Camryn Mullin

Problem 2

CFL condition: $\frac{v\,dt}{dx} \leq 1$ sol of form $f(t,x) = \xi^t e^{ikx}$

$$\frac{f(t+dt,x) - f(t-dt,x)}{2dt} = -v\,\frac{f(t,x+dx) - f(t,x-dx)}{2dx}$$

$$\frac{\xi^{t+dt} e^{ikx} - \xi^{t-dt} e^{ikx}}{2dt} = -v\,\frac{\xi^t e^{ik(x+dx)} - \xi^t e^{ik(x-dx)}}{2dx}$$

$$\xi^{\cancel{t}+dt} \cancel{e^{ikx}} - \xi^{\cancel{t}-dt} \cancel{e^{ikx}} = -\frac{v\,dt}{dx}\left(\cancel{\xi^t} e^{ik(\cancel{x}+dx)} - \cancel{\xi^t} e^{ik(\cancel{x}-dx)}\right)$$

$$\xi^{dt} - \xi^{-dt} = -\frac{v\,dt}{dx}\left(e^{ikdx} - e^{-ikdx}\right)$$

$$\xi^{2dt} - 1 = -\frac{v\,dt}{dx}\,\xi^{dt}\, 2i\sin(kdx)$$

$$\xi^{2dt} + 2i\frac{v\,dt}{dx}\sin(kdx)\,\xi^{dt} - 1 = 0 \qquad (\text{let } dt = 1)$$

$$\xi = -\frac{i\,v\,dt}{dx}\sin(kdx) \pm \sqrt{1 - \left(\frac{v\,dt}{dx}\sin(kdx)\right)^2}$$

if $|\xi| = 1$, energy stable

if $\frac{v\,dt}{dx} > 1$ then $\sqrt{1 - \left(\frac{v\,dt}{dx}\sin(kdx)\right)^2}$ complex and $|\xi| > 1$

if $\frac{v\,dt}{dx} \leq 1$ then $\sqrt{1 - \left(\frac{v\,dt}{dx}\sin(kdx)\right)^2}$ real

$$|\xi|^2 = \cancel{\left(\frac{v\,dt}{dx}\sin(kdx)\right)^2} + 1 - \cancel{\left(\frac{v\,dt}{dx}\sin(kdx)\right)^2}$$

$$|\xi| = 1$$

$$\boxed{\therefore \quad \frac{v\,dt}{dx} \leq 1 \text{ necessary!}}$$

2.

a) $V = \frac{-1}{4\pi\varepsilon_0} \rho \ln(r)$ $\nabla^2 V = \rho$ $\rho = V - \bar{V}_{neighbours}$

$V[1,0] = 0$, $V[2,0] = -0.6931...$

$\ln(1)$ $\ln(2)$

b) charge can be seen to be symmetric along box side

c) potential is nearly constant inside box (ranges from 0.995 - 1.030)

As expected the field is highest inside the box where the charge is and drops off as $-\ln(r)$ away from the box

get a cm and plot cm