# Summary: GLS Data Inversion

- Approaches to inverting data share relationships

$$Ku = b \quad \text{Model Equation}$$
$$\delta = d - Su \quad \text{Data-Model Misfit}$$

and same general objective to minimize $\delta$ by estimating $b$.

- Eliminating $u$ and wanting small misfit

$$\delta = d - SK^{-1}b \Rightarrow SK^{-1}b \simeq d$$

Residual of this eqn is the data-model misfit

- SVD directly on $SK^{-1}b = d$ minimizes $Var(\delta)$ highlights small singular values as amplifying noise
- Alternately minimize

$$\Omega = b^T W_b b + \delta^T W_\delta \delta$$

leads to several methods for satisfying the same first order equations for the same extremum of $\Omega$

- $K$ is sparse, well-conditioned FE matrix in $n$ equations

- Direct GLS uses generalized normal equations to invert a full $n \times n$ matrix with no guarantee of its condition number

- An augmented objective function can be formed by embedding the model equations through Lagrange multipliers where the first order equations for the extremum are kept separately and solved either with direct or iterative methods

- Direct methods use Representers (responses to unit misfits) when $m \ll n$ as an efficient option requiring computation of $m$ Representers (each needing the equivalent of 2 model solutions forward and adjoint) and inversion of an $m \times m$ dense matrix

- OR, uses Predictors (responses to unit forcing) in an analogous way when $n \ll m$.

- Iterative methods are possible using gradient descent in b-space where each iteration requires a forward and adjoint model run. Slow convergence can offset speed per iteration

- All results are linear in the data $b = Bd$ and each method has a different practical approach to calculating $B$. that are algebraically identical

- Weight matrices are ideally equally to in inverse covariance matrices