Artificial Intelligence

Hai Thi Tuyet Nguyen

Outline

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CHAPTER 7
QUANTIFYING UNCERTAINTY

7.1 Acting Under Uncertainty

7.1 Uncertainty

Let action A_t = leave for airport t minutes before flight
Will A_t get me there on time?

Problems:
1) partial observability (road state, other drivers' plans, etc.)
2) noisy sensors (traffic reports)
3) uncertainty in action outcomes (flat tire, etc.)
4) immense complexity of modelling and predicting traffic

Conclusion:
1) hard to conclude: "A25 will get me there on time"
2) weak conclusions: "A25 will get me there on time if there's no accident on the bridge and it doesn't rain and my tires remain intact etc etc."

Methods for handling uncertainty:
Given the available evidence, A_{25} will get me there on time with probability 0.04

7.1 Probability

- Probability provides a way of summarizing the uncertainty that comes from our laziness and ignorance
    - Laziness: it is too much work to list the complete set of antecedents or consequents, too hard to use such rules.
    - Ignorance: We have no complete theory for the domain, lack of relevant facts, initial conditions, etc.
- Probabilities relate propositions to one's own state of knowledge

  $P(A_{25}|\text{no reported accidents}) = 0.06$
- Probabilities of propositions change with new evidence:

  $P(A_{25}|\text{no reported accidents, 5 a.m.}) = 0.15$

7.1 Making decisions under uncertainty

Suppose I believe the following:

$P(A_{25}$ gets me there on time$|\ldots) = 0.04$

$P(A_{90}$ gets me there on time$|\ldots) = 0.70$

$P(A_{120}$ gets me there on time$|\ldots) = 0.95$

$P(A_{1440}$ gets me there on time$|\ldots) = 0.9999$

Which action to choose?

Depends on my preferences for missing flight vs. airport cuisine, etc.

- Utility theory is used to represent and infer preferences;
  "Utility" ~ the quality of being useful
- Decision theory = utility theory + probability theory
  - an agent is rational if and only if it chooses the action that yields the highest expected utility, averaged over all the possible outcomes of the action; ~ maximum expected utility (MEU).

7.2 Basic Probability Notation

7.2 Probability basics

- Begin with a set Ω - the sample space
  e.g., 6 possible rolls of a die.
  $\omega \in \Omega$ is a sample point/possible world/atomic event

- A probability space or probability model is a sample space with an assignment $P(\omega)$:
  $0 \leq P(\omega) \leq 1$ for every $\omega \in \Omega$ and $\Sigma_{\omega} P(\omega) = 1$
  e.g., $P(1)=P(2)=P(3)=P(4)=P(5)=P(6)=1/6$

- An event A is any subset of Ω:
  $P(A) = \Sigma_{\{\omega \in A\}} P(\omega)$
  E.g., $P(\text{die roll} < 4) = P(1) + P(2) + P(3) = 1/6 + 1/6 + 1/6 = 1/2$

7.2 Random variables

- Variables in probability theory are called random variables and their names begin with an uppercase letter, e.g., Total, Die_1,
  Every random variable has a domain - the set of possible values it can take on.
  The domain of Total for two dice is the set {2,...,12}
  The domain of Die_1 is {1,...,6}
- P induces a probability distribution for any random variable X:
  $P(X=x_i) = \Sigma_{\{\omega:X(\omega)=x_i\}}P(\omega)$
  e.g., P (Odd = true) = P (1) + P (3) + P (5) = 1/6 + 1/6 + 1/6 = 1/2

7.2 Propositions

A proposition as the event (set of sample points) under some conditions

Given Boolean random variables A and B:

event a = set of sample points where $A(\omega) = true$

event $\neg a$ = set of sample points where $A(\omega) = false$

event $a \wedge b$ = points where $A(\omega) = true$ and $B(\omega) = true$

In AI applications, the sample points are defined by the values of a set of random variables.

With Boolean variables, sample point = propositional logic model

e.g., $A=true$, $B=false$, or $a \wedge \neg b$.

Proposition = disjunction of atomic events in which it is true

e.g., $(a \vee b) \equiv (\neg a \wedge b) \vee (a \wedge \neg b) \vee (a \wedge b)$

$\Rightarrow P(a \vee b) = P(\neg a \wedge b) + P(a \wedge \neg b) + P(a \wedge b)$

7.2 Syntax for propositions

Propositional or Boolean random variables

e.g., Cavity (do I have a cavity?)

