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<u>Questions 1</u>

The generator from C creates points distributed in rows/column.
See fig C-random.png

The python generator does not have this problem
See Python-random.png

My computer runs into the same problem with C
See Mine-random.png

Question 2

exponential:

$\frac{dx}{dt} = -x \Rightarrow x = e^{-t}$

$PDF = \frac{dx}{dt} = -e^{-t}$

$CDF = \int PDF(t)\,dt = e^{-t}$ $CDF(t) = q$ where $q \in 0,1$

now $t = CDF^{-1}(q)$

for an exponential CDF: $e^{-t} = q$

$t = -\ln(q)$

lorentzian:

$PDF = \frac{1}{1+x^2}$ $CDF = \int \frac{1}{1+x^2}\,dx = \arctan(x) = q$

$\therefore x = \tan(q)$ $y = \frac{1}{1+x^2}$ $\sqrt{\frac{1}{y} - 1} = x$

gaussian: $PDF = e^{-\frac{x^2}{2}}$

$CDF = \frac{\sqrt{\pi}\, erf(\frac{x}{\sqrt{2}})}{\sqrt{2}} = q$

$\therefore X = erf^{-1}\left(\frac{q\sqrt{2}}{\sqrt{\pi}}\right)\sqrt{2}$

power law:

$PDF \sim t^{-\alpha}$ some index (1,2,..)

$CDF = \int_1^{\tau} t^{-\alpha}\,dt = \frac{t^{1-\alpha}}{1-\alpha}\Big|_1^{\tau} = \frac{\tau^{1-\alpha} - 1}{1-\alpha}$ want $CDF(\infty) = 1$

$CDF = \frac{\frac{1-\tau^{1-\alpha}}{1-\alpha}}{\underbrace{1/1-\alpha}_{CDF(\infty)}} = 1-\tau^{1-\alpha} = q \longrightarrow \tau = (1-q)^{1/1-\alpha} \sim q^{1/1-\alpha}$

It can be seen by the curves that Gaussian is not a valid bounding distribution as it crosses below the exponential

The lorentzian and power law do not have this issue in the defined region, though the power is more clearly above

The efficiencies were:

lor: ~81% → most closely matches shape of exp

gauss: ~19% → needed a high amplitude

power: ~18% → curve deviates from shape of exp at small x

Question 3

use $r = v/u$

Set $0 \le u \le \sqrt{p(\frac{v}{u})}$ if $0 \le u \le 1$ then $\sqrt{p(\frac{v}{u})} = 1$

for the exponential distribution $p(x) = \alpha e^{-\alpha x}$

$$u \le \sqrt{\alpha e^{-\alpha r}} \rightarrow u^2 = \alpha e^{-\alpha r}$$

$$\ln\left(\frac{u^2}{\alpha}\right) = -\alpha r = -\alpha \frac{v}{u}$$

$$v = -\frac{u}{\alpha} \ln\left(\frac{u^2}{\alpha}\right)$$

$$\therefore \quad 0 \le v \le -\frac{u}{\alpha} \ln\left(\frac{u^2}{\alpha}\right)$$

$$\text{if } \sqrt{\alpha e^{-\alpha r}} = 1 \rightarrow \alpha e^{-\alpha r} = 1$$

$$-\alpha r = \ln\left(\frac{1}{\alpha}\right)$$

$$v = -\frac{1}{\alpha} \ln\left(\frac{1}{\alpha}\right)$$

The generator is ~81% efficient