# Ideal Observers for the Estimation of Disparity in Random-Pixel Stereograms

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## Motivation

Independently and identically distributed (IID) textures, whose pixels have gray levels that are statistically independent and identically distributed, are routinely used to study how the visual system estimates and discriminates depth/disparity.

However, it is unknown how standard computational models of depth estimation (e.g., cross-correlation, squared-error, normalized cross-correlation) compare to the Bayesian ideal observer.

We derived the ideal observer to evaluate how sub-optimal are the standard computational models and to understand the optimal computations for the task.

## Stimuli

IID textures.

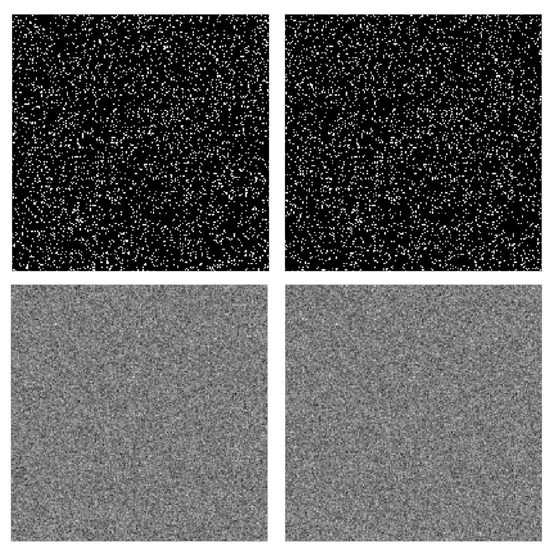
$$S_{i,j} \sim f(x;\boldsymbol{\theta})$$

$$P(\mathbf{S}) = \prod_{i,j} f(S_{i,j};\boldsymbol{\theta})$$

Horizontal Disparity.

$$S'_{i-d,j} = S_{i,j}$$

Left ($\mathbf{S}$) Right ($\mathbf{S}'$)



Binary texture (p = 0.1)

$$f(x;\boldsymbol{\theta}) = B(x; s_l, s_h, p)$$

White texture

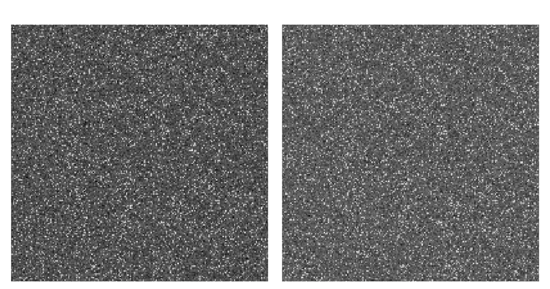
$$f(x;\boldsymbol{\theta}) = G(x; 0, \sigma_s)$$

New random sample of texture on each trial, $\mathbf{S}$

To limit maximum performance we added independent Gaussian pixel noise that was uncorrelated in the left and right images, $N_{i,j}$

$$L_{i,j} = S_{i,j} + N_{i,j}$$

$$R_{i,j} = S'_{i,j} + N_{i,j}$$



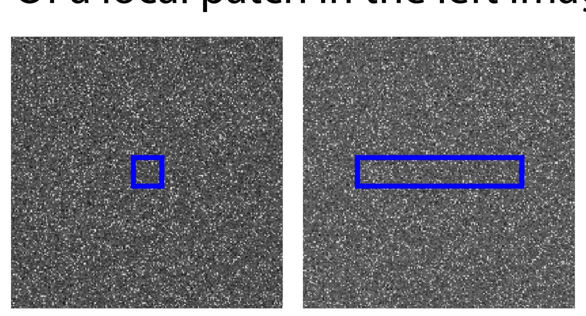
Binary Texture with Gaussian noise

$$N_{i,j} \sim G(0,\sigma_n)$$

## Task

Estimate the absolute disparity, $d$

Of a local patch in the left image, $\mathbf{l}$



Binary Texture with Gaussian noise

Blue square in left image is $\mathbf{l}$

Blue rectangle in right image is search region that covers all possible locations that corresponding left patch might be located, $\mathbf{r}$

