Dartmouth College
Thayer School of Engineering

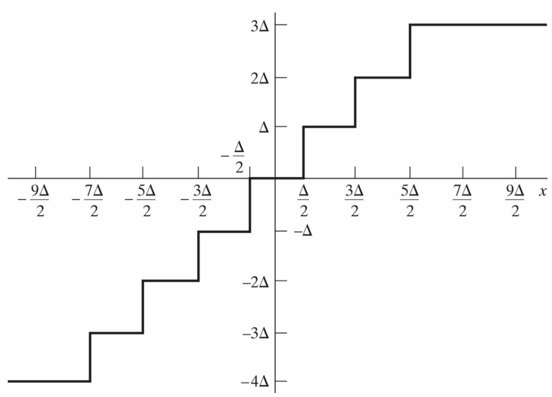
Signal Processing

Problem Set #2

This assignment is worth 50 points. Show all of your work when solving problems and answer conceptual questions clearly. You may discuss these problems with others after you have made an initial start by yourself. List the names of any collaborators and acknowledge all sources and tools that you used outside the normal course readings (such as other books, websites, computing tools, etc.).

In this problem set, you will explore discrete-time signals and systems which provide a foundation for the signal processing topics to come later in this course. You will work with signals in the time domain and in the frequency domain using the discrete-time Fourier transform and the z-transform.

1. *Mean-square value of quantization error.* The input-output relationship of quantization in an analog-to-digital converter is shown on the figure below. The input signal on the x-axis is rounded to the nearest multiple of the step size Δ and output is on the vertical axis.



The input is a random voltage and the output is rounded to the nearest multiple of Δ volts. The quantization error, $e[n]$, then is always between $\pm\Delta/2$. If we consider this error to be a uniform random variable between the values, $-\Delta/2$ and $\Delta/2$, determine the performance of the quantizer by calculating the mean of the error and the mean-square value (or power in the error signal).

2. *Autocorrelation and variance of a random process.*The sequences $s[n]$, $x[n]$, and $w[n]$ are sample sequences of wide-sense stationary random processes where

$$s[n] = x[n]w[n]$$

The sequences $x[n]$ and $w[n]$ are zero-mean and statistically independent. The autocorrelation function of $w[n]$ is

$$E\{w[n]w[n+m]\} = \sigma_w^2 \delta[m]$$

and the variance of $x[n]$ is σ_x^2. Show that $s[n]$ is white, with variance $\sigma_x^2 \sigma_w^2$

3. *Autocorrelation functions and their Fourier transforms.* Consider a random signal

$$x[n] = s[n] + e[n]$$

where both $s[n]$ and $e[n]$ are independent zero-mean stationary random signals with autocorrelation functions $\phi_{ss}[m]$ and $\phi_{ee}[m]$ respectively.

   (a) Determine the expressions for $\phi_{xe}[m]$ and its power spectral density $\Phi_{xe}(e^{j\omega})$.

   (b) Determine the expressions for $\phi_{xs}[m]$ and its power spectral density $\Phi_{xs}(e^{j\omega})$.

4. *Power spectral density of an output process.* Consider the LTI system shown in the figure below. The input to this system, $e[n]$, is a stationary zero-mean white-noise signal with average power σ_e^2. The first system is a backward-difference system as defined by:

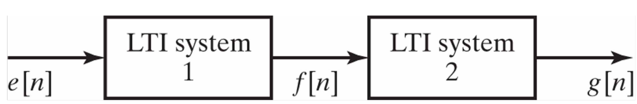
$$f[n] = e[n] - e[n-1]$$

which is equivalent to a frequency response of

$$H_1(e^{j\omega}) = 1 - e^{-j\omega}$$

The second system is an ideal lowpass filter with frequency response:

$$H_2(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$



   (a) Determine an expression for $\Phi_{ff}(e^{j\omega})$, the power spectral density of $f[n]$ using two different approaches: first by finding the autocorrelation of $f[n]$ and then taking the Fourier transform. Second, by using the power spectral density of the input process and the frequency response of the first system. Verify that they are equivalent.

   (b) Determine an expression for $\Phi_{gg}(e^{j\omega})$, the power spectral density of $g[n]$.

   (c) Integrate this power spectral density to find an expression for the average power of the output, σ_g^2.