Cavity = true is a proposition, also written cavity

Discrete random variables (finite or infinite)

e.g., Weather is one of ⟨sunny, rain, cloudy, snow⟩

Weather = rain is a proposition

Values must be exhaustive and mutually exclusive

Continuous random variables (bounded or unbounded)

e.g., Temp = 21.6; also allow, e.g., Temp < 22.0.

Arbitrary Boolean combinations of basic propositions

7.2 Prior probability

Prior or unconditional probabilities of propositions

e.g., P(Cavity = true) = 0.1 and P(Weather = sunny) = 0.72

correspond to belief prior to arrival of any (new) evidence

Probability distribution gives values for all possible assignments:

P(Weather) = $\langle 0.72, 0.1, 0.08, 0.1 \rangle$ (normalized, i.e., sums to 1)

Joint probability distribution for a set of r.v.s gives the probability of every atomic event on those r.v.s (i.e., every sample point)

P(Weather, Cavity) = a 4×2 matrix of values:

$Weather =$	$sunny$	$rain$	$cloudy$	$snow$
$Cavity = true$	0.144	0.02	0.016	0.02
$Cavity = false$	0.576	0.08	0.064	0.08

7.2 Conditional probability

Conditional or posterior probabilities

e.g., $P(cavity|toothache) = 0.8$ i.e., given that toothache is all I know

If we know more, e.g., cavity is also given, then we have

$P(cavity|toothache, cavity) = 1$

New evidence may be irrelevant, allowing simplification, e.g.,

$P(cavity|toothache, 49ersWin) = P(cavity|toothache) = 0.8$

7.2 Conditional probability

Definition of conditional probability:

$$P(a|b) = \frac{P(a \wedge b)}{P(b)} \text{ if } P(b) \neq 0$$

Product rule gives an alternative formulation:

$$P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a)$$

A general version holds for whole distributions,

e.g., $\mathbf{P}(Weather, Cavity) = \mathbf{P}(Weather|Cavity)\mathbf{P}(Cavity)$

Chain rule is derived by successive application of product rule:

$$\begin{aligned} \mathbf{P}(X_1,\ldots,X_n) &= \mathbf{P}(X_1,\ldots,X_{n-1})\, \mathbf{P}(X_n|X_1,\ldots,X_{n-1}) \\ &= \mathbf{P}(X_1,\ldots,X_{n-2})\, \mathbf{P}(X_{n1}|X_1,\ldots,X_{n-2})\, \mathbf{P}(X_n|X_1,\ldots,X_{n-1}) \\ &= \ldots \\ &= \Pi^n_{i=1} \mathbf{P}(X_i|X_1,\ldots,X_{i-1}) \end{aligned}$$

7.3 Inference Using Full Joint Distributions

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Start with the joint distribution:

	toothache		¬ *toothache*	
	catch	¬ *catch*	*catch*	¬ *catch*
cavity	**.108**	**.012**	**.072**	**.008**
¬ *cavity*	**.016**	**.064**	**.144**	**.576**

For any proposition φ, sum the atomic events where it is true:

$$P(\varphi) = \Sigma_{\omega:\omega\models\varphi} P(\omega)$$

P (toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

7.3 Inference Using Full Joint Distributions

Start with the joint distribution:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

For any proposition φ, sum the atomic events where it is true:

$P(\varphi) = \Sigma_{\omega:\omega\models\varphi} P(\omega)$

P (cavity ∨ toothache) = 0.108+0.012+0.072+0.008+0.016+0.064 = 0.28

7.3 Inference Using Full Joint Distributions

Start with the joint distribution:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	**.108**	**.012**	**.072**	**.008**
¬ cavity	**.016**	**.064**	**.144**	**.576**

Can also compute conditional probabilities:

$$
\begin{aligned}
P(cavity \mid toothache) &= \frac{P(cavity \wedge toothache)}{P(toothache)} \\
&= \frac{0.108 + 0.012}{0.108 + 0.012 + 0.016 + 0.064} = 0.6
\end{aligned}
$$

7.3 Normalization

X: a single variable (Cavity in the example)
E: the list of evidence variables (Toothache in the example), **e**: the list of their observed values
Y: be the remaining unobserved variables (Catch in the example)

The query is P(X|e) and can be evaluated as
$P(X|\mathbf{e}) = \alpha P(X,\mathbf{e}) = \alpha \sum_{y}^{ys} P(X,\mathbf{e},y)$

ys: all possible combinations of values of the unobserved variables Y

General idea: compute distribution on query variable
by **fixing evidence variables** and **summing over unobserved variables**

