$$\langle\ (\ )\ \rangle \equiv \int_{x_1}^{x_2} (\ )\,dx$$

$$\phi_1 = \frac{x_2 - x}{h} \qquad\qquad \frac{d\phi_1}{dx} = \frac{-1}{h}$$

$$\phi_2 = \frac{x - x_1}{h} \qquad\qquad \frac{d\phi_2}{dx} = \frac{1}{h}$$

$$\langle 1 \rangle = h$$

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = \frac{h}{2}$$

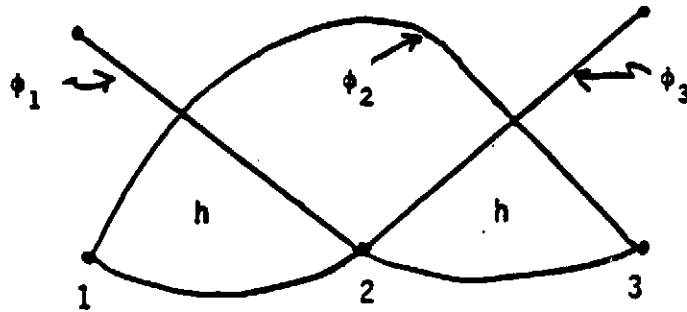$$\langle \phi_1 \phi_2 \rangle = \frac{h}{6}$$

$$\langle \phi_1 \phi_1 \rangle = \langle \phi_2 \phi_2 \rangle = \frac{h}{3}$$

$$\langle \phi_1^3 \rangle = \langle \phi_2^3 \rangle = \frac{h}{4}$$

$$\langle \phi_1^2 \phi_2 \rangle = \langle \phi_2^2 \phi_1 \rangle = \frac{h}{12}$$

$$\left\langle a(x)\,\frac{d\phi_1}{dx} \right\rangle = \frac{-1}{h}\langle a(x) \rangle$$

$$\left\langle a(x)\,\frac{d\phi_2}{dx} \right\rangle = \frac{1}{h}\langle a(x) \rangle$$

$$x_2 = 0$$

$$x_3 = h$$
$$\langle \; (\;) \; \rangle \equiv \int_{x_1 = -h}^{x_3 = h} (\;) \, dx$$

$$\dot{\phi}_1 \equiv \frac{d\phi_1}{dx}$$

$$\phi_1 = \frac{x(x-h)}{2h^2} \qquad\qquad \dot{\phi}_1 = \frac{(2x-h)}{2h^2}$$

$$\phi_2 = \frac{-(x+h)(x-h)}{h^2} \qquad\qquad \dot{\phi}_2 = \frac{-2x}{h^2}$$

$$\phi_3 = \frac{(x+h)x}{2h^2} \qquad\qquad \dot{\phi}_3 = \frac{(2x+h)}{2h^2}$$

$$\langle \phi_1 \rangle = h/3 \qquad\qquad \langle \dot{\phi}_1 \rangle = -1 \qquad\qquad \langle \phi_1 \dot{\phi}_1 \rangle = -1/2$$

$$\langle \phi_2 \rangle = 4h/3 \qquad\qquad \langle \dot{\phi}_2 \rangle = 0 \qquad\qquad \langle \phi_1 \dot{\phi}_2 \rangle = 2/3$$

$$\langle \phi_3 \rangle = h/3 \qquad\qquad \langle \dot{\phi}_3 \rangle = 1 \qquad\qquad \langle \phi_1 \dot{\phi}_3 \rangle = -1/6$$

$$\langle \phi_1 \phi_2 \rangle = 2h/15 \qquad\qquad \langle \dot{\phi}_1 \dot{\phi}_2 \rangle = -4/3h \qquad\qquad \langle \phi_2 \dot{\phi}_1 \rangle = -2/3$$

$$\langle \phi_2 \phi_3 \rangle = 2h/15 \qquad\qquad \langle \dot{\phi}_2 \dot{\phi}_3 \rangle = -4/3h \qquad\qquad \langle \phi_2 \dot{\phi}_2 \rangle = 0$$

$$\langle \phi_3 \phi_1 \rangle = -h/15 \qquad\qquad \langle \dot{\phi}_1 \dot{\phi}_3 \rangle = 1/6h \qquad\qquad \langle \phi_2 \dot{\phi}_3 \rangle = 2/3$$

$$\langle \phi_1 \phi_1 \rangle = 4h/15 \qquad\qquad \langle \dot{\phi}_1 \dot{\phi}_1 \rangle = 7/6h \qquad\qquad \langle \phi_3 \dot{\phi}_1 \rangle = 1/6$$

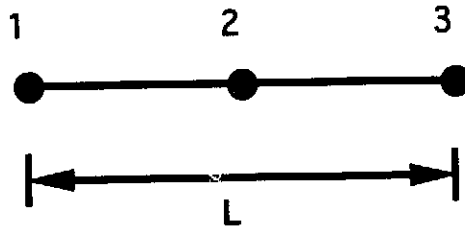$$\langle \phi_2 \phi_2 \rangle = 16h/15 \qquad\qquad \langle \dot{\phi}_2 \dot{\phi}_2 \rangle = 8/3h \qquad\qquad \langle \phi_3 \dot{\phi}_2 \rangle = -2/3$$

$$\langle \phi_3 \phi_3 \rangle = 4h/15 \qquad\qquad \langle \dot{\phi}_3 \dot{\phi}_3 \rangle = 7/6h \qquad\qquad \langle \phi_3 \dot{\phi}_3 \rangle = 1/2$$

SOME MORE INTEGRATION FORMULARS FOR QUADRATIC ELEMENTS

$$\left\langle \phi_1^{\,3} \right\rangle = \left\langle \phi_3^{\,3} \right\rangle = \frac{39L}{420}$$

$$\left\langle \phi_1^{\,2}\, \phi_2 \right\rangle = \left\langle \phi_2\, \phi_3^{\,2} \right\rangle = \frac{20L}{420}$$

$$\left\langle \phi_1^{\,2}\, \phi_3 \right\rangle = \left\langle \phi_1\, \phi_3^{\,2} \right\rangle = -\frac{3L}{420}$$

$$\left\langle \phi_1\, \phi_2^{\,2} \right\rangle = \left\langle \phi_2^{\,2}\, \phi_3 \right\rangle = \frac{16L}{420}$$

$$\left\langle \phi_1 \phi_2 \phi_3 \right\rangle = -\frac{8L}{420}$$

$$\left\langle \phi_2^{\,3} \right\rangle = \frac{192L}{420}$$