(i) 0.76, 0.32, 0.54.

(ii) Finally, we have $(1 : 0.86), (2 : 0.71), (3 : 1.00), (4 : 0.29), (5 : 1.00), (6 : 0.52), (7 : 0.14), (8 : 0.00), (9 : 0.38), (10 : 0.00)$, so nodes with indexes $[1, 2, 3, 5, 6]$ will belong to class "+" and nodes with indexes $[4, 7, 8, 9, 10]$ will belong to class "-".

(i)

$$
\begin{aligned}
b_1(x_1) &= \frac{1}{Z}\phi_i(x_i)m_{21}(x_1) \\
&= \frac{1}{Z}\phi_1(x_1)\Sigma_{x_2}\phi_2(x_2)\psi_{21}(x_2,x_1)m_{32}(x_2)m_{42}(x_2) \\
&= \frac{1}{Z}\phi_1(x_1)\Sigma_{x_2}\phi_2(x_2)\psi_{21}(x_2,x_1)(\Sigma_{x_3}\phi_3(x_3)\psi_{32}(x_3,x_2)\Sigma_{x_4}\phi_4(x_4)\psi_{42}(x_4,x_2)) \\
&= \frac{1}{Z}\Sigma_{x_2}\Sigma_{x_3}\Sigma_{x_4}\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\psi_{21}(x_2,x_1)\psi_{32}(x_3,x_2)\psi_{42}(x_4,x_2)
\end{aligned}
$$

(ii) According to the properties of grapical model, we have

$$
\begin{aligned}
p(x_1|y_1,y_2,y_3,y_4) &= \frac{1}{Z}\Sigma_{x_2}\Sigma_{x_3}\Sigma_{x_4}p(x_1,x_2,x_3,x_4|y_1,y_2,y_3,y_4) \\
&= \frac{1}{Z}\Sigma_{x_2}\Sigma_{x_3}\Sigma_{x_4}\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\psi_{21}(x_2,x_1)\psi_{32}(x_3,x_2)\psi_{42}(x_4,x_2) \\
&= b_1(x_1)
\end{aligned}
$$

BTW, if you are not familiar with grapical model, you can refer to the lecture *Probabilistic Graphical Models* or *Graphical Models and Belief Propagation* in MIT 6.869 Advances in Computer Vision.

(iii) The results are as below, for which the caculation is done by hand with cmd assitance of Python,

$$b_1(x_1) = \frac{1}{Z}\begin{bmatrix}1.42681\\1.299\end{bmatrix}$$

$$b_2(x_2) = \frac{1}{Z}\begin{bmatrix}2.4871\\0.24681\end{bmatrix}$$

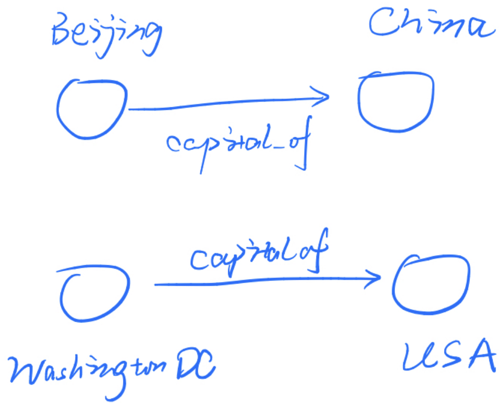
$$b_3(x_3) = \frac{1}{Z}\begin{bmatrix}0.418\\2.300881\end{bmatrix}$$

$$b_4(x_4) = \frac{1}{Z}\begin{bmatrix}0.231781\\2.4871\end{bmatrix}$$

$$b_5(x_5) = \frac{1}{Z}\begin{bmatrix}1.12009\\0.31009\end{bmatrix}$$

As we can see, x_2, y_4 is influenced by y_2, y_4 most respectively. ψ_{12}, ψ_{34} indicate that node 1 and node 2, and node 3 and node 4 tend to be the same slightly. And ψ_{23}, ψ_{35} indicate that node 2 and node 3, and node 3 and node 5 tend to be the opposite with high possibility. So the results are as expected.