7.3 Normalization

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	**.108**	**.012**	**.072**	**.008**
¬ cavity	**.016**	**.064**	**.144**	**.576**

P(cavity|toothache) can be viewed as a normalization constant α

P(Cavity|**toothache**) = α P(Cavity, **toothache**)

= α[P(Cavity,**toothache**,catch) + P(Cavity,toothache,¬catch)]

= α[⟨0.108,0.016⟩ + ⟨0.012,0.064⟩]

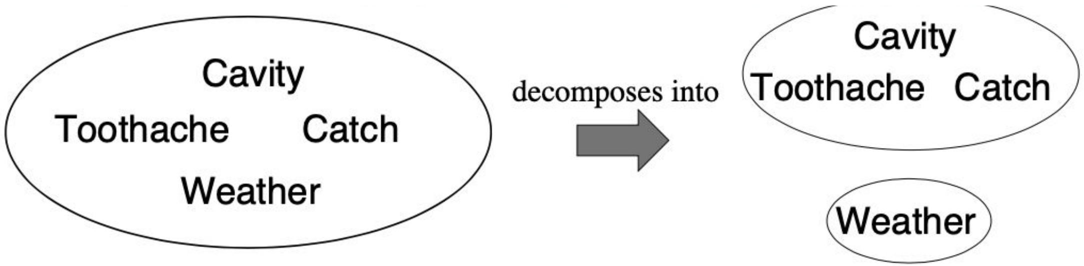
= α⟨0.12,0.08⟩=⟨0.6,0.4⟩

// Cavity as a single variable

// Toothache as evidence variable

// Catch as unobserved variables

7.3 Independence



For indoor dentistry, at least, it seems safe to say that the weather does not influence the dental variables => Dental variables are independent of weather.

A and B are independent iff $P(A|B) = P(A)$ or $P(B|A) = P(B)$ or $P(A, B) = P(A)P(B)$

P(Toothache, Catch, Cavity, Weather) = P(Toothache, Catch, Cavity)P(Weather)

Absolute independence is powerful but rare

Dentistry is a large field with hundreds of variables, none of which are independent.

What to do?

7.4 Bayes' Rule And Its Use

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Product rule $P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a)$

Bayes' rule

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

or in distribution form

$$\mathbf{P}(Y|X) = \frac{\mathbf{P}(X|Y)\mathbf{P}(Y)}{\mathbf{P}(X)} = \alpha\mathbf{P}(X|Y)\mathbf{P}(Y)$$

7.4 Bayes' Rule And Its Use

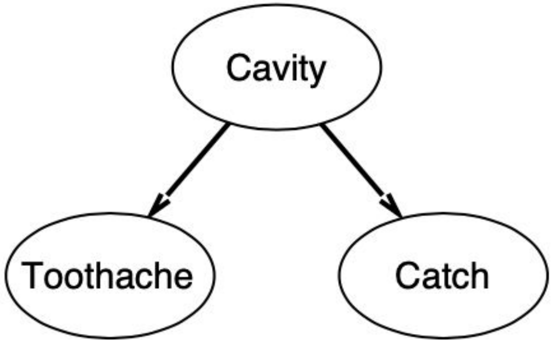
P(effect | cause) quantifies the relationship in the **causal** direction,

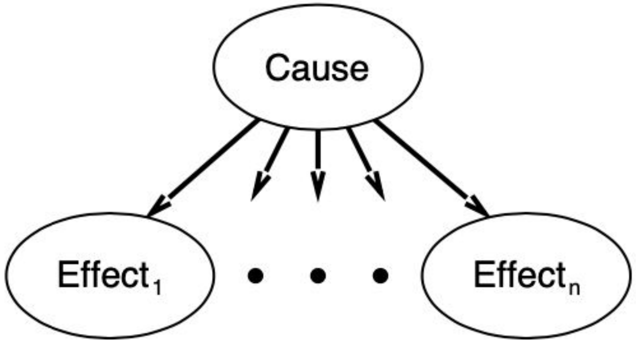
P(cause | effect) describes the **diagnostic** direction

Useful for assessing **diagnostic probability** from **causal probability**

$$P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$

$P(Cause, Effect_1, \ldots, Effect_n) = P(Cause)\ \Pi_i\ P(Effect_i \mid Cause)$





7.4 Using Bayes' rule: Combining evidence

$P(Cavity \mid toothache \wedge catch) = \alpha P(toothache \wedge catch \mid Cavity) P(Cavity)$

