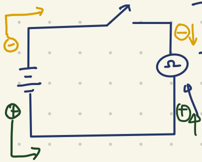
① when switch on → electric charge (C) will flow
→ $1.602 \cdot 10^{-19}$ C ⊖ charge in electron



→ $1.602 \cdot 10^{-19}$ C ⊕ charge in proton ⊕
⊕ ⊖ meet in bulb and release energy

$$\boxed{i = \frac{dq}{dt} \rightarrow q = \int_{t_0}^{t} i(t)\,dt}$$

② There are two current
→ Direct current (DC)
→ Alternating current (AC)

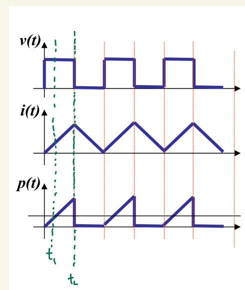
③ Direction of Current 方向可不同
($\xrightarrow{+1A} = \xleftarrow{-1A}$) 只要 reverse the sign of current

Energy & Voltage

→ voltage / potential difference across element
(energy required to move a unit charge through)

A height
electron
+ve
voltage between two point
B Ground level -ve

→ Energy (provide capacity to do work) Joule (J)
→ Power (time rate expending / absorbing energy) Watt (W)



Power (W)

$$p(t) = \frac{dE}{dt} = \frac{dE}{dq}\frac{dq}{dt} = v(t) \cdot i(t)$$

Energy (J)

$$E(t) = \int_{t_0}^{t} p(t)\,dt = \int_{t_0}^{t} v(t)\, i(t)\, dt$$

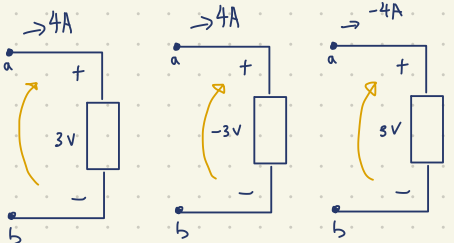
Average Power (W)

$$P_{average} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt$$

Polarity of Power

→ Passive sign convention

(Active source vs Passive source)



+ve

to

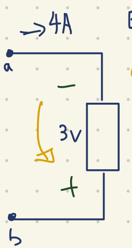
−ve

P = +12W (absorb)

= −12W (supplied)

= −12W (supplied)

↓



Blue arrow & Orang arrow same (not ok)

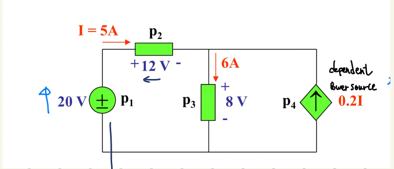
① reverse V = −3V · 4A = −12W

② reverse I = 3V · −4A = −12W

Law of conversation energy

→ Power supplied = Power absorbed

→ Sum of power is circuit alway 0 $\Sigma p = 0$



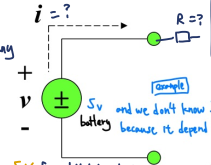
P_1 : 20V · −5A = −100W

P_2 : 12V · 5A = 60W

P_3 : 8 · 6 A = 48W

P_4 : 8 · (0.2 · I) = 8 · 0.2 · 5 = −8W

$P_1 + P_2 + P_3 + P_4 = 0$



Connect how many load, 5V still 5V

Ideal

Non Ideal Independent voltage source

↓

If connect 2, 100, 1000, the 5v drop to 4.99, 4.96, little bit, then it is not Ideal (real world case)

example

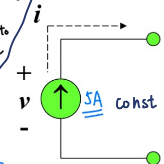
and we don't know I because it depend to the load connected.

5V fix and Not dependent

the voltage is independent and not change due to change of flow

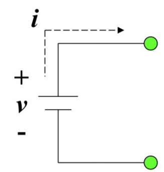
Independent voltage source

(const. or time-varying voltage)



const and not dependent to everything

Independent current source



Independent constant DC voltage source

- Voltage source
  - Voltage defined.
  - Current depends on load connected to source.
- Current source
  - Current defined.
  - Voltage depends on load connected to source.

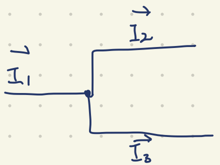
$\boxed{\text{Resistor}}$

→ $v = iR$ $\boxed{\text{Ohm' law}}$
- → only applied in close circuit.
- → Open circuit, $R=\infty$, $i=0$
- → short circuit, $R=0$, $i \to \infty$

→ $P = vi = i^2R = \frac{v^2}{R}$
$= vi = v^2 G = \frac{i^2}{G}$

G for conductance

For node circuit



$I_1 = I_2 + I_3$
(in) (out)

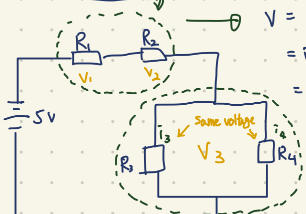
→ $\boxed{\text{voltage divider}}$

① Resistor in series (假設無 R_3 R_4 V_3)