Take this graph as an example,



Let all of $\mathbf{e}$ to be

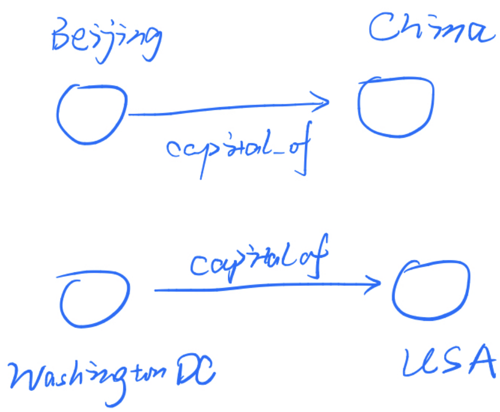
$$\begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}$$

and ℓ to be

$$\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

We would make the objective to be 0, but obviously the embeddings make no sense.

Take this graph as an example,



Let all of $\mathbf{e}$ to be

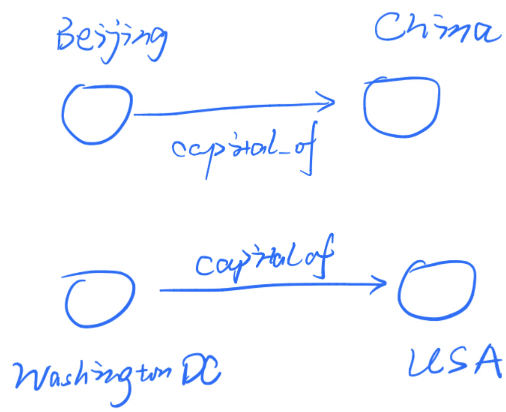
$$\begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}$$

and ℓ to be

$$\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

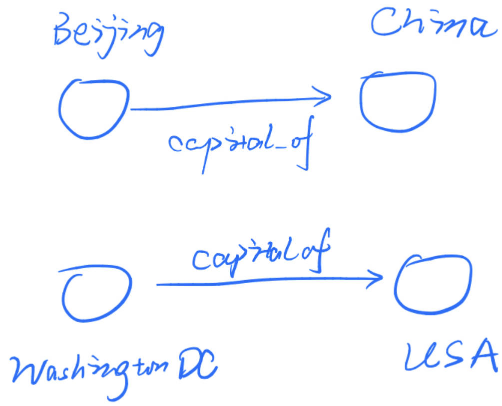
We would make the objective to be 0, but obviously the embeddings make no sense.

Take this graph as an example,



When we don't normalize the entity embeddings, the algorithm tends to trivially change the $||e||_2$ to optimize to the objective. In this example, in this case, one possible generation is that $||e||_2$ for *China* and *Beijing* tends to be very small and $||e||_2$ for *USA* and *Washington, DC* tends to be very large, or vice versa.

Take this graph as an example,



after we add a new node *Earth* and the relation from **China** and *USA*, $a_country_of$, it's impossible to find a proper embedding of $a_country_of$ and *Earth* to meet with the conditions in a 2-d embedding space, since *Beijing* $\leftarrow$ *China* and *Washington, DC* $\leftarrow$ *USA* are two parallel lines (vectors) in the embedding space.

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Information sheet
CS224W: Machine Learning with Graphs

Assignment Submission Fill in and include this information sheet with each of your assignments. This page should be the last page of your submission. Assignments are due at 11:59pm and are always due on a Thursday. All students (SCPD and non-SCPD) must submit their homework via GradeScope (http://www.gradescope.com). Students can typeset or scan their homework. Make sure that you answer each (sub-)question on a separate page. That is, one answer per page regardless of the answer length. Students also need to upload their code on Gradescope. Put all the code for a single question into a single file and upload it.

Late Homework Policy Each student will have a total of *two* late periods. *Homework are due on Thursdays at 11:59pm PT and one late period expires on the following Monday at 11:59pm PT*. Only one late period may be used for an assignment. Any homework received after 11:59pm PT on the Monday following the homework due date will receive no credit. Once these late periods are exhausted, any assignments turned in late will receive no credit.

Honor Code We strongly encourage students to form study groups. Students may discuss and work on homework problems in groups. However, each student must write down their solutions independently, i.e., each student must understand the solution well enough in order to reconstruct it by him/herself. Students should clearly mention the names of all the other students who were part of their discussion group. Using code or solutions obtained from the web (GitHub/Google/previous year's solutions etc.) is considered an honor code violation. We check all the submissions for plagiarism. We take the honor code very seriously and expect students to do the same.

Your name: ________________________________

Email: ____________________ **SUID:** ________________

Discussion Group: ________________________________

I acknowledge and accept the Honor Code.

(Signed) ________________________________