

ELEG2202 Homework3

Q1. 1) Find the phasor corresponding to the following signal:

(a) $i(t) = -5 \sin(50t + 20^\circ) \text{ mA}$

2) Obtain the sinusoidal signal corresponding to the following phasor:

(a) $V_1 = 6 + j8 \text{ V}, \omega = 20$

Q2. A network consisting of an independent current source and a dependent current source is shown in Fig. 1, please find the Thevenin equivalent circuit as seen from node a and b.

Hints: you can use the Method 2 for R_{th} determination.

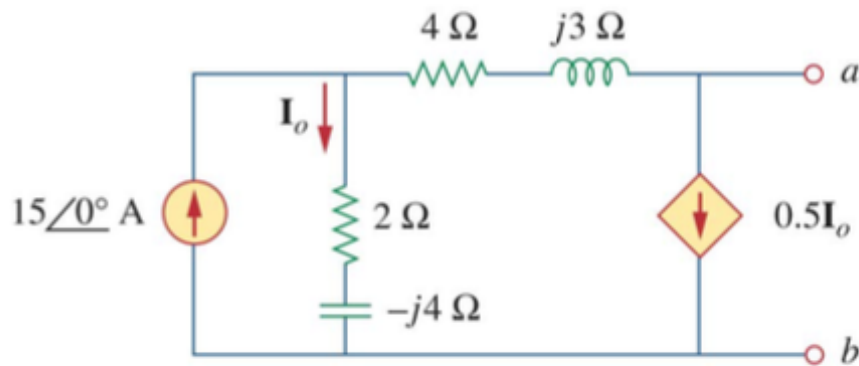


Figure 1

Q3. Find I_o using Norton equivalent circuit seen from node a, b.

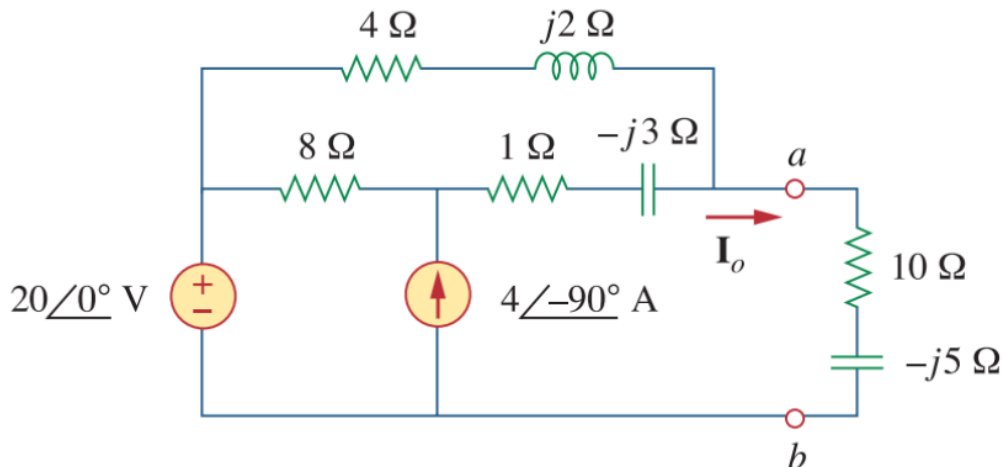


Figure 2

Q4. The voltage across a load is $v(t) = 10\cos(\omega t + 20^\circ) \text{ V}$ and the current

through the element in the direction of the voltage drop is

$i(t) = 2\sin(\omega t - 40^\circ)$ A. Find:

- (a) the complex and apparent powers,
- (b) the average power and the load impedance.

Q1)

Q1. 1) Find the phasor corresponding to the following signal:

(a) $i(t) = -5 \sin(50t + 20^\circ) \text{ mA}$

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a) $\hat{i}(t) = -5 \sin(50t + 20^\circ) \text{ mA}$
 $= 5 \cos(50t + 20^\circ + 90^\circ) = 5 \angle 110^\circ \text{ mA}$

2) $V_1 = 6 + j8 \text{ V} = 10 \angle 53.13^\circ$
 $\omega = 40$
 $\therefore V_1(t) = 10 \cos(\omega t + 53.13^\circ)$
 $= 10 \cos(40t + 53.13^\circ)$

Q2. A network consisting of an independent current source and a dependent current source is shown in Fig. 1, please find the Thevenin equivalent circuit as seen from node a and b.