## The Bayesian Ideal Observer

The parameters of the probability distribution of IID texture, the internal noise, and the prior over disparity were assumed to be known to the observer.

$$P(d \mid \mathbf{l},\mathbf{r}) \propto P(\mathbf{l},\mathbf{r} \mid d)P(d)$$

$$\boxed{\hat{d}_{opt} = \underset{d}{\operatorname{argmin}} \sum_{i,j\in\mathbf{l}} \left[ -lnP(L_{i,j}, R_{i-d,j} \mid d) + lnP(R_{i-d,j} \mid d) - lnP(d) \right]}$$

the joint distribution of corresponding pixel values — the marginal distribution of pixel values — prior probability distribution over disparity

For any IID texture, if these distributions are known the optimal estimate can obtained.

### White texture with Gaussian noise

Marginal distribution:

$$L_{i,j} \sim G(0,\sigma_t)$$

$$R_{i,j} \sim G(0,\sigma_t)$$

$$\sigma_t = \sqrt{\sigma_s^2 + \sigma_n^2}$$

Joint distribution:

A multivariate Gaussian with the covariance of $\sigma_s^2$

### Binary texture with Gaussian noise

Marginal distribution:

A mixture of Gaussians

$$P(R_{i-d,j}) = G(R_{i-d,j} \mid s_l, \sigma_n)\ (1-p) + G(R_{i-d,j} \mid s_h, \sigma_n)\ p$$

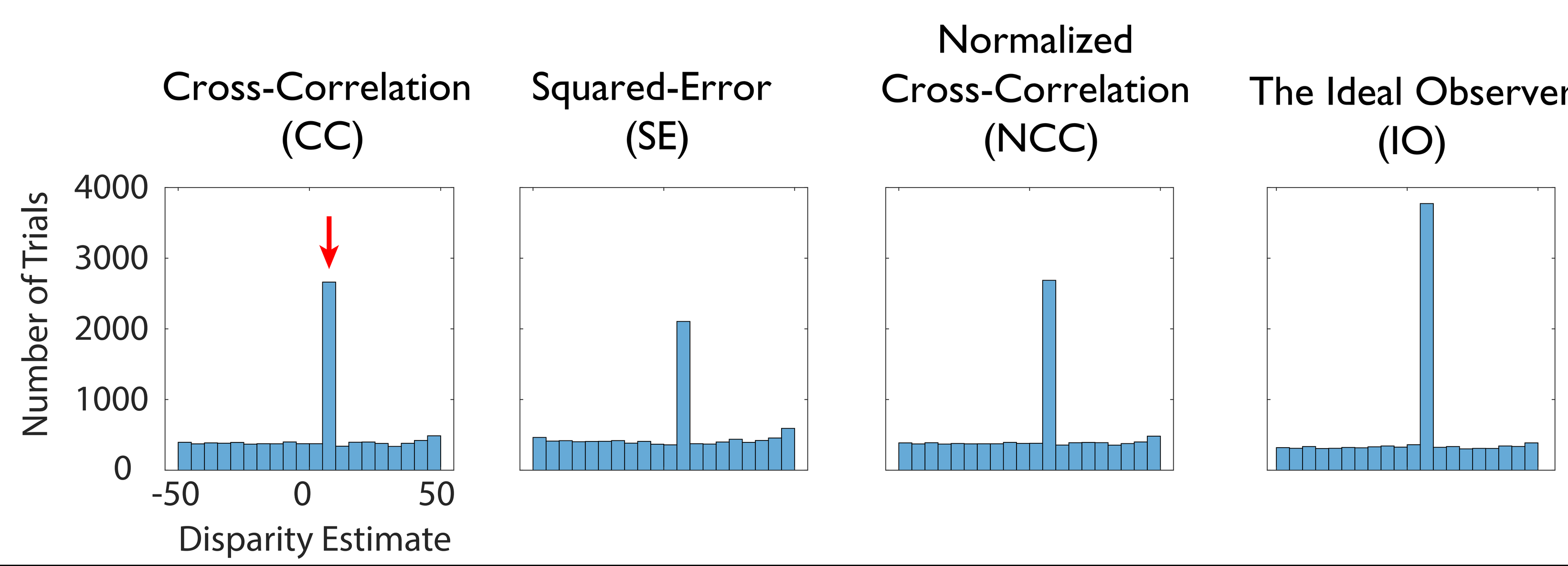
Joint distribution:

