Table 4.1 SOME FUNDAMENTAL SOLUTIONS

	Equation	*Fundamental solution*
(i) *One-dimensional equations*		
Laplace	$\frac{d^2u^*}{dx^2}+\delta_0=0$	$u^*=r/2 \quad r=\lvert x\rvert$
Helmholtz	$\frac{d^2u^*}{dx^2}+\lambda^2u^*+\delta_0=0$	$u^*=-\frac{1}{2\lambda}\sin(\lambda r)$
Wave equation	$c^2\frac{\partial^2u^*}{\partial x^2}-\frac{\partial^2u^*}{\partial t^2}+\delta_0\delta(t)=0$	$u^*=\frac{1}{2c}H(ct-r)$ where H is the Heaviside or unit step function
Diffusion equation	$\frac{\partial^2u^*}{\partial x^2}-\frac{1}{k}\frac{\partial u^*}{\partial t}+\delta_0\delta(t)=0$	$u^*=-\frac{H(t)}{\sqrt{4\pi kt}}\exp\left(-\frac{r^2}{4kt}\right)$
Convection/ decay equation	$\frac{\partial u^*}{\partial t}+\bar{u}\frac{\partial u^*}{\partial x}+\beta u^*+\delta_0\delta(t)=0$	$u^*=-e^{-\beta r/\bar{u}}\,\delta\left(t-\frac{r}{\bar{u}}\right)$
(ii) *Two-dimensional equations*		$r=\sqrt{x_1^2+x_2^2}$
Laplace	$\frac{\partial^2u^*}{\partial x_1^2}+\frac{\partial^2u^*}{\partial x_2^2}+\delta_0=0$	$u^*=\frac{1}{2\pi}\ln\left(\frac{1}{r}\right)$
Helmholtz	$\frac{\partial^2u^*}{\partial x_1^2}+\frac{\partial^2u^*}{\partial x_2^2}+\lambda^2u^*+\delta_0=0$	$u^*=\frac{1}{4i}H_0^{(2)}(\lambda r)$ where H is a Hankel function
D'Arcy	$k_1\frac{\partial^2u^*}{\partial x_1^2}+k_2\frac{\partial^2u^*}{\partial x_2^2}+\delta_0=0$ (orthotropic case)	$u^*=\frac{1}{\sqrt{k_1k_2}}\frac{1}{2\pi}\ln\left(\frac{1}{r_0}\right)$ where $r_0=\left(\frac{x_1^2}{k_1}+\frac{x_2^2}{k_2}\right)^{1/2}$
Wave equation	$c^2\left(\frac{\partial^2u^*}{\partial x_1^2}+\frac{\partial^2u^*}{\partial x_2^2}\right)-\frac{\partial^2u^*}{\partial t^2}+\delta_0\delta(t)=0$	$u^*=-\frac{H(ct-r)}{2\pi c(c^2t^2-r^2)}$
Plate equation	$\left(\frac{\partial^2}{\partial t^2}-\mu^2\nabla^4\right)u^*+\delta_0\delta(t)=0$ $\nabla^4=(\nabla^2)^2$ in two dimensions	$u^*=+\frac{H(t)}{4\pi\mu}S_i\left(\frac{r}{4\mu t}\right)$ S_i is the integral sine function $S_i(u)=-\int_u^\infty\frac{\sin v}{v}\,dv$
Reduced plate equation	$k_p=\omega/\mu$ $(\nabla^4-k_p^4)\,u^*+\delta_0=0$	$u^*=-\frac{1}{8ik_p^2}\left(H_0^{(2)}(k_pr)-\frac{2i}{\pi}K_0(k_pr)\right)$ where K_0 is an elliptic function
Navier's equation (Kelvin solution)	$\frac{\partial\sigma_{jk}^*}{\partial x_j}+\delta_l=0$ (point load in direction l)	Displacement in direction k $u_k^*=U_{lk}^*e_k$ $=\frac{[(3-4v)\ln(1/r)\delta_{kl}+r_{,k}r_{,l}]e_l}{8\pi G(1-v)}$ Traction in direction k $p_k^*=p_{kl}^*e_l$ $=-\frac{1}{r}\left(\frac{dr}{dn}[(1-2\nu)\delta_{lk}+2r_{,l}r_{,k}]-(1-2\nu)(n_lr_{,k}-n_kr_{,l})\right)\frac{e_l}{4\pi(1-\nu)}$

Table 4.1 (*continued*)

Equation		Fundamental solution
(iii) Three-dimensional equations		
Navier's equation (isotropic, homogeneous, Kelvin solution)	$\frac{\partial \sigma_{jk}^*}{\partial x_j} + \delta_l = 0$ (point load in direction l)	Displacement in direction k $u_k^* = U_{lk}^* e_l$ $u_{lk}^* = \frac{1}{16\pi G(1-\nu)}\left(\frac{3-4\nu}{r}\tilde{\delta}_{lk} + \frac{\partial r}{\partial x_l}\frac{\partial r}{\partial x_k}\right)$ Traction in direction k $p_i^* = p_{ji}^* e_j$ $p_j^* = p_{ij}^* e_l$ $= -\frac{1}{8\pi(1-\nu^2)r^2}\left(\frac{\partial r}{\partial n}[(1-2\nu)\tilde{\delta}_{ij} + 3r_{,i}r_{,j}] + (1-2\nu)(njr_{,i} - n_i r_{,j})\right)e_j$
(iii) Three-dimensional equations		$r = \sqrt{x_1^2 + x_2^2 + x_3^2}$
Laplace	$\frac{\partial^2 u^*}{\partial x_1^2} + \frac{\partial^2 u^*}{\partial x_2^2} + \frac{\partial^2 u^*}{\partial x_3^2} + \delta_0 = 0$	$u^* = 1/4\pi r$
Helmholtz	$\frac{\partial^2 u^*}{\partial x_1^2} + \frac{\partial^2 u^*}{\partial x_2^2} + \frac{\partial^2 u^*}{\partial x_3^2} + \lambda^2 u^* + \delta_0 = 0$	$u^* = \frac{1}{4\pi r}\exp(-\mathrm{i}\lambda r)$
D'Arcy	$k_1\frac{\partial^2 u^*}{\partial x_1^2} + k_2\frac{\partial^2 u^*}{\partial x_2^2} + k_3\frac{\partial^2 u^*}{\partial x_3^2} + \delta_0 = 0$	$u^* = \frac{1}{\sqrt{k_1 k_2 k_3}}\frac{1}{4\pi}\frac{1}{r_0}$ where $r_0 = \sqrt{\frac{x_1^2}{k_1} + \frac{x_2^2}{k_2} + \frac{x_3^2}{k_3}}$
Wave equation	$c^2\nabla^2 u^* - \frac{\partial^2 u^*}{\partial t^2} + \delta_0\delta(t) = 0$	$u^* = \frac{\delta(t-(r/c))}{4\pi r}$