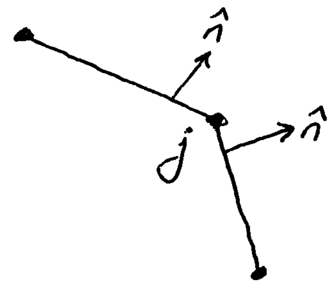
Treatment of Corners

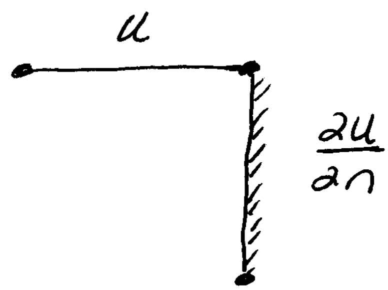
- Discretized boundary w/ linear elements... $\hat{n}$ discontinuous (element to element)



- Usually don't worry about it, compute $\frac{\partial u}{\partial n_j}$ as nodal value; BEM doesn't need to define $\hat{n}_j$ explicitly; Do this when surface is intended to be smooth

- But, some problems have explicit corners ... often having u specified on one side and $\frac{\partial u}{\partial n}$ on the other... how to proceed?

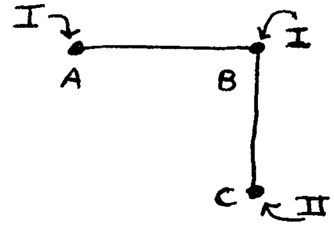
e.g.



generally, u continuous but $\frac{\partial u}{\partial n}$ is not

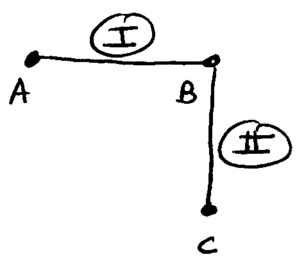
A number of strategies possible:

Strategy I: Ignore the Problem (makes $\frac{\partial u}{\partial n}$ continuous)



- Use node-based BCs (u and $\frac{\partial u}{\partial n}$)
- continuous bases (u and $\frac{\partial u}{\partial n}$)
- $\frac{\partial u}{\partial n}_B$ some kind of average
- ϕ_B, G_B unambiguous
- IF Type I/Type II corner let one condition over-ride (Type I usually)

Strategy II: Implicitly enforce discontinuous $\frac{\partial u}{\partial n}$ at element level



- use node-based <u>and</u> element-based BC classifications
- 2 BCs (u and $\frac{\partial u}{\partial n}$) at B
  element-based $\hat{n}$ dictates $\frac{\partial u}{\partial n}$
- On assembly:

Contributes to matrix assembly for $\frac{\partial u}{\partial n}$ variables at A + B nodes

Type I element: enforce u, $\frac{\partial u}{\partial n_I}$ unknown

Type II element: enforce $\frac{\partial u}{\partial n_{II}}$ <u>and</u> u <u>if</u> Type I node exists

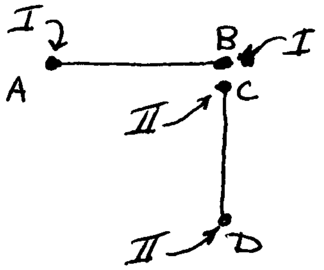
Contributes to matrix assembly for u variables at B + C nodes (C only, if B is Type I node)

- at Type I/II corner, leaves $\frac{\partial u_B}{\partial n_I}$ in list of unknowns ... coefficient for $\frac{\partial u_B}{\partial n_I}$ from Type I element <u>only</u> ... in effect have u_B, $\frac{\partial u_B}{\partial n_I}$, $\frac{\partial u_B}{\partial n_{II}}$ at corner w/ u_B and $\frac{\partial u_B}{\partial n_{II}}$ known

  $\frac{\partial u_B}{\partial n_I}$ only exists in element I; $\frac{\partial u_B}{\partial n_{II}}$ only exists in element II

- Features:
  (i) only need logical decision making at element level
  (ii) similar care during evaluation of interior values
  (iii) No difficulty w/ G_B, but ϕ_B unclear
  (iv) doesn't work at Type I/Type I corner (single G_B, but 2 unknowns $\frac{\partial u_B}{\partial n_I}$, $\frac{\partial u_B}{\partial n_{II}}$)