// toothache and catch are independent given the presence or the absence of a cavity

// conditional independence of toothache and catch given Cavity

$P(toothache \wedge catch \mid Cavity) = P(toothache \mid Cavity)P(catch \mid Cavity)$

$\Leftrightarrow P(Cavity \mid toothache \wedge catch) = \alpha P(toothache \mid Cavity) P(catch \mid Cavity) P(Cavity)$

7.5 Wumpus World

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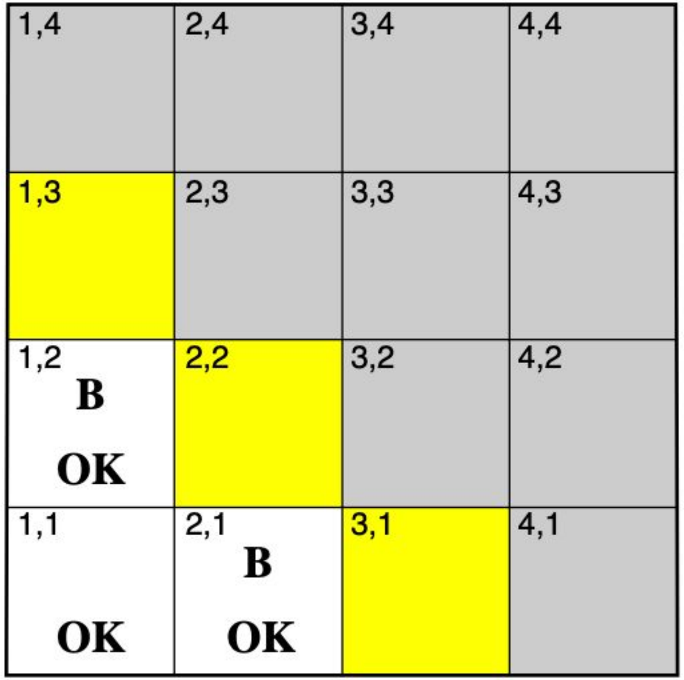
P_{ij} = true iff [i, j] contains a pit
B_{ij} = true iff [i, j] is breezy
Include only $B_{1,1}$, $B_{1,2}$, $B_{2,1}$ in the probability model



7.5 Specifying the probability model

The full joint distribution: $\mathbf{P}(P_{1,1}, \ldots, P_{4,4}, B_{1,1}, B_{1,2}, B_{2,1})$
Apply product rule: $\mathbf{P}(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,1}, \ldots, P_{4,4})\, \mathbf{P}(P_{1,1}, \ldots, P_{4,4})$
(Do it this way to get $P(Effect|Cause)$.)

First term: 1 if pits are adjacent to breezes, 0 otherwise
Second term: pits are placed randomly, probability 0.2 per square

7.5 Observations and query

We know the following facts:

$b = \neg b_{1,1} \wedge b_{1,2} \wedge b_{2,1}$

$known = \neg p_{1,1} \wedge \neg p_{1,2} \wedge \neg p_{2,1}$

Query is $P(P_{1,3} \mid known, b)$

Define $Unknown = P_{ij}$s other than $P_{1,3}$ and $known$

For inference by enumeration, we have

$$P(P1,3|known, b) = \alpha\Sigma_{unknown} P(P_{1,3}, unknown, known, b)$$

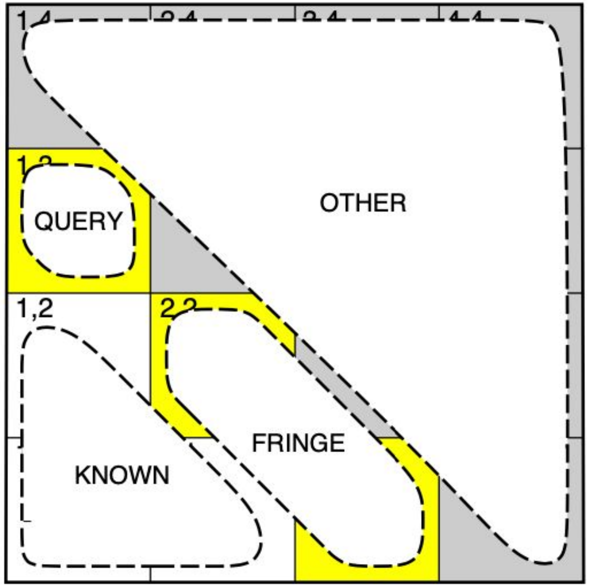
7.5 Using conditional independence

Basic insight: observations are conditionally independent of other hidden squares given neighbouring hidden squares

Define $\text{Unknown} = \text{Fringe} \cup \text{Other}$

$P(b|P_{1,3},\text{Known},\text{Unknown}) = P(b|P_{1,3},\text{Known},\text{Fringe})$

Manipulate query into a form where we can use this!