$V = V_1 + V_2$
$= iR_1 + iR_2$
$= i\,R_{eq}$

For $v_1 = \frac{R_1}{R_1+R_2}$ $v_2 = \frac{R_2}{R_1+R_2}$

V_3

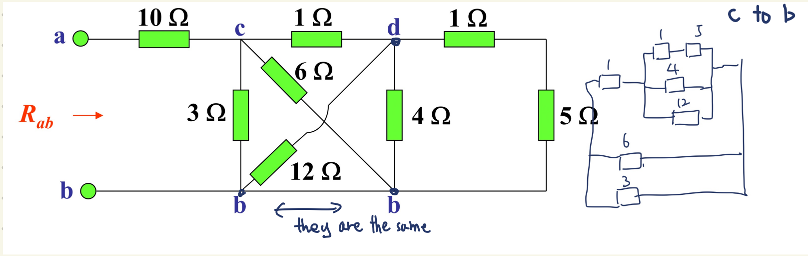


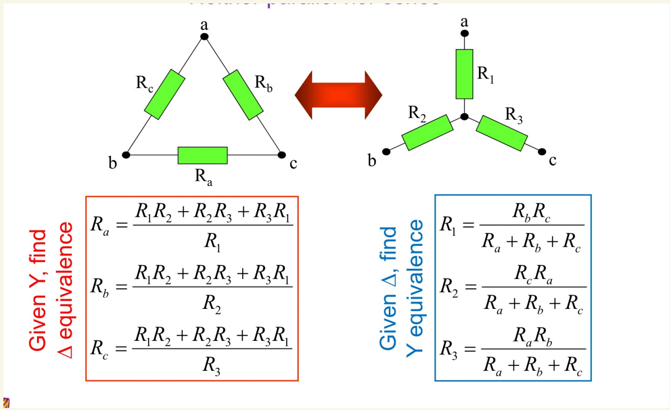
$\boxed{\text{current divider}}$

$i_{in} = i_3 + i_4$
$= \frac{V}{R_3} + \frac{V}{R_4}$
$\frac{1}{R_{eq}} = \frac{1}{R_3} + \frac{1}{R_4}$

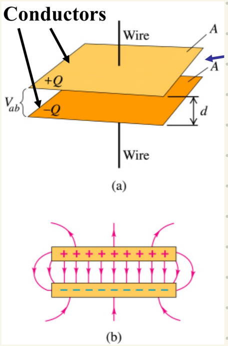
睇 input node d to b 有 1, 5, 4, 12

c to b 有 1, 6, 3





Capacitor 電容 → storge charges (Energy)



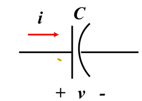
$Q = CV$

C capacitance in (F) farad

$C = \varepsilon A / d$

area distance

permittivity of material / dielectric between plate



charge voltage

$$i = \frac{dq}{dt} = C\frac{dv}{dt} \quad \text{since} \quad q = Cv$$

→ $\boxed{i = C\frac{dv}{dt}}$ common use

→ $p = v \cdot i = vC\frac{dv}{dt}$

$E(t) = \int_{-\infty}^{t} p(t)\,dt$

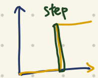
$= \int_{-\infty}^{t} C \cdot v \frac{dv}{dt} \cdot dt$

$= C\int_{-\infty}^{t} v(t)\,dv$

Energy stored : $E(t) = \frac{1}{2}C \cdot v^2 = \frac{Q^2}{2C}$ → assuming $v(-\infty) = 0$

① when v is constant (DC),
$i = 0$ (since capacitor behave like open circuit)

② v can't change in step because it result $i \to \infty$

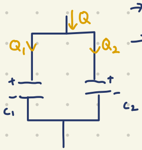


→ $\frac{dv}{dt} = \infty$

☆ Energy in Capacitor tranfer energy faster than energy in battery. (due to chemical reaction in battery slow down)

☆ Energy store in Capacitor lower than a battery

In Parallel



→ can store more charges

→ $Q_1 = C_1 V$ $Q_2 = C_2 V$

$Q = Q_1 + Q_2$

$\therefore Q/V = C_1 + C_2 = C$

In series

$\left(\frac{C_1 C_2}{C_1 + C_2}\right)$

By $V_1 = \frac{Q}{C_1}$ $V_2 = \frac{Q}{C_2}$

$V = V_1 + V_2$

$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{V}{Q} = \frac{1}{C}$

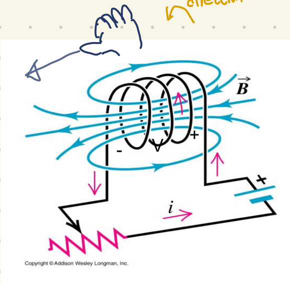
$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$C = \frac{C_1 C_2}{C_1 + C_2}$

Inductor

right hand Rule

direction of current



inductance (Henry (H))

$$v = L \frac{di}{dt}$$

where $L = \dfrac{N^2 \mu A}{l}$

No. of turn

Permeability of core

Cross-sectional area

length of flux length

Core → 

Core

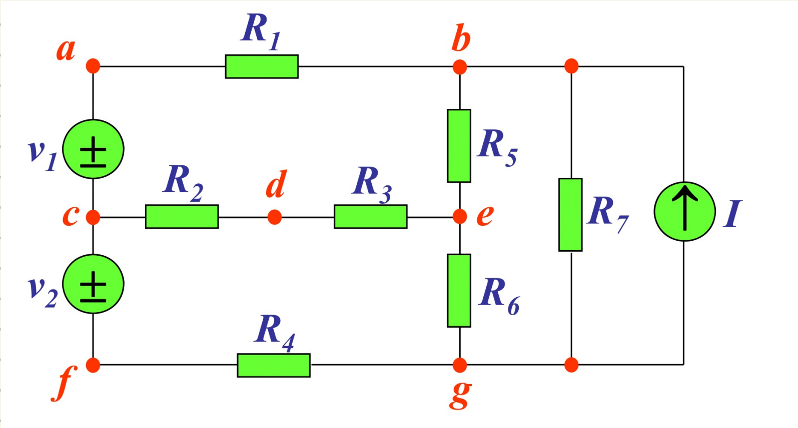
Node

node : point of two or move circuit join (a, b, c, d, e, f, g)

Essential node : point of three or more circuit element joined (b, c, e, g)

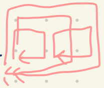
Branch : circuit element between 2 node

Essential Branch : circuit element between 2 essential node (R_6)



Path : 路 that element not include more than once

Loop : closed path with no node pass more than once

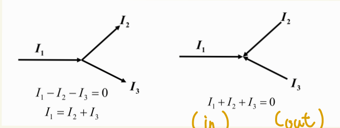


11個

Mesh : a loop that doesn't enclose any other loop

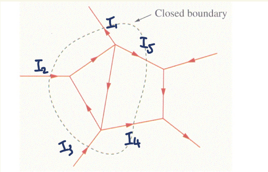
4個

KCL (Kirchhoff's Current Law)



(in) (out) (in) (out)

$\text{Current}_{in} = \text{Current}_{out}$

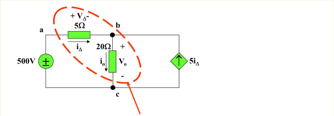


$$\boxed{\sum_N i_n = 0}$$

For this case : (in) $I_2 + I_3$ = (out) $I_1 + I_4 + I_5$

KVL (Kirchhoff's voltage law)

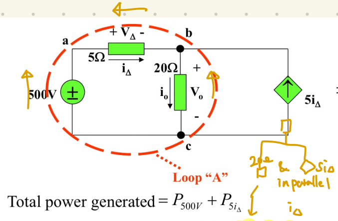
→ Algebraic sum of all voltages around a closed path / a loop



Can the voltage divider's rule be applied here?