A mixture of two dimensional Gaussians where the means are $[s_l\ \ s_l]$ and $[s_h\ \ s_h]$. They are composed of independent one dimensional Gaussians.

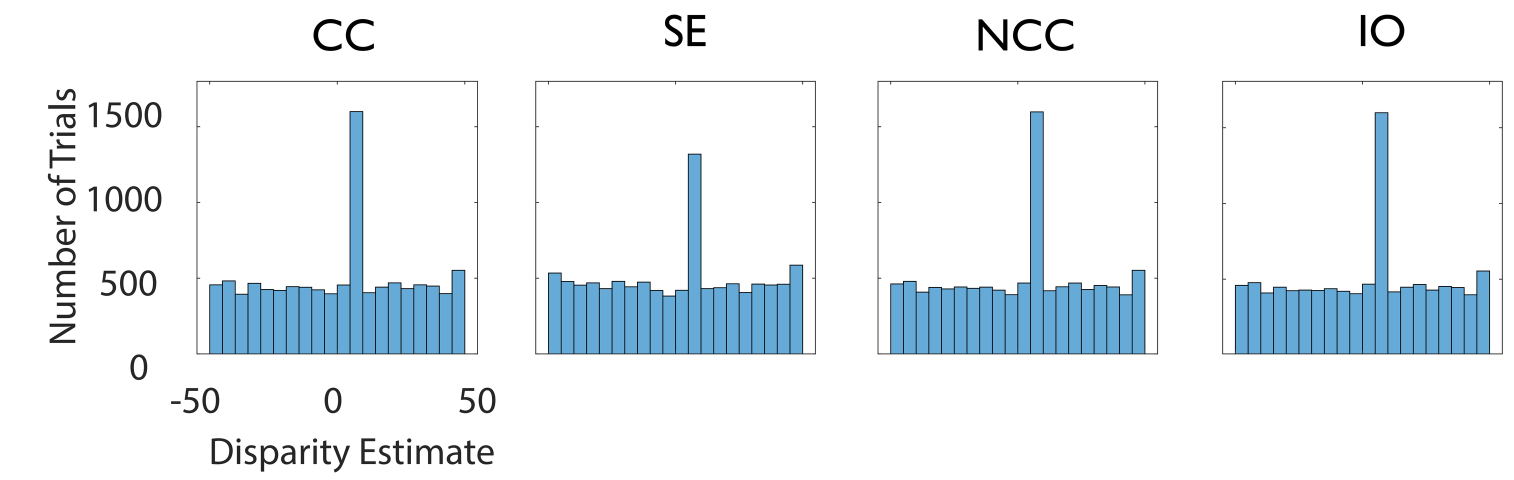
$$P(R_{i-d,j}, L_{i,j}) = G(L_{i,j} \mid s_l, \sigma_n)\ G(R_{i-d,j} \mid s_l, \sigma_n)\ (1-p) + G(L_{i,j} \mid s_h, \sigma_n)\ G(R_{i-d,j} \mid s_h, \sigma_n)\ p$$

