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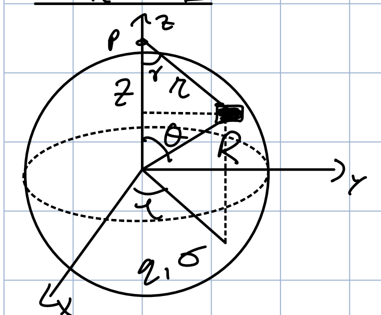
Problem 1

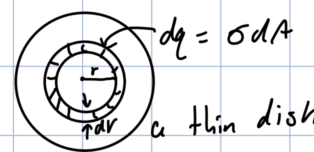



$\frac{1}{4\pi\varepsilon_0}$

$$\vec{E} = k_e \int \frac{dq}{r^2}\,\hat{r}$$

$r = \sqrt{R^2+z^2-2Rz}$

$$dq = \sigma\, dA = \sigma R^2 \sin\theta\, d\theta\, d\varphi$$




$$\hat{r} = \cos\gamma = \frac{z - R\cos\theta}{r}$$

$$E = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi} \frac{\sigma R^2 \sin\theta\,(z - R\cos\theta)}{(R^2+z^2-2Rz\cos\theta)^{3/2}}\, d\theta\, d\varphi$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\, 2\pi\sigma R^2 \int_0^{\pi} \frac{\sin\theta\,(z-R\cos\theta)}{(R^2+z^2-2Rz\cos\theta)^{3/2}}\, d\theta}$$

$u = \cos\theta,\ \frac{du}{d\theta} = -\sin\theta \qquad -du = \sin\theta\, d\theta$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\int_{-1}^{1} \frac{(-du)(z-Ru)}{(R^2+z^2-2Rzu)^{3/2}}$$

$R^2+z^2-2Rzu = v$

$-2Rz = \frac{dv}{du}$

$du = \frac{dv}{-2Rz}$

$z - Ru = \frac{v - R^2 z^2}{2z} + z$

$= \frac{z^2 - R^2 + v}{2z}$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\int_{(R+z)^2}^{(R-z)^2} \left(\frac{dv}{-2Rz}\right) \frac{z^2 - R^2 + v}{v^{3/2}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\int_{(R-z)^2}^{(R+z)^2} \left(\frac{1}{4Rz^2}\right)\left((z^2-R^2)\,v^{-3/2} + v^{-1/2}\right) dv$$

$$= \frac{1}{RRz^2}\left[(z^2-R)^2\left(-2v^{-1/2}\right) + 2v^{1/2}\right]_{(R-z)^2}^{(R+z)^2}$$

$$= \frac{1}{2Rz^2}\left((R^2-z^2)\frac{1}{\sqrt{v}} + \sqrt{v}\right)\Bigg|_{|R-z|^2}^{|R+z|^2}$$

$$= \frac{1}{2Rz^2}\left((R^2-z^2)\left(\frac{1}{R+z} - \frac{1}{|R-z|}\right) + (R+z) - |R-z|\right)$$

$$= \frac{1}{2RZ^2}\left(R-Z - \frac{R^2-Z^2}{|R-Z|} + R+Z - |R-Z|\right)$$

$$= \frac{1}{2RZ^2}\left(2R - \left(\frac{R^2-Z^2 + |R-Z|^2}{|R-Z|}\right)\right) \quad R^2-2RZ+Z^2$$

$$= \frac{1}{2RZ^2}\left(2R - \left(\frac{2R^2-2RZ}{|R-Z|}\right)\right) = \frac{1}{Z^2}\left(1 - \frac{R-Z}{|R-Z|}\right)$$

$$\boxed{E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{Z^2}\left(1 - \frac{R-Z}{|R-Z|}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma R^2}{Z^2}\left(1 - \frac{(R-Z)}{|R-Z|}\right)}$$

1.) $Z > R$:

$$E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{Z^2}\left(1 - \underbrace{\cancel{\frac{R-Z}{Z-R}}}_{(-1)}\right) = \frac{\sigma R^2}{\varepsilon Z^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{Z^2}$$

2) $Z < R$:

$$E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{Z^2}\left(1 - \underbrace{\cancel{\frac{R-Z}{R-Z}}}_{1}\right)$$

$E = 0$ (inside sphere)

There is a singularity at $Z = R$. Quad does not care but my integrator needs to skip at that point

2) need to save all x, y from each call and only recalculate fun(x) for x vals that are new. Number of times fun(x) is called gets counted

for $e^x$, $x(0,1)$ 78 function calls were saved and for $\frac{1}{1+x^2}$, $x(-100,100)$ 3102 calls were saved

3) model $\log_2$ from 0.5 to 2 accuracy region better than $10^{-6}$

$T_n(x) = \cos(n\theta)$ when $x = \cos\theta$

$\theta = \arccos(x)$

$T_n(x) = \cos(n \arccos(x)) \rightarrow (-1, 1)$

$p(x) = c_0 + c_1 \cdot T_1(x) + \dots + c_n T_n(x)$

$$\log_e a = \frac{\log_2 a}{\log_2 e}$$

The chebyshev has smaller error, but only by a fractional amount. But when working with $\log_e$, the legrange had better error than chebyshev by a factor of $10^3$ for rms and $10^4$ for maximum error