No, because they are not in series.

(即使睇落係 in series, in series 條件係 all in same current, but $i_\Delta \neq i_o$ in 2 resistor)



V=IR

By KVL around loop "A",

$$-500 + i_\Delta(5) + i_o(20) = 0$$

$$\Rightarrow \quad i_\Delta + 4i_o = 100 \quad ...(1)$$

By KCL at node "b",

$$\overset{in}{i_\Delta} - \overset{out}{i_o} + \overset{in}{5i_\Delta} = 0$$

$$\Rightarrow \quad i_o = 6i_\Delta ...(2)$$

Put (2) into (1):

$$i_\Delta + 4(6i_\Delta) = 100$$

$$\Rightarrow \quad i_\Delta = 4A$$

$$\therefore \quad i_o = 24A$$

$$V_\Delta = 4(5) = 20V$$

$$V_o = 24(20) = 480V$$

Total power generated $= P_{500V} + P_{5i_\Delta}$

$= 500(4) + 480(5)(4)$

$= 11600W$

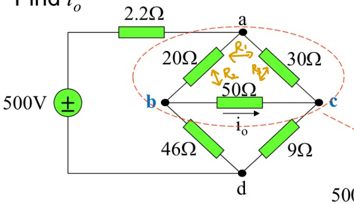
Total power dissipated $= P_{5\Omega} + P_{20\Omega}$

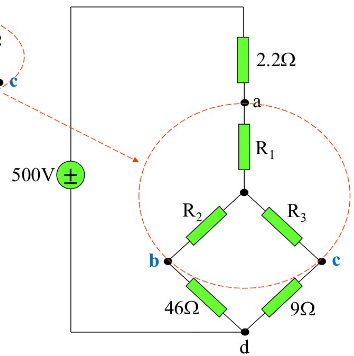
$= 20(4) + 480(24)$

$= 11600W$

Example 3.2

Find i_o



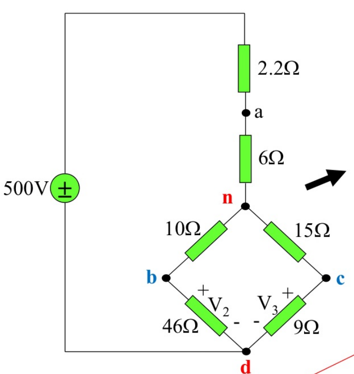


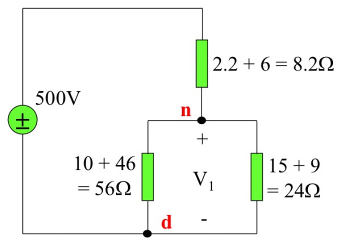
$$R_1 = \frac{20(30)}{20+30+50} = 6\Omega$$

$$R_2 = \frac{20(50)}{20+30+50} = 10\Omega$$

$$R_3 = \frac{30(50)}{20+30+50} = 15\Omega$$

Example 3.2





$$V_1 = \frac{56 // 24}{8.2 + 56 // 24}(500) = 336V$$

$$V_2 = \frac{46}{10+46}(336) = 276V$$

$$V_3 = \frac{9}{15+9}(336) = 126V$$

$$i_o = \frac{V_2 - V_3}{50} = 3A$$

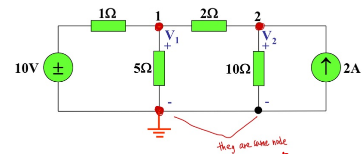
Voltage divider

Nodal Analysis (KCL use repeatly)

Procedure

Steps:

1. Identify all essential nodes (●); no. of nodes = n_e. (3)
2. Choose and label the reference node with ⏚ (usually it connects most branches, but theoretically arbitrary).
3. Assign node voltage variable (with respect to the reference node) to each essential node. (V_1, V_2) V_3 no need (因 $V_3 = 0$) reference node
4. Write one ***KCL*** equation for each essential node to give $(n_e - 1)$ equations in total.
5. Solve the linear equations.



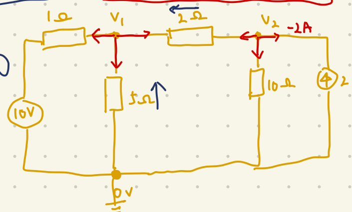
By V i

[illegible] all term start with V_1

$\boxed{\text{Draw all current as flowing out}}$

KCL at node 1

$$\frac{(V_1 - 10)}{1} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{2} = 0$$



KCL at node 2

$$\frac{V_2 - 0}{10} + \frac{V_2 - V_1}{2} - 2 = 0$$

$$17V_1 - 5V_2 = 100$$

$$-5V_1 + 6V_2 = 20$$

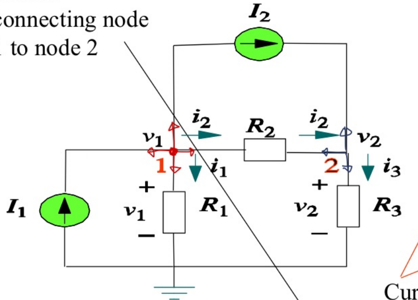
$$\begin{pmatrix} 17 & -5 \\ -5 & 6 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 100 \\ 20 \end{pmatrix}$$

$V_1 = 9.09\,V$

$V_2 = 10.91\,V$

Example 3.4

Conductance of branch connecting node 1 to node 2



$-I_1 + \frac{V_1 - V_2}{R_2} + I_2 + \frac{V_1 - 0}{R_1} = 0$

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = I_1 - I_2$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} = I_2$$

Current out of node 1/2

Current into node 1/2

Rewrite

都係 node 1 connect 既 R

R connected between node 1 & 2

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Total conductance from each branch connected to node 1

Total conductance from each branch connected to node 2

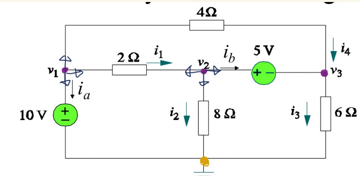
R connect between node 2, 1)

都係 node 2 connect 既 R

Important!