Strategy III: Explicitly enforce discontinuous $\frac{\partial u}{\partial n}$



- return to simple node-based BC classifications

- assemble with distinct nodes B and C ("double nodes")

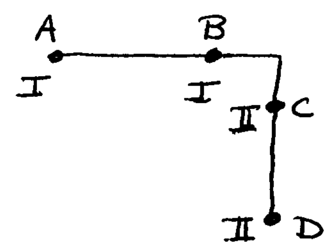
- merge B and C to get single G_{BC}

- explicitly enforce $u_B = u_C$ in "sparse" row

- at Type I/II corner, leaves $\frac{\partial u_B}{\partial n}$ and u_C as unknowns; "extra" row in matrix used to satisfy $u_B = u_C$.

- Features:
  - (i) same as Strategy II, but with "extra" node so ϕ_B, ϕ_C unambiguous, BCs node-based
  - (ii) creates extra "place-holder" in matrix so $\frac{\partial u}{\partial n}$ can be explicitly discontinuous... but requires additional constraints to ensure u continuous.
  - (iii) doesn't work for Type I/Type I corner (No obvious relationship between $\frac{\partial u_B}{\partial n}$, $\frac{\partial u_C}{\partial n}$)

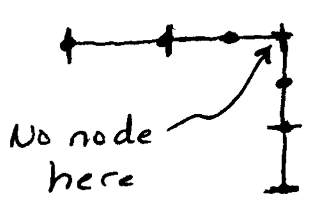
Strategy IV: Avoid the Corner



- Nodes B + C physically distinct
- No issues about $\hat{n}$ at B or C
- G_B, G_C distinct... two separate BE equations
- Works for all BC combinations

But u and $\frac{\partial u}{\partial n}$ discontinuous at corner!

Book: use constant elements at corner



Summary:

(i) No corner in continuum $\Rightarrow$ Strategy I

(ii) Type I/Type II (or Type II/Type II) corner
$\Rightarrow$ Strategy II

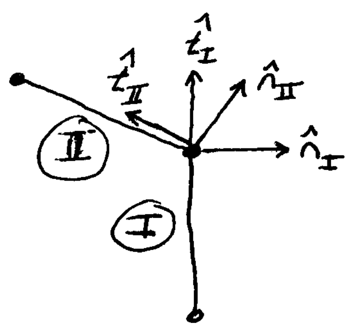
(iii) Type I/Type I corner $\Rightarrow$ Strategy IV

OR

Strategy III w/ relationship between $\frac{\partial u_B}{\partial n}$ and $\frac{\partial u_c}{\partial n}$ invented

OR

Compute $\frac{\partial u}{\partial n}$ from $\frac{\partial u}{\partial t}$ directly (Sládek and Sládek paper)



$$\hat{t}_I = (\hat{t}_I \cdot \hat{n}_{II})\,\hat{n}_{II} + (\hat{t}_I \cdot \hat{t}_{II})\,\hat{t}_{II} \qquad (1)$$

$$\hat{n}_I = (\hat{n}_I \cdot \hat{n}_{II})\,\hat{n}_{II} + (\hat{n}_I \cdot \hat{t}_{II})\,\hat{t}_{II} \qquad (2)$$

from (1): $$\hat{n}_{II} = \frac{\hat{t}_I - (\hat{t}_I \cdot \hat{t}_{II})\,\hat{t}_{II}}{(\hat{t}_I \cdot \hat{n}_{II})}$$

So $$\hat{n}_I = (\hat{n}_I \cdot \hat{n}_{II})\left[\frac{\hat{t}_I - (\hat{t}_I \cdot \hat{t}_{II})\,\hat{t}_{II}}{\hat{t}_I \cdot \hat{n}_{II}}\right] + (\hat{n}_I \cdot \hat{t}_{II})\,\hat{t}_{II}$$

then $$\frac{\partial u}{\partial n_I} = \nabla u \cdot \hat{n}_I = \left(\frac{\hat{n}_I \cdot \hat{n}_{II}}{\hat{t}_I \cdot \hat{n}_{II}}\right)\frac{\partial u}{\partial t_I} + \left[\hat{n}_I \cdot \hat{t}_{II} - \frac{(\hat{t}_I \cdot \hat{t}_{II})(\hat{n}_I \cdot \hat{n}_{II})}{(\hat{t}_I \cdot \hat{n}_{II})}\right]\frac{\partial u}{\partial t_{II}}$$

Compute from given Type I data!