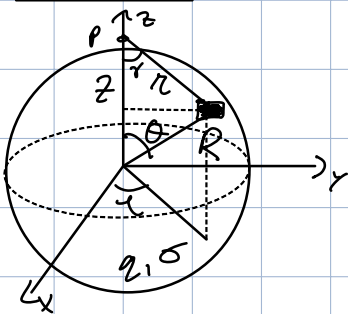


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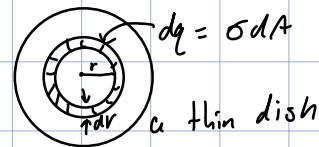
Problem 1



$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r} \rightarrow \frac{1}{4\pi\epsilon_0}$$

$$r = \sqrt{R^2 + z^2 - 2Rz\cos\theta}$$

$$dq = \sigma dA = \sigma R^2 \sin\theta d\theta d\phi$$



$$\vec{r} = \cos\gamma = \frac{z - R\cos\theta}{r}$$

$$E = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^\pi \frac{\sigma R^2 \sin\theta (z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}} d\theta d\phi$$

$$E = \frac{1}{4\pi\epsilon_0} 2\pi\sigma R^2 \int_0^\pi \frac{\sin\theta (z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}} d\theta$$

$$u = \cos\theta, \frac{du}{d\theta} = -\sin\theta \quad -du = \sin\theta d\theta$$

$$= \frac{2\pi\sigma R^2}{4\pi\epsilon_0} \int_{-1}^1 \frac{(-du)(z - Ru)}{(R^2 + z^2 - 2Rzu)^{3/2}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\epsilon_0} \int_{(R-z)^2}^{(R+z)^2} \left(\frac{dV}{-2Rz} \right) \frac{z^2 - Rz^2 u}{V^{3/2}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\epsilon_0} \int_{(R-z)^2}^{(R+z)^2} \left(\frac{1}{4Rz^2} \right) \left((z^2 - Rz^2) V^{-3/2} + V^{-1/2} \right) dV$$

$$= \frac{1}{4Rz^2} \left[(z^2 - Rz^2) (-2V^{-1/2}) + 2V^{1/2} \right] \Big|_{(R-z)^2}^{(R+z)^2}$$

$$= \frac{1}{2Rz^2} \left((R^2 - z^2) \frac{1}{\sqrt{V}} + \sqrt{V} \right) \Big|_{(R-z)^2}^{(R+z)^2}$$

$$= \frac{1}{2Rz^2} \left((R^2 - z^2) \left(\frac{1}{R-z} - \frac{1}{R+z} \right) + (R+z) - (R-z) \right)$$

$$R^2 + z^2 - 2Rzu = V$$

$$-2Rz = \frac{dV}{du}$$

$$du = \frac{dV}{-2Rz}$$

$$z - Ru = \frac{V - R^2 z^2}{2z} + z$$

$$= \frac{z^2 - R^2 + V}{2z}$$

$$= \frac{1}{2Rz^2} \left(R-z - \frac{R^2-z^2}{|R-z|} + R+z - |R-z| \right)$$

$$= \frac{1}{2Rz^2} \left(2R - \left(\frac{R^2-z^2 + (R-z)^2}{|R-z|} \right) \right) R^2 - 2Rz + z^2$$

$$= \frac{1}{2Rz^2} \left(2R - \left(\frac{2R^2 - 2Rz}{|R-z|} \right) \right) = \frac{1}{z^2} \left(1 - \frac{R-z}{|R-z|} \right)$$

$$\boxed{E = \frac{\sigma R^2}{2\epsilon_0} \frac{1}{z^2} \left(1 - \frac{R-z}{|R-z|} \right) = \frac{1}{4\pi\epsilon_0} \frac{2\pi\sigma R^2}{z^2} \left(1 - \frac{(R-z)}{|R-z|} \right)}$$

1.) $z > R$:

$$E = \frac{\sigma R^2}{2\epsilon_0} \frac{1}{z^2} \left(1 - \frac{R-z}{z-R} \right) \xrightarrow{(-1)} = \frac{\sigma R^2}{\epsilon_0 z^2} = \frac{1}{4\pi\epsilon_0} \frac{4\pi R^2 \sigma}{z^2}$$

2.) $z < R$:

$$E = \frac{\sigma R^2}{2\epsilon_0} \frac{1}{z^2} \left(1 - \frac{R-z}{R-z} \right) \xrightarrow{1}$$

$E = 0$ (inside sphere)

There is a singularity at $z=R$. Quad does not care but my integrator needs to skip at that point

2) need to save all x, y from each call and only recalculate $\text{func}(t)$ for x vals that are new. Number of times $\text{func}(t)$ is called gets counted

3) model \log_2 from 0.5 to 2 accuracy region better than 10^6

$$T_n(x) = \cos(n\theta) \quad \text{when } x = \cos\theta$$

$$\theta = \arccos(x)$$

$$T_n(x) = \cos(n \arccos(x)) \rightarrow (-1, 1)$$

$$p(x) = c_0 + c_1 \cdot T_1(x) + \dots + c_n T_n(x)$$

$$\log_e a = \frac{\log_2 a}{\log_2 e}$$

The chebyshev has smaller error, but only by a fractional amount