- So far, only current sources connected to essential nodes. (voltage source not connect to essential node)
- Procedure has to change slightly with voltage sources connected to essential nodes.

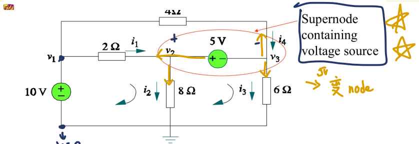
Find Supernode!



Applying KCL at node 1: $i_a + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0$

Applying KCL at node 2: $\frac{V_2 - V_1}{2} + \frac{V_2}{8} + i_b = 0$

Applying KCL at node 3: $\frac{V_3 - V_1}{4} - i_b + \frac{V_3}{6} = 0$



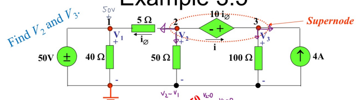
Applying KCL at supernode: $\frac{V_2 - V_1}{2} + \frac{V_2}{8} + \frac{V_3 - V_1}{4} + \frac{V_3}{6} = 0$

Note within supernode: $V_2 - V_3 = 5$

Also note at node 1: $V_1 = 10$

Example 3.5

Find V_2 and V_3.

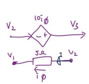


Applying KCL at supernode: $\frac{V_2 - V_1}{5} + \frac{V_2}{50} + \frac{V_3}{100} - 4 = 0 \quad ...(1)$

$V_1 = 50$

Note within supernode:

$V_3 = V_2 + 10 i_\phi \quad ...(2)$



Where $i_\phi = \frac{V_2 - 50}{5} \quad ...(3)$

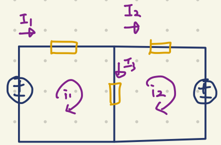
Solving leads to: $i_\phi = 2A$

$V_2 = 60V, V_3 = 80V$

Mesh

→ loop without other loop inside (Not work with current source in mesh)

KVL

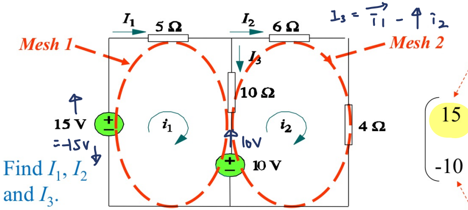


$I_1 = i_1$

$I_2 = i_2$

$I_3 = i_1 - i_2$

(work only when voltage source in each mesh)



Find I_1, I_2 and I_3.

Total resistance in mesh 1

$$\begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

Total resistance in mesh 2

Shared resistance between mesh 1 and mesh 2

Apply KVL to the meshes: (當 i_1 ↻ ∴ $v = -15$

Mesh 1: $-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$ ...(1)

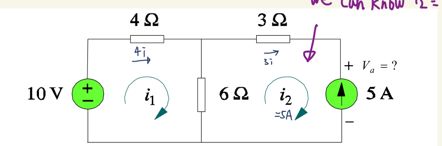
Mesh 2: $-10 + 10(i_2 - i_1) + 6i_2 + 4i_2 = 0$ ...(2)

Solving leads to:

$$i_1 = i_2 = I_1 = I_2 = 1A, \quad I_3 = 0A$$

With current source,

we can know $i_2 = 5A$



$4i_1 - 10V + 6(i_1 - i_2) = 0$ ①

$3i_2 + 6(i_2 - i_1) + V_a = 0$ ②

By $i_2 = 5A$,

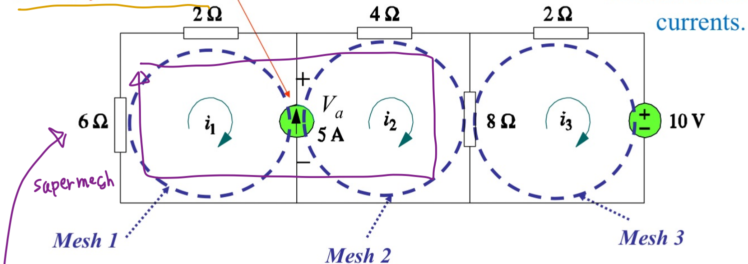
⋮

Current source shared by two meshes

Example 3.6

Common current source shared by two meshes

Find all mesh currents.