Hints: you can use the Method 2 for R_{TH} determination.

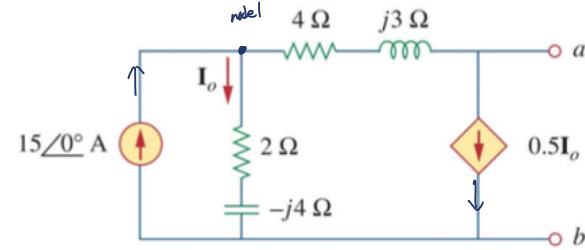


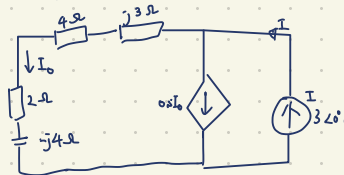
Figure 1

Let node 1 be position in Fig.

By KCL,
 $15 = I_o + 0.5 I_o$
 $I_o = 10 \text{ A}$

By KVL
 $-I_o(2 - j4) + 0.5 I_o(4 + j3) + V_{TH} = 0$
 $\therefore V_{TH} = 10(2 - j4) - 5(4 + j3)$
 $= -j55$
 $= 55 \angle -90^\circ \text{ V}$

For Z_{TH} , remove independent source and connect 3 A constant source.



By KCL,
 $3 = I_o + 0.5 I_o$
 $I_o = 2 \text{ A}$

By KVL
 $V_s = I_o(4 + j3 + 2 - j4)$
 $= 2(6 - j)$

$\therefore Z_{TH} = \frac{V_s}{I_s} = \frac{2(6 - j)}{3} = 4 - j\frac{2}{3} \Omega$

Q3. Find I_o using Norton equivalent circuit seen from node a, b.

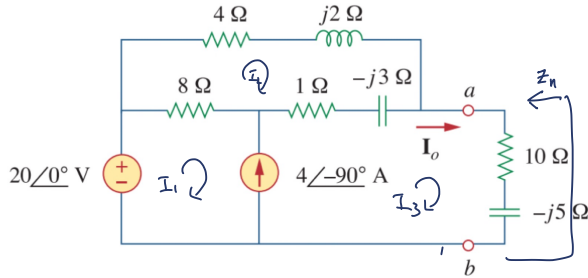
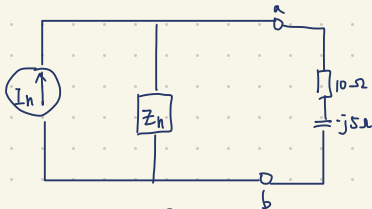


Figure 2

$$\begin{aligned} \text{For } Z_n, \quad Z_n &= (8 + 1 - j3) \parallel (4 + j2) \\ &= (9 - j3)(4 + j2) / (9 - j3 + 4 + j2) \\ &= 42 + j6 / 13 - j1 \\ &= 3.176 + j0.706 \end{aligned}$$

For I_o , circuit like



$$\begin{aligned} I_o &= I_N \cdot \left(\frac{Z_n}{Z_n + 10 - j5} \right) \\ &= (4.198 \angle -32.68^\circ) \cdot \left(\frac{3.176 + j0.706}{13.176 - j5 + j0.706} \right) \\ &= (4.198 \angle -32.68^\circ) (0.202 + j0.1195) \\ &= 0.9853 \angle -2.1^\circ \text{ A} \end{aligned}$$

By KVL in mesh 2,

$$\begin{aligned} 8(I_2 - I_1) + (4 + j2)(I_2) + (1 - j3)(I_2 - I_3) &= 0 \\ -8I_1 + (8 + 4 + j2 + 1 - j3)I_2 - (1 - j3)I_3 &= 0 \\ -8I_1 + (13 - j1)I_2 - (1 - j3)I_3 &= 0 \rightarrow \textcircled{1} \end{aligned}$$