7.5 Using conditional independence

$$\mathbf{P}(P_{1,3}|known, b) = \alpha \sum_{unknown} \mathbf{P}(P_{1,3}, unknown, known, b)$$

$$= \alpha \sum_{unknown} \mathbf{P}(b|P_{1,3}, known, unknown)\mathbf{P}(P_{1,3}, known, unknown)$$

$$= \alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|known, P_{1,3}, fringe, other)\mathbf{P}(P_{1,3}, known, fringe, other)$$

$$= \alpha \sum_{fringe} \sum_{other} \mathbf{P}(b|known, P_{1,3}, fringe)\mathbf{P}(P_{1,3}, known, fringe, other)$$

$$= \alpha \sum_{fringe} \mathbf{P}(b|known, P_{1,3}, fringe) \sum_{other} \mathbf{P}(P_{1,3}, known, fringe, other)$$

$$= \alpha \sum_{fringe} \mathbf{P}(b|known, P_{1,3}, fringe) \sum_{other} \mathbf{P}(P_{1,3})P(known)P(fringe)P(other)$$

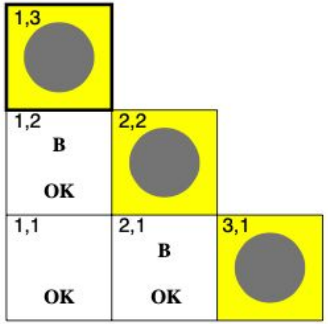
$$= \alpha\, P(known)\mathbf{P}(P_{1,3}) \sum_{fringe} \mathbf{P}(b|known, P_{1,3}, fringe)P(fringe) \sum_{other} P(other)$$

$$= \alpha' \,\mathbf{P}(P_{1,3}) \sum_{fringe} \mathbf{P}(b|known, P_{1,3}, fringe)P(fringe)$$

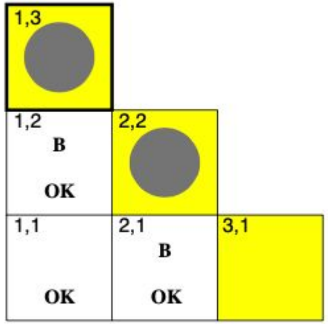
7.5 Using conditional independence

$P(P_{1,3}|known, b) = \alpha' \langle 0.2(0.04 + 0.16 + 0.16), 0.8(0.04 + 0.16) \rangle \approx \langle 0.31, 0.69 \rangle$

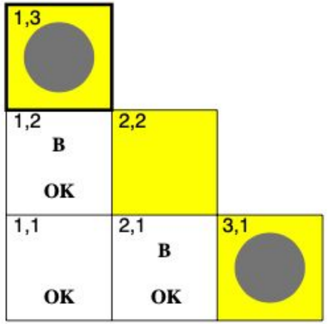
$P(P_{2,2}|known, b) \approx \langle 0.86, 0.14 \rangle$



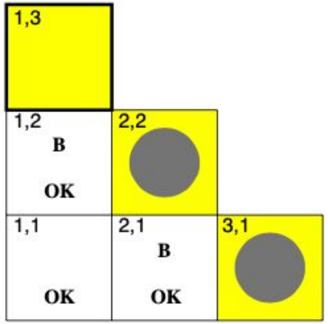
0.2 x 0.2 = 0.04



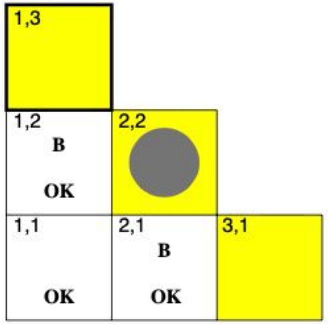
0.2 x 0.8 = 0.16



0.8 x 0.2 = 0.16



0.2 x 0.2 = 0.04



0.2 x 0.8 = 0.16