✗ Apply KVL to mesh 1: $6i_1 + 2i_1 + V_a = 0$

✗ Apply KVL to mesh 2: $-V_a + 4i_2 + 8(i_2 - i_3) = 0$

✓ Apply KVL to mesh 3: $8(i_3 - i_2) + 2i_3 + 10 = 0$ → ③

✓ Current source constraint: $i_2 - i_1 = 5$ → ②

✓ Apply KVL to supermesh: $6i_1 + 2i_1 + 4i_2 + 8(i_2 - i_3) = 0$ → ①

$V_a = ?$

唔用有 Va 既 equation

→ $i_2 = 5 + i_1$

∴ $8i_1 + 12(5 + i_1) - 8i_3 = 0$

$20i_1 - 8i_3 = 60$

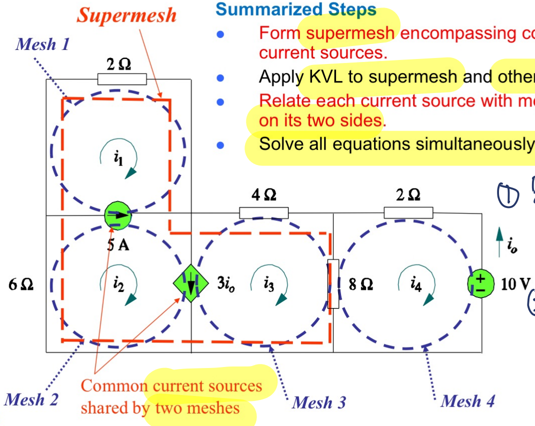
$10i_3 - 8i_2 = 10$

$10i_3 - 8i_1 = 50$

↓

⋮

Mesh Analysis with Supermesh



Summarized Steps

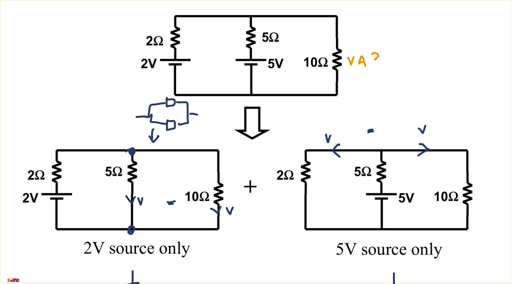
- Form supermesh encompassing common current sources.
- Apply KVL to supermesh and other meshes.
- Relate each current source with mesh currents on its two sides.
- Solve all equations simultaneously.

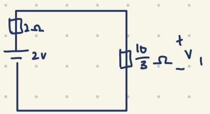
① 只用 supermesh
& other mesh

② 代找 $\bar{i}$ given solve equation

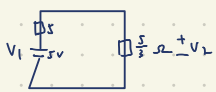
Part 4: → Linear circuit

Superposition → 分做 2 个 subcircuit





$V_1 = \frac{10/3}{2+\frac{10}{3}} \cdot 2 = 1.25V$

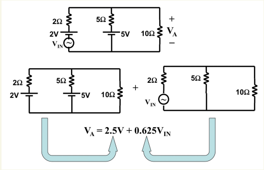


$V_2 = \frac{10/6}{5+10/6} \cdot 5V = 1.25V$

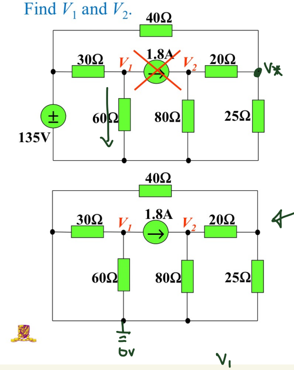
$\therefore V_{10\Omega} = V_1 + V_2 = 2.5V$ //

→ can do longer working than nodal / mesh analyses

case ① : can mix AC & DC circuit



Find V_1 and V_2.



Consider 135-V voltage source only:

voltage divider

parallel V the same

$$\underline{V_1} = \frac{60(135)}{30+60} = 90V$$

$$V_2 = \frac{80}{20+80} \cdot \frac{(20+80)//25(135)}{[(20+80)//25]+40}$$

$= 36V$

Consider 1.8-A current source only:

$$\boxed{V_1 = -1.8(30//60) = -36V}$$

$V_2 = 1.8[80//(25//40+20)]$

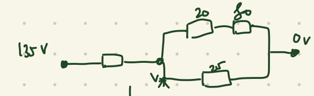
$= 44.16V$

Total:	$V_1 = 90 - 36 = 54V$
	$V_2 = 36 + 44.16 = 80.16V$

V = IR

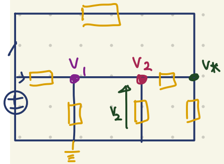
V_1

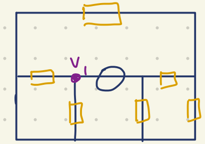
∵ Current same in series

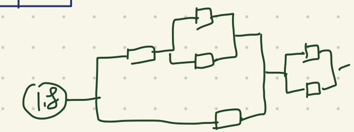


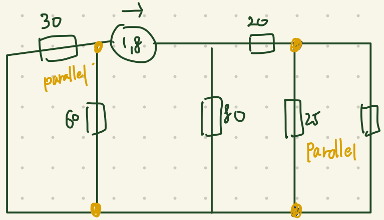
$V_2 = V_* \cdot \frac{80}{20+80}$

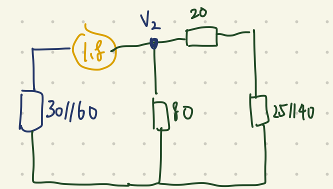
$= \frac{100//25}{100//25+40} \cdot \frac{80}{20+80} \cdot (135$

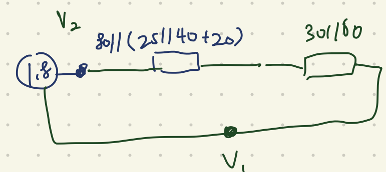




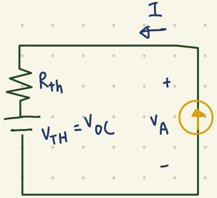






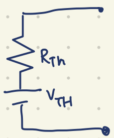


$-1.8 \cdot (30//60)$



$\boxed{V_A = V_{OC} + R_{Th} \cdot I}$

$V_{TH} = V_{OC} \doteq$ Open circuit voltage

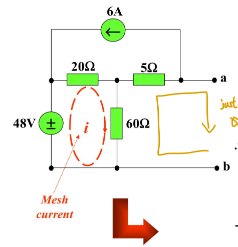


$R_{Th} = V_{TH} / I_{sc} = V_{oc} / I_{sc}$

In case short-circuit current too high, need a R to

$R_{Th} = \frac{V_{oc} - V}{I}$

Method 1 (No dependent source) ☆



Set all sources to zero,

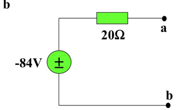
$R_{Th} = 5 + 20 // 60 = 5 + 15 = 20\Omega$

