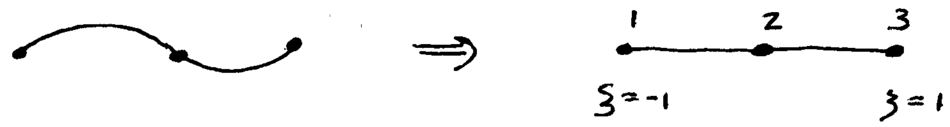
Quadratic Elements



$$\phi_1 = \tfrac{1}{2}\xi(\xi-1)$$
$$\phi_2 = (1-\xi)(1+\xi)$$
$$\phi_3 = \tfrac{1}{2}\xi(1+\xi)$$

- Local Transformation:

$$x(\xi) = \sum x_i \phi_i(\xi) \qquad \frac{dx}{d\xi} = \sum x_i \frac{d\phi_i}{d\xi}$$
$$y(\xi) = \sum y_i \phi_i(\xi) \qquad \frac{dy}{d\xi} = \sum y_i \frac{d\phi_i}{d\xi}$$

$$\int (\ ) \, ds = \int (\ ) \, |J| \, d\xi$$

$$\hookrightarrow \frac{ds}{d\xi} = \left(\left(\frac{dx}{d\xi}\right)^2 + \left(\frac{dy}{d\xi}\right)^2 \right)^{1/2}$$

- Normal/Tangential Vectors:

$$ds\,\hat{s} = dx\,\hat{x} + dy\,\hat{y}$$

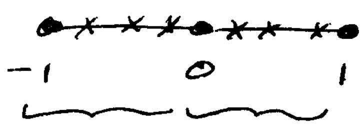
$$\underbrace{\frac{ds}{d\xi}}_{|J|}\hat{s} = \frac{dx}{d\xi}\hat{x} + \frac{dy}{d\xi}\hat{y} \Rightarrow \hat{s} = \frac{1}{|J|}\left(\frac{dx}{d\xi}\hat{x} + \frac{dy}{d\xi}\hat{y}\right)$$

$$\hat{n} = \hat{s} \times \hat{z} = \frac{1}{|J|}\left(\frac{dy}{d\xi}\hat{x} - \frac{dx}{d\xi}\hat{y}\right)$$

- Singular Integrals
  - $|J|$ not available in polynomial form
  - use special Gauss pts for log singularities (see Text pp 270-271) on interval $[0,1]$

Mid-element node: no change

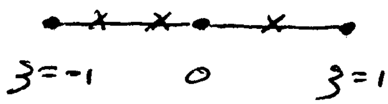
e.g.



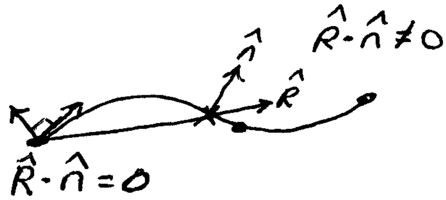
3pt quadrature on each side

Left/Right Corner: Map $\underbrace{-1 \to 1 \text{ onto } 0 \to 1}_{\frac{\xi_k + 1}{2}}$

e.g.



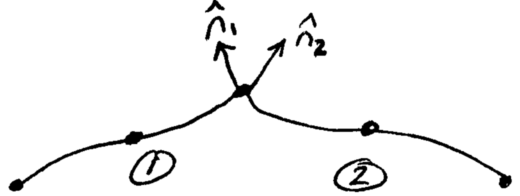
- $\int \frac{\partial G_i}{\partial n} \phi_j \neq$ since $\hat{R}\cdot\hat{n} \neq 0$ so cannot ignore this contribution, but <u>not singular</u>



Use standard quadrature

- Computing α_i
  - $\alpha_i = 180°$ for mid-element-nodes
  - corner nodes: Compute $\hat{n}_1, \hat{n}_2$

    Proceed as in linear elements, i.e. compute $\hat{n}_1 \times \hat{n}_2$, $\hat{n}_1 \cdot \hat{n}_2$



(alternately, use $\hat{s}_1$, $\hat{s}_2$)