Dartmouth College
Thayer School of Engineering

Signal Processing

Problem Set #4

This assignment is worth 50 points. Please start each new problem on a new page. Show all of your work when solving problems and answer any conceptual questions clearly and concisely. After you have made an initial start by yourself, you may discuss these problems with others, but copying work from others is not allowed. You may not use any solutions that address the specific question asked. List the names of any collaborators and acknowledge all sources and tools that you used outside the normal course readings (such as other books, websites, computing tools, etc.).

1. *Pole-zero design of digital filter.* A digital filter is characterized by the following properties:

   1. It is a highpass filter and has one pole and one zero.
   2. The pole is at a distance $r = 0.9$ from the origin of the z-plane.
   3. Constant signals do not pass through the system

   (a) Plot the pole-zero pattern of the filter and determine its system function $H(z)$ and frequency response $H(e^{j\omega})$.

   (b) Normalize the frequency response $H(e^{j\omega})$ so that $H(e^{j\pi}) = 1$ by finding the appropriate constant k that will ensure this desired response at frequency $\omega = \pi$.

   (c) Determine the output of the system if the input is

   $$x[n] = 2\cos\left(\frac{\pi}{6}n + 45^\circ\right)$$

   (d) Determine the difference equation (input-output relation in the time domain).

2. *Pole-zero plot of LTI systems.* A causal LTI system has the system function

$$H(z) = \frac{(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})(1 + 1.1765z^{-1})}{(1 - 0.9e^{j\pi/3}z^{-1})(1 - 0.9e^{-j\pi/3}z^{-1})(1 + 0.85z^{-1})}$$

(a) Write the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$ of this system. A computing tool can be using to convert the roots of the polynomials to the coefficients of each polynomial in z^{-1} From this form of the system function the numerator represent the input terms and the denominator represents the output terms.

(b) Plot the pole-zero diagram and indicate the region of convergence (ROC) for the system function.

(c) Make a carefully labeled sketch of $|H(e^{j\omega})|$ using the pole-zero locations to explain.

(d) Explain whether the following statements are true or false about this system.

i. The system is stable.

ii. The impulse response approaches a nonzero constant for large n.

iii. Because the system function has a pole at angle $\pi/3$, the magnitude of the frequency response has a peak at approximately $\omega = \pi/3$.

iv. The system is a minimum phase system.

v. The system has a causal and stable inverse.

3. *Inverse systems.* Consider the cascade of an LTI system with its inverse system. For the inverse system:

$$H(e^{j\omega})H_i(e^{j\omega}) = 1$$

The impulse response of the first system is $h[n] = \delta[n] + 2\delta[n-1]$.

(a) Determine the impulse response $h_i[n]$ of a stable inverse system for $h[n]$. Is the inverse system causal?

(b) Now consider the more general case where $h[n] = \delta[n] + \alpha\delta[n-1]$. Under what conditions on α will there exist an inverse system that is both causal and stable?

4. *Minimum-phase systems.* For each of the following system functions, specify a minimum-phase system function $H_{min}(z)$ such that the frequency-response magnitudes of the two systems are equal: $|H(e^{j\omega})| = |H_{min}(e^{j\omega})|$. Note that a scale factor may be introduced when the pole or zero is reflected inside the unit circle.

   (a)
   $$H(z) = \frac{1 - 2z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

   (b)
   $$H(z) = \frac{(1 - 3z^{-1})(1 - \frac{1}{4}z^{-1})}{(1 - \frac{3}{4}z^{-1})(1 - \frac{4}{3}z^{-1})}$$

5. *Linear-phase and generalized linear-phase systems.* Consider the class of linear-phase discrete-time filters whose frequency response has the form

$$H(e^{j\omega}) = |H(e^{j\omega})|e^{-j\alpha\omega}$$

where $|H(e^{j\omega}|$ is a real and nonnegative function of ω and α is a real constant. Also consider the class of generalized linear-phase filters of the form

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha\omega+j\beta}$$

where $A(e^{j\omega})$ is a real (could be negative) function of ω, and α, β are real constants.

For each of the filters with impulse response in the figure below, determine whether it is a generalized linear phase filter. If it is, then find $A(e^{j\omega})$, α, and β. For these filters, indicate whether or not it also meets the more stringent requirement for being a linear phase filter.