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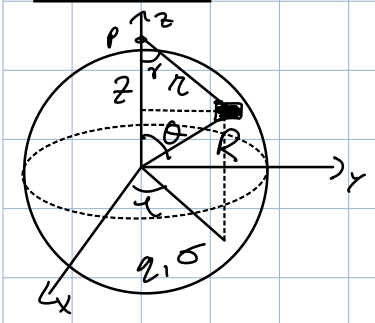
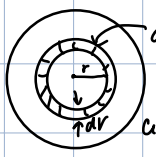
Problem 1



$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r} \qquad \frac{1}{4\pi\varepsilon_0} \to k_e$$

$$r \to R = \sqrt{R^2 + z^2 - 2Rz}$$

$$dq = \sigma dA = \sigma R^2 \sin\theta \, d\theta \, d\varphi$$



a thin dish

$$\hat{r} = \cos\gamma = \frac{z - R\cos\theta}{R}$$

$$E = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi} \frac{\sigma R^2 \sin\theta\,(z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}\, d\theta\, d\varphi$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma R^2 \int_0^{\pi} \frac{\sin\theta\,(z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}\, d\theta}$$

$u = \cos\theta, \quad \frac{du}{d\theta} = -\sin\theta \quad -du = \sin\theta\, d\theta$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\int_{-1}^{1} \frac{(-du)(z - Ru)}{(R^2 + z^2 - 2Rzu)^{3/2}}$$

$R^2 + z^2 - 2Rzu = V$

$-2Rz = \frac{dV}{du}$

$du = \frac{dV}{-2Rz}$

$z - Ru = \frac{V - R^2 z^2}{2z} + z = \frac{z^2 - R^2 + V}{2z}$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\int_{(R-z)^2}^{(R+z)^2} \left(\frac{dV}{-2Rz}\right)\frac{z^2 - R^2 + V}{V^{3/2}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\int_{(R-z)^2}^{(R+z)^2} \left(\frac{1}{4Rz^2}\right)\left((z^2 - R^2)V^{-3/2} + V^{-1/2}\right) dV$$

$$= \frac{1}{8Rz^2}\left[(z^2 - R^2)^2(-2V^{-1/2}) + 2V^{1/2}\right]_{(R-z)^2}^{(R+z)^2}$$

$$= \frac{1}{2Rz^2}\left((R^2 - z^2)\frac{1}{\sqrt{V}} + \sqrt{V}\right)_{(R-z)^2}^{(R+z)^2}$$

$$= \frac{1}{2Rz^2}\left((R^2 - z^2)\left(\frac{1}{R+z} - \frac{1}{|R-z|}\right) + (R+z) - |R-z|\right)$$

$$= \frac{1}{2Rz^2}\left(R-z-\frac{R^2-z^2}{|R-z|}+R+z-|R-z|\right)$$

$$= \frac{1}{2Rz^2}\left(2R-\left(\frac{R^2-z^2+(R-z)^2}{|R-z|}\right)\right) \quad R^2-2Rz+z^2$$

$$= \frac{1}{2Rz^2}\left(2R-\left(\frac{2R^2-2Rz}{|R-z|}\right)\right) = \frac{1}{z^2}\left(1-\frac{R-z}{|R-z|}\right)$$

$$\boxed{E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1-\frac{R-z}{|R-z|}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma R^2}{z^2}\left(1-\frac{(R-z)}{|R-z|}\right)}$$

1.) $z > R$:

$$E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1-\frac{R-z}{z-R}\right) \quad (-1) \qquad = \frac{\sigma R^2}{\varepsilon z^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{z^2}$$

2) $z < R$:

$$E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1-\frac{R-z}{R-z}\right) \quad 1$$

$E = 0$ (inside sphere)

There is a singularity at $z = R$. Quad does not care but my integrator needs to skip at that point

2) need to save all x,y from each call and only recalculate fun(x) for x vals that are new. Number of times fun(x) is called gets counted

for e^x, $x(0,1)$ 78 function calls were saved and for $\frac{1}{1+x^2}$, $x(-100,100)$ 3102 calls were saved

3) model $\log_2$ from 0 to 1 accuracy region better than 10^{-6}

$T_n(x) = \cos(n\theta)$ when $x = \cos\theta$

$\theta = \arccos(x)$

$T_n(x) = \cos(n \arccos(x))$ $\longrightarrow (-1, 1)$

$p(x) = c_0 + c_1 \cdot T_1(x) + \ldots + c_n T_n(x)$

$$\log_e a = \frac{\log_2 a}{\log_2 e}$$

$X = m \cdot 2^e \longrightarrow m = \frac{x}{2^e}$, $e = \log_2\left(\frac{x}{m}\right)$

$y = \log_2(x) = \log_2(m 2^e) = \log_2(m) + \log_2(2^e)$

$= \log_2(m) + e\log_2(2)$

$y = \log_2(m) + e$

The chebyshev has smaller error, but only by a fractional amount. But when working with $\log_e$, the legrange had better error than chebyshev by a factor of 10^3 for rms and 10^4 for maximum error