By KCL in Supermesh (mesh 1 + mesh 3),

$$\begin{aligned} -I_1 - 4 \angle 90^\circ + I_3 &= 0 \\ I_3 &= I_1 + 4 \angle 90^\circ \\ -I_1 + 0I_2 + I_3 &= j4 \rightarrow \textcircled{2} \end{aligned}$$

By KVL in Supermesh,

$$\begin{aligned} -20 \angle 0^\circ + 8(I_1 - I_2) + (1 - j3)(I_3 - I_2) &= 0 \\ 8(I_1) + (8 - 1 + j3)I_3 + (1 - j3)I_3 - 20 &= 0 \\ 8(I_1) + (j3 - 9)I_2 + (1 - j3)I_3 &= 20 \rightarrow \textcircled{3} \end{aligned}$$

$$\therefore \begin{bmatrix} -8 & (13 - j1) & (j3 - 1) \\ -1 & 0 & 1 \\ 8 & (j3 - 9) & (1 - j3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ j4 \\ 20 \end{bmatrix}$$

$$\begin{aligned} \Delta &= \begin{vmatrix} -8 & (13 - j1) & (j3 - 1) \\ -1 & 0 & 1 \\ 8 & (j3 - 9) & (1 - j3) \end{vmatrix} \\ &= (-8) \begin{vmatrix} 0 & 1 \\ j3 - 9 & 1 - j3 \end{vmatrix} - (13 - j1) \begin{vmatrix} -1 & 1 \\ 8 & 1 - j3 \end{vmatrix} + (j3 - 1) \begin{vmatrix} -1 & 0 \\ 8 & j3 - 9 \end{vmatrix} \\ &= -8(9 - j3) - (13 - j1)(-9 + j3) + (j3 - 1)(9 - j3) \end{aligned}$$

$$\begin{aligned} &= 42 + j6 \\ \Delta &= \begin{vmatrix} -8 & 13 - j1 & 0 \\ -1 & 0 & j3 \\ 8 & j3 - 9 & 10 \end{vmatrix} \\ &= 162 - j74 \end{aligned}$$

$$I_3 = \frac{42 + j6}{42 + j6} = 3.533 - j2.667 \text{ A}$$

$$I_N = I_3 = 4.198 \angle -32.68^\circ$$

$$10 \angle 20^\circ$$

Q4. The voltage across a load is $v(t) = 10\cos(\omega t + 20^\circ)\text{V}$ and the current

through the element in the direction of the voltage drop is

$$i(t) = 2\sin(\omega t - 40^\circ)\text{A. Find: } 2 \cos(\omega t - 130^\circ)$$

(a) the complex and apparent powers,

(b) the average power and the load impedance.

$$\begin{aligned} a) \quad v(t) &= 10 \angle 20^\circ \\ i(t) &= 2 \sin(\omega t - 40^\circ) = 2 \cos(\omega t - 90^\circ - 40^\circ) = 2 \angle -130^\circ \end{aligned}$$

$$\begin{aligned} \text{Complex power} = S &= VI^* \\ &= 10 \angle 20^\circ \cdot 2 \angle +130^\circ \\ &= 20 \angle 150^\circ \\ &= 20 \cos(150^\circ) + j 20 \sin(150^\circ) \\ &= -17.32 + j10 \text{ //} \end{aligned}$$

$$\begin{aligned} \text{Apparent Power} = |S| &= \sqrt{(-17.32)^2 + (10)^2} \\ &= 19.99 \text{ VA //} \end{aligned}$$

$$\begin{aligned} b) \quad \text{average power} &= VI \cos(\theta_v - \theta_i) \\ &= (10)(2) \cos(20^\circ - (-130^\circ)) \\ &= 20 \cos(150^\circ) \\ &= -17.32 \text{ W //} \end{aligned}$$

load impedance

$$V = IZ$$

$$Z = \frac{V}{I}$$

$$\begin{aligned} &= \frac{10 \angle 20^\circ}{2 \angle +130^\circ} \\ &= 5 \angle 150^\circ \text{ //} \end{aligned}$$