approximations to the seasonal variations of wind and solar power generation  $W(t)$  and  $S(t)$  and load  $L(t)$ :

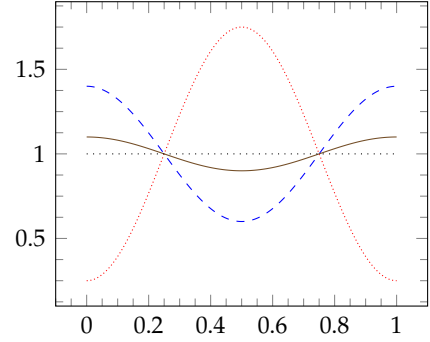
$$W(t) = 1 + A_W \cos \omega t$$

$$S(t) = 1 - A_S \cos \omega t$$

$$L(t) = 1 + A_L \cos \omega t$$

The time series are normalized to  $\langle W \rangle = \langle S \rangle = \langle L \rangle := \frac{1}{T} \int_0^T L(t) dt = 1$ , and the constants have the values

$$\begin{aligned} \omega &= \frac{2\pi}{T} & T &= 1 \text{ year} \\ A_W &= 0.4 & A_S &= 0.75 & A_L &= 0.1 \end{aligned}$$



**Figure 1:** Seasonal variations of wind and solar power generation  $W(t)$  --- and  $S(t)$  ·····, and load  $L(t)$  — around the mean 1 ·····.

(a) What is the seasonal optimal mix  $\alpha$ , which minimizes

$$\langle [\alpha W(t) + (1 - \alpha)S(t) - L(t)]^2 \rangle = \frac{1}{T} \int_0^T [\alpha W(t) + (1 - \alpha)S(t) - L(t)]^2 dt,$$

(b) How does the optimal mix change if we replace  $A_L \rightarrow -A_L$ ?

(c) Now assume that there is a seasonal shift in the wind signal

$$W(t) = 1 + A_W \cos(\omega t - \phi).$$

Express the optimal mix  $\alpha$  as a function of  $\phi$ .

(d) A constant conventional power source  $C(t) = 1 - \gamma$  is now introduced. The mismatch then becomes

$$\Delta(t) = \gamma [\alpha W(t) + (1 - \alpha)S(t)] + C(t) - L(t).$$

Analogously to (a), find the optimal mix  $\alpha$  as a function of  $0 \leq \gamma \leq 1$ , which minimizes  $\langle \Delta^2 \rangle$ .