KVL around indicated mesh with a-b open-circuit,

$-48 + 20(i+6) + i60 = 0$

$i = -0.9A$

$\therefore V_{Th} = (-0.9)(60) + (-6)(5) = -84V$



1. Find KVL

$2i - 9 + 1i + 3v_x = 0$

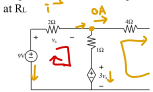
$3i + 3v_x = 9 \rightarrow ①$

$2i = v_x \rightarrow ②$

$i = 1$

$v_x = 2$

Step 1. find Vth, make a open circuit at RL



$2i + i + 3v_x = 9$

$2i = v_x$ → $1.1 + 3 \cdot V_x = 7$

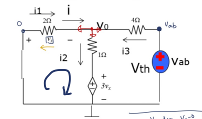
$V_{th} = i + 3v_x$

$V_{th} = 7$

Maximum power obtained only when $R_L = R_{Th}$

$P_{max} = \frac{V_{th}^2}{4R_{th}} = 2.9W$

Step 2. find Rth, **turn off independent source and suppose a voltage source at Vab**



$v_0 - 3v_x + \frac{v_0 - v_{ab}}{4} + \frac{v_0}{2} = 0$

$v_0 = -v_x$

$i_3 = \frac{v_{ab} - v_0}{4}$

$R_{th} = \frac{v_{ab}}{i_3} = \frac{38}{9}$

$0 = \frac{V_0 - 3v_x}{1} + \frac{V_0 - 0}{2} + \frac{V_0 - V_{ab}}{4}$

$V_0 = -V_x$ ($\because$ V_x and V_0)

$4V_0 + \frac{V_0}{2} + \frac{V_0}{4} - \frac{V_{ab}}{4} = 0$

$19V_0 = V_{ab}$

$i_3 = \frac{V_{ab} - V_0}{4}$

$4i_3 = 18V_0$

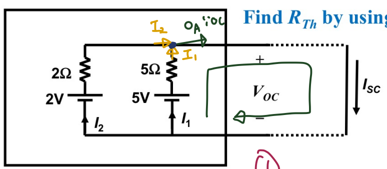
$i_3 = 4.5V_0 = \frac{4.5}{19} V_{ab}$

2. KCL

Method 3

Example 4.2

Open circuit condition



Find R_{Th} by using Method 3

By KCL

$I_2 = -I_1$

By KVL,

$2 + 2I_2 - 5 - 5I_1 = 0$

$7I_1 = 5 - 2$

$I_1 = \frac{3}{7} A$

② Short-circuit

$I_1 = \frac{5}{5} = 1$ A

$I_2 = \frac{2}{2} = 1$ A

$I_{SC} = I_1 + I_2 = 2$ A

① Open-circuit

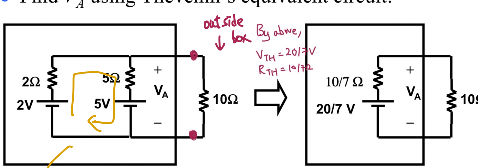
$I_1 = -I_2 = \frac{5-2}{5+2} = \frac{3}{7}$ A

$V_{OC} = 5 - 5 \times \frac{3}{7} = \frac{20}{7}$ V

$\boxed{V_{OC} = V - V_{5\Omega}}$ 下减上

$\therefore \quad R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{10}{7}\ \Omega$

- Find V_A using Thevenin's equivalent circuit.



Application of voltage divider formula

$V_a = \frac{10}{10 + \frac{10}{7}} * \frac{20}{7} V = 2.5V$

$2i + 5i - 2 - 5 = 0$

$7i = 7$

$i = 1$

Method 1: set V as 0,

$R_{TH} = \frac{1}{\frac{1}{2} + \frac{1}{5}} = \frac{10}{7}$

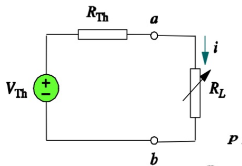
- For Thevenin's circuit below, maximum power transfer occurs when tested with load resistance

$$\boxed{R_L = R_{Th}}$$

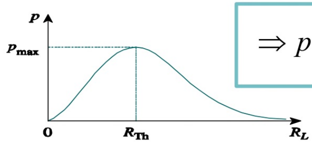
$i = \frac{V_{Th}}{R_{Th}+R_L}$ when $R_L = R_{TH}$ max

- Proof: Power delivered to load

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$



$$\frac{\partial p}{\partial R_L} = 0 \quad \text{and} \quad \frac{\partial^2 p}{\partial R_L^{\,2}} < 0$$

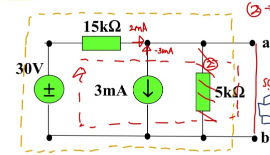


$$\boxed{\Rightarrow p = \frac{v_{Th}^2}{4R_L} \quad \text{at } R_L = R_{Th}}$$

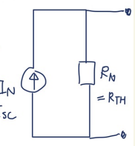
Example 4.5

Find Norton's equivalent circuit between terminals a and b.

② → 因sc, Ω 無需略



→ to get



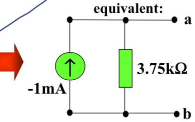
$\frac{30}{15k} = 2mA$

I_{ab}(short circuit) = 2mA – 3mA

I_{sc} = – 1mA

R_N = 15k//5k
= 3.75k

Norton's equivalent:



∴ I_N & R_N parallel

Example 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

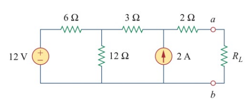


Figure 4.50
For Example 4.13.

Solution:
We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a-b*. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain

$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

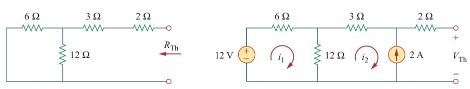


Figure 4.51
For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals *a-b*, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22\text{ V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\text{ W}$$

sin wave

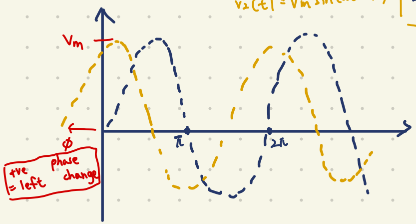
Amplitude

$v_1(t) = V_m \sin(\omega t) = V_m \sin\omega(t+T) = v_1(t+T)$

$v_2(t) = V_m \sin(\omega t + \phi)$

period $\boxed{T = \frac{2\pi}{\omega} = \frac{1}{f}}$

Angular frequency $\omega = 2\pi f$



Impedance of Resistor

Time domain: $v(t) = R \times i(t)$

Phasor domain: $\mathbf{V} = R \times \mathbf{I}$

Impedance: $\mathbf{Z_R} = \mathbf{V}/\mathbf{I} = \boxed{R} =$ resistance

Impedance of Capacitor

Time domain: $i(t) = \underset{\text{capacitance}}{C} \times \frac{d}{dt}(v(t))$

Phasor domain: $\mathbf{I} = C \times j\omega(\mathbf{V})$

(Still remember $\frac{d}{dt} \to j\omega$ in phasor domain)

