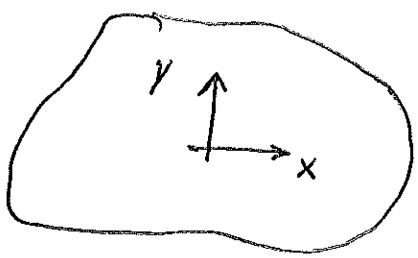
Treatment of point sources (sinks)

Typical Parabolic System: 2-D

$$c \frac{\partial u}{\partial t} = \nabla \cdot K \nabla u + V \cdot \nabla u + ku + \sigma$$

Source strength: $\frac{\text{stuff}}{\text{Time}}$

ex: Heat Transfer

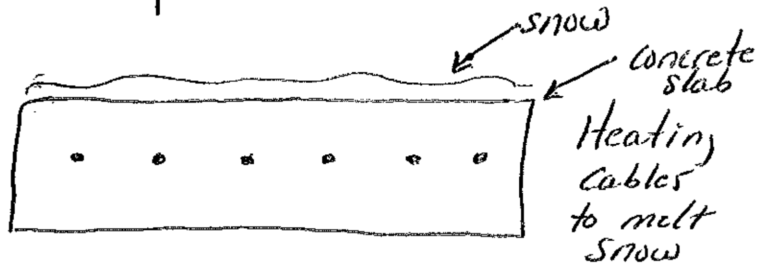


σ: distributed e.g. microwaves in cooking $\sigma(x,y)$ W/cm^3

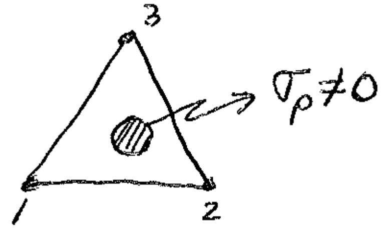
σ: Point Source (Line Source) effectively only at 1 point e.g. metal rod, heating cable Hot water pipe through the plane

In all cases:

$$\left\langle c \frac{\partial \hat{u}}{\partial t} \phi \right\rangle = \cdots \langle \sigma, \phi_i \rangle$$

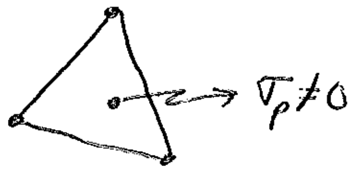


Look at the pipe:



$\langle \sigma_p \phi_i \rangle = 0$ unless "i" is in the element with the pipe

If pipe small relative to element



$$\langle \sigma_p \phi_i \rangle \approx \phi_i(x_p, y_p) \langle \sigma_p \rangle^e$$

Want limit as pipe $\to 0$, but W/cm constant

$$\langle \sigma_p \phi_i \rangle = \phi_i(x_p\, y_p) \langle \sigma_p \rangle^e \qquad (\approx \text{ becomes } =)$$

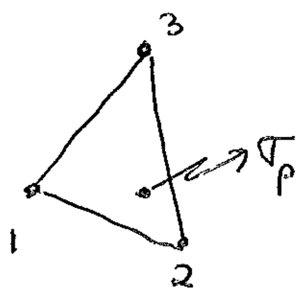
This is the case where $\sigma_p(x,y) = \underbrace{\sigma^*}_{\text{strength : W/cm} \Rightarrow \sigma^* = \langle \sigma \rangle} \delta(x_p, y_p)$

By defn: $\delta(x_p, y_p) = 0$ everywhere except

$= \infty$ at (x_p, y_p)

and $\langle \delta(x_p, y_p) \rangle^e = 1$ <u>if</u> (x_p, y_p) in element e

So:



$$\Rightarrow \begin{aligned} \langle \sigma_p \phi_1 \rangle &= \sigma^* \phi_1(x_p, y_p) \\ \langle \sigma_p \phi_2 \rangle &= \sigma^* \phi_2(x_p, y_p) \\ \langle \sigma_p \phi_3 \rangle &= \sigma^* \phi_3(x_p, y_p) \end{aligned}$$

$$\sigma^* \underbrace{\sum \phi_i(x_p, y_p)}_{=1} = \sigma^*$$

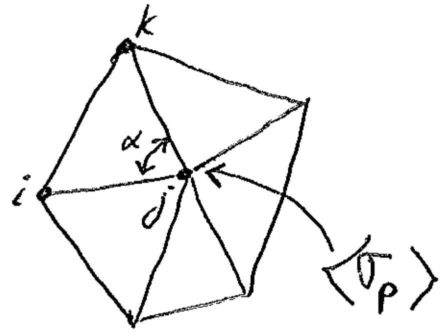
- Total input is correct
- allocation to closest nodes is greatest

Easiest case : σ_p is located at Node j

$$\Rightarrow \langle \sigma_p \phi_i \rangle = \sigma^* \quad \text{if } i=j$$
$$= 0 \quad \text{otherwise } i \neq j$$

all source in Galerkin equation #j ... Simply add σ^* to RHS

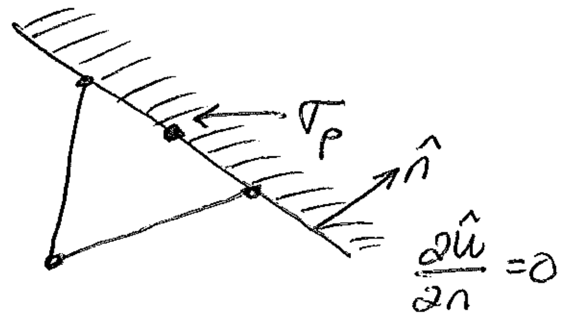
Formally, at the element level



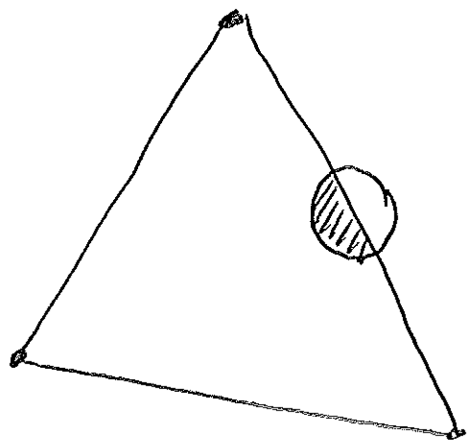
$$\{b^e\} = \frac{\alpha}{360}\,\sigma^* \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

in each element, but sums to σ^* after all elements considered ∴ No need to calculate α. Insert σ^* after element assembly

Symmetry Boundary



enlarged:



only ½ of σ_p goes into an element – balance occurs in "image" region

$$\langle \sigma_p \phi_i \rangle^e = \underbrace{\langle \sigma_p \rangle^e}_{\frac{1}{2}\sigma^*} \phi_i (x_p, y_p)$$