Question 3

For the circuit shown in Figure 3, prove that if $\Delta R \ll R$ (<< means much smaller than), output voltage v_o can be approximated as (ideal op-amp can be assumed):

$$v_o \approx \frac{R_f}{R^2}\left(\frac{R+R_f}{R+2R_f}\right)(-\Delta R)v_{in}$$

(30 marks)

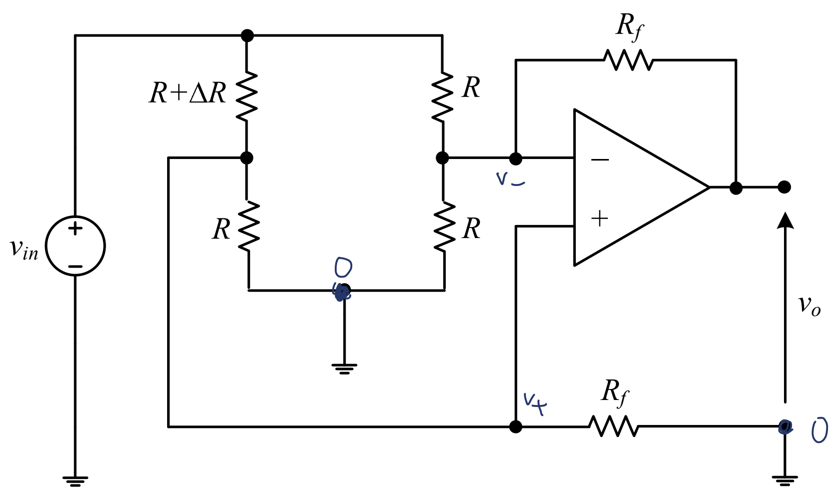

Figure 3

For V_- and V_+, ideal amplier $V_- = V_+$

At node V_+, by KCL,

$$\frac{V_+ - V_{in}}{R+\Delta R} + \frac{V_+ - 0}{R_f} + \frac{V_+ - 0}{R} = 0$$

$$V_+ = \frac{V_{in}}{R+\Delta R}\left(\frac{1}{R+\Delta R} + \frac{1}{R_f} + \frac{1}{R}\right)^{-1}$$

$$= \frac{V_{in}}{R+\Delta R}\left(\frac{R_f R + (R+\Delta R)R + (R+\Delta R)R_f}{(R+\Delta R)(R_f)(R)}\right)^{-1}$$

$$= \frac{V_{in}\,\cancel{(R+\Delta R)}\,(R_f)(R)}{\cancel{(R+\Delta R)}(R_f R + (R+\Delta R)R + (R+\Delta R)R_f)}$$

At node V_-,

$$\frac{V_- - V_{in}}{R} + \frac{V_- - 0}{R} + \frac{V_- - V_{out}}{R_f} = 0$$

$$\frac{1}{R}(2V_- - V_{in}) + \frac{V_-}{R_f} - \frac{V_{out}}{R_f} = 0$$

$$V_-\left(\frac{2}{R} + \frac{1}{R_f}\right) = \frac{V_{out}}{R_f} + \frac{V_{in}}{R}$$

$$V_-(2R_f + R) = V_{out}R + V_{in}R_f$$

$$V_{out} = \frac{V_-(2R_f + R)}{R} - \frac{V_{in}R_f}{R}$$

$$V_- = \frac{V_{out}R + V_{in}R_f}{2R_f + R}$$

$$\because V_+ = V_- = \frac{V_{in}(R_f)(R)}{(R_fR + (R+\Delta R)R + (R+\Delta R)R_f)}$$

$$\therefore V_{out} = \frac{\frac{V_{in}(R_f)(\cancel{R})}{(R_fR + (R+\Delta R)R + (R+\Delta R)R_f)}(2R_f + R)}{\cancel{R}} - \frac{V_{in}R_f}{R}$$

$$= (V_{in})\left(\frac{2R_f^2 + R_fR}{(R_fR + (R+\Delta R)R + (R+\Delta R)R_f)} - \frac{R_f}{R}\right)$$

$$= (V_{in}) \frac{2R_f^2R + R_fR^2 - R_f^2R - R^2R_f - \Delta R R R_f - RR_f^2 - \Delta R R_f^2}{R\ (R_fR + (R+\Delta R)R + (R+\Delta R)R_f)}$$

$$= (V_{in}) \left(\frac{-\Delta R R_f R - \Delta R R_f^2}{R\ (R_fR + (R+\Delta R)R + (R+\Delta R)R_f)} \right)$$

$$v_o \approx \frac{R_f}{R^2}\left(\frac{R+R_f}{R+2R_f}\right)(-\Delta R)v_{in}$$

$$= (V_{in})(-\Delta R) \frac{R_f(R+R_f)}{R(R_fR + (R+\Delta R)R + (R+\Delta R)R_f)}$$

$\therefore$ if $\Delta R \ll R$, $(R+\Delta R) = R$

$$= (V_{in})(-\Delta R) \frac{R_f(R+R_f)}{R(R_fR + (R)R + (R)R_f)}$$

$$= (V_{in})(-\Delta R) \frac{R_f}{R^2} \left(\frac{R+R_f}{R_f+R+R_f} \right)$$

$$= \frac{R_f}{R^2}\left(\frac{R+R_f}{R+2R_f}\right)(-\Delta R)(V_{in})$$

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