Impedance: $\mathbf{Z_C} = \mathbf{V}/\mathbf{I} = \frac{1}{j\omega C} = \boxed{-\frac{j}{\omega C}}$

Reactance: $X_C = -\frac{1}{\omega C}$

V lags **I** by 90° or **I** leads **V** by 90° because of $-j$

Impedance of Inductor

Time domain: $v(t) = L \times \frac{d}{dt}(i(t))$

Phasor domain: $\mathbf{V} = L \times j\omega(\mathbf{I})$

(Still remember $\frac{d}{dt} \to j\omega$ in phasor domain)

Impedance: $\mathbf{Z_L} = \mathbf{V}/\mathbf{I} = \boxed{j\omega L}$

Reactance: $X_L = \omega L$

V leads **I** by 90° or **I** lags **V** by 90° because of $+j$

$$\mathbf{Z} = R + jX$$

Resistance: $R = Re(\mathbf{Z})$

Reactance: $X = Im(\mathbf{Z}) = \begin{cases} positive \to inductive \\ negative \to capacitive \end{cases}$

Example 5.1

(In Series)

- $R = 15\Omega; C = 800\mu F; L = 0.2H$
- $\omega_o = 50 rad/s$
- Then $Z_R = 15\Omega$

$Z_C = \frac{1}{j\omega_o C} = -j25\Omega$

$Z_L = j\omega_o L = j10\Omega$

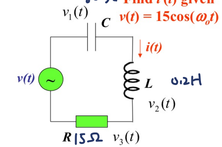
$Z = 15 - j15$ $R + jX$ $= 15\sqrt{2}\angle -45°$

magnitude $= 15\sqrt{2}$
phase $= -45°$

- If $v(t) = 15\cos\omega_o t \Rightarrow \mathbf{V} = 15\angle 0°$
- Then $\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{15}{15\sqrt{2}\angle -45°} = \frac{1}{\sqrt{2}}\angle 45°$

$i(t) = \frac{1}{\sqrt{2}}\cos(\omega_o t + \frac{\pi}{4})$

Find $i(t)$ given $v(t) = 15\cos(\omega_o t)$

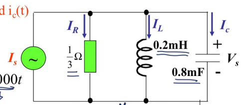


Example 5.2

Determine $v_s(t)$, $i_L(t)$ and $i_c(t)$

Then $I_s = 30\angle 0°$

$i_s(t) = 30\cos 5000t$ $=\omega_o$

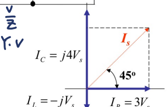


$I = \frac{V}{Z} = Y \cdot V$

$Y_R = 3S = \left(\frac{1}{R}\right) = \frac{1}{\frac{1}{3}} \to I_R = 3V_s$

$Y_L = \frac{1}{j\omega_o L} = -jS \to I_L = -jV_s$

$Y_C = j\underset{5000}{\omega_o}\underset{0.8\cdot 10^{-3}}{C} = j4S \to I_C = j4V_s$



$\underset{30\angle 0°}{I_s} = I_R + I_L + I_C = (3 + j3)V_s$

$v_s(t) = \frac{10}{\sqrt{2}}\cos(5000t - \frac{\pi}{4})$	$i_L(t) = \frac{10}{\sqrt{2}}\cos(5000t - \frac{3\pi}{4})$	$i_c(t) = 20\sqrt{2}\cos(5000t + \frac{\pi}{4})$

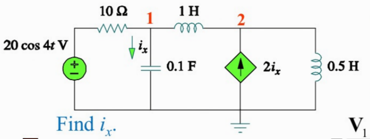
$V_s = \frac{30}{3+j3}$

$= 5 - 5j$

$= \frac{10}{\sqrt{2}}\angle -45°$

Find i @ Nodal

Example 5.3 – Nodal Analysis



Find i_x.

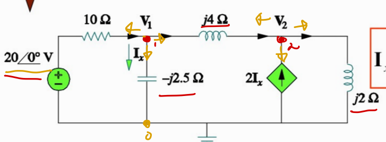
$20\cos(4t) \Rightarrow \boxed{20\angle 0^o}, \omega_o = 4rad/s$

Inductor value

$1H \quad \Rightarrow j\omega_o L = j4\Omega$ $\quad j \cdot 4 \cdot 1$

$0.5H \quad \Rightarrow j\omega_o L = j2\Omega$ $\quad j \cdot 4 \cdot 0.5$

$0.1F \quad \Rightarrow 1/j\omega_o C = -j2.5\Omega$ $\left(\frac{1}{j \cdot 4 \cdot 0.1}\right)$ $= -j\frac{1}{wc}$



KCL at node 1: $\boxed{V_1}$

$$\left(\frac{1}{10}+\frac{1}{-j2.5}+\frac{1}{j4}\right)\mathbf{V}_1-\frac{\mathbf{V}_2}{j4}=\frac{20}{10}$$

$\rightarrow (V_1-20)\cdot\frac{1}{10}+\frac{(V_1-0)}{-j2.5}+\frac{(V_1-V_2)}{j4}=0$

KCL at node 2:

$$-\frac{\mathbf{V}_1}{j4}+\left(\frac{1}{j4}+\frac{1}{j2}\right)\mathbf{V}_2=2\mathbf{I}_x \text{ and } \boxed{\mathbf{I}_x=\frac{\mathbf{V}_1}{-j2.5}}$$

$\rightarrow \frac{V_2-V_1}{j4}+\frac{V_2-0}{j2}+-2I_x=0$

$\boxed{\rightarrow V_1 = I_x R \Rightarrow I_x = \frac{V_1}{-2.5j}}$

$$\Rightarrow \begin{bmatrix} 0.1+j0.15 & j0.25 \\ 0.55 & 0.75 \end{bmatrix}\begin{pmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{pmatrix}=\begin{pmatrix}2\\0\end{pmatrix}$$

$$\begin{pmatrix} 0.1+0.15j & -0.25j \\ -0.55j & 0.75j \end{pmatrix}\begin{pmatrix}V_1\\ V_2\end{pmatrix}=\begin{pmatrix}2\\0\end{pmatrix}$$

Solving:

$\mathbf{V}_1 = 18.97\angle 18.43^o V$

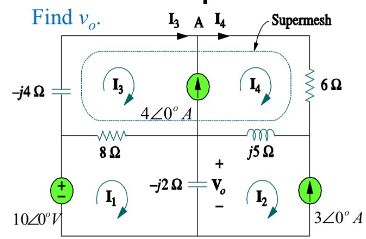
$\mathbf{V}_2 = 13.91\angle 198.43^o V$

$$\boxed{\mathbf{I}_x=\frac{18.97\angle 18.43^o}{2.5\angle -90^o}=7.59\angle 108.4^o}$$

$$\boxed{\Rightarrow i_x(t)=7.59\cos(4t+108.4^o)A}$$

Example 5.4 – Mesh Analysis

Find v_o.



Mesh 1:

$$(8-j2)\mathbf{I}_1+j2\mathbf{I}_2-8\mathbf{I}_3=10$$

Mesh 2: No way to kvl

$$\mathbf{I}_2=-3$$

Supermesh:

$$-8\mathbf{I}_1-j5\mathbf{I}_2+(8-j4)\mathbf{I}_3+(6+j5)\mathbf{I}_4=0$$

Current source constraint between mesh 3 and 4: $\mathbf{I}_4=\mathbf{I}_3+4$

From Mesh 1 and Mesh 2:

$$(8-j2)\mathbf{I}_1-8\mathbf{I}_3=10+j6$$

From Mesh 2 and Supermesh:

$$-8\mathbf{I}_1+(14+j)\mathbf{I}_3=-j15-24-j20$$

Then:

$$\begin{bmatrix} 8-j2 & -8 \\ -8 & 14+j \end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_3\end{bmatrix}=\begin{bmatrix}10+j6\\ -24-j35\end{bmatrix}$$

$$\mathbf{I}_1=\frac{(14+j)(10+j6)+8(-24-j35)}{(8-j2)(14+j)-64}$$

$$\therefore \mathbf{I}_1=\frac{-58-j186}{50-j20}=3.618\angle 274.5^o A$$
$$=0.284-j3.607A$$

$$\mathbf{V}_o=-j2(\mathbf{I}_1-\mathbf{I}_2)=-j2(3.284-j3.607)$$
$$=9.756\angle 222.32^o V$$

circuit operates at $\omega = 50$ rad/s.

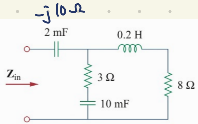
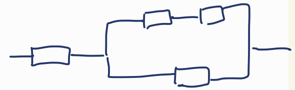


Figure 9.23
For Example 9.10.

Solution:
Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor
$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the10-mF capacitor
$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor



Then

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$
$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$
$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$
$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{\text{in}} = 3.22 - j11.07\ \Omega$$

Example 9.11

Determine $v_o(t)$ in the circuit of Fig. 9.25.

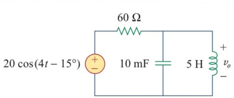


Figure 9.25
For Example 9.11.

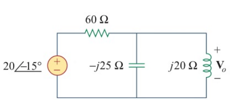
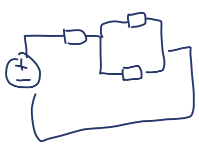


Figure 9.26
The frequency domain equivalent of the circuit in Fig. 9.25.



Solution:
To do the analysis in the frequency domain, we must first transform the time domain circuit in Fig. 9.25 to the phasor domain equivalent in Fig. 9.26. The transformation produces

$$v_s = 20\cos(4t - 15°) \quad \Rightarrow \quad \mathbf{V}_s = 20\underline{/-15°}\ \text{V}, \qquad \omega = 4$$
$$10\ \text{mF} \quad \Rightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\ \Omega$$
$$5\ \text{H} \quad \Rightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

Let

$\mathbf{Z}_1$ = Impedance of the 60-Ω resistor
$\mathbf{Z}_2$ = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

Then $\mathbf{Z}_1 = 60\ \Omega$ and

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

By the voltage-division principle,

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\underline{/-15°})$$
$$= (0.8575\underline{/30.96°})(20\underline{/-15°}) = 17.15\underline{/15.96°}\ \text{V}$$

We convert this to the time domain and obtain

$$v_o(t) = 17.15\cos(4t + 15.96°)\ \text{V}$$

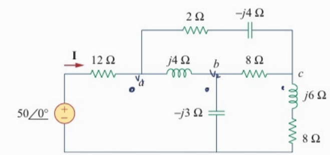
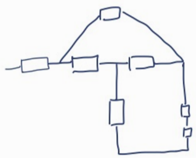


Figure 9.28
For Example 9.12.



Solution:
The delta network connected to nodes a, b, and c can be converted to the Y network of Fig. 9.29. We obtain the Y impedances as follows using Eq. (9.68):

$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\ \Omega$$
$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$

The total impedance at the source terminals is

$$\mathbf{Z} = 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)$$
$$= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)$$
$$= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3}$$
$$= 13.6 + j1 = 13.64\underline{/4.204°}\ \Omega$$

The desired current is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\underline{/0°}}{13.64\underline{/4.204°}} = 3.666\underline{/-4.204°}\ \text{A}$$

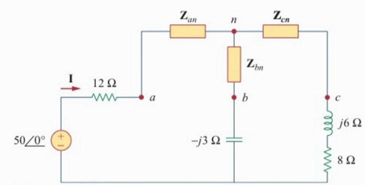


Figure 9.29
The circuit in Fig. 9.28 after delta-to-wye transformation.