## Comparison with the Standard Observer Models

Binary texture

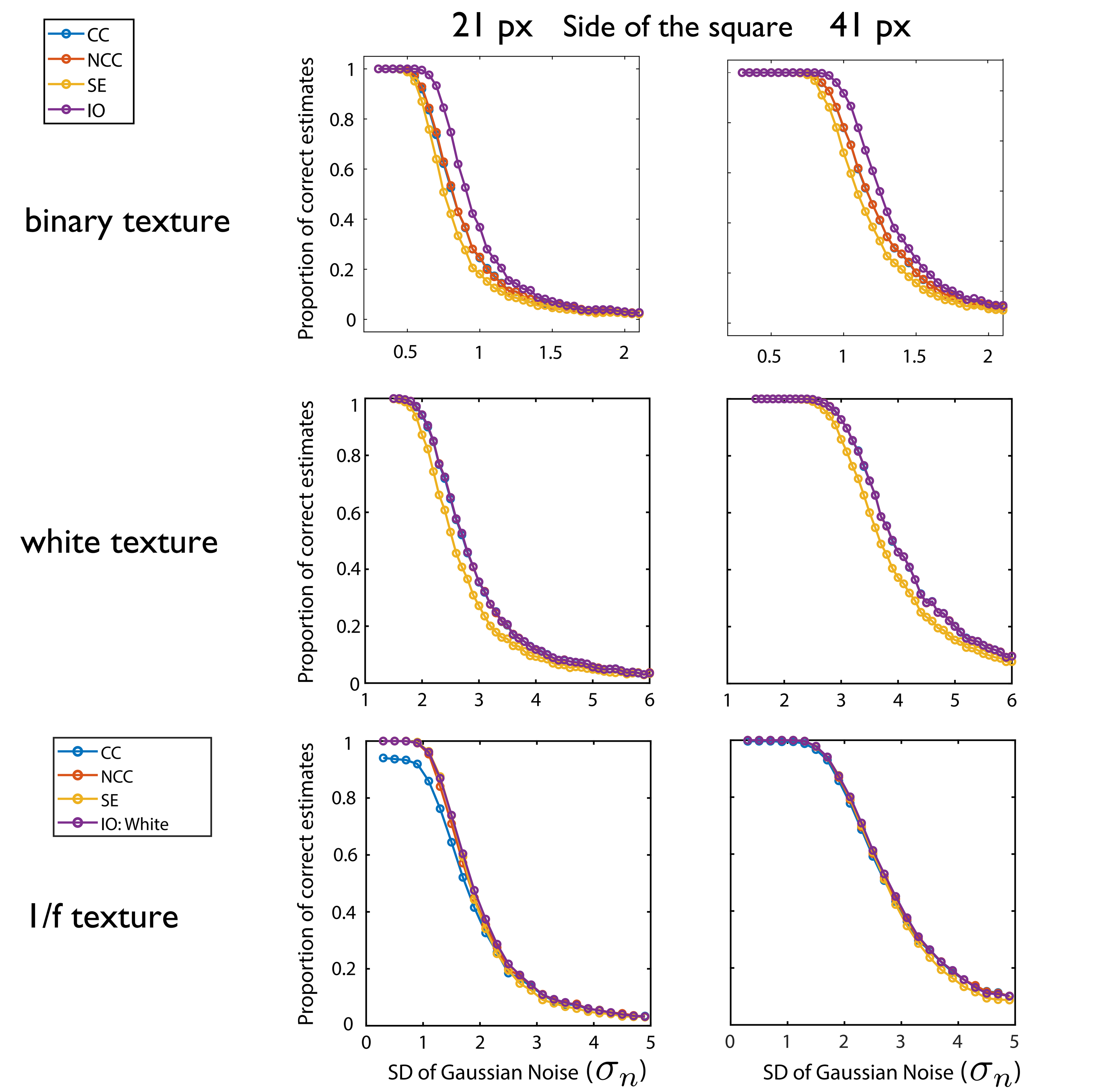



White texture




As our measure of performance, we used the proportion of trials where the disparity was correctly estimated.

## Performance as a function of Gaussian noise level




For internal noise that has the characteristics of 1/f noise and is uncorrelated in the left and right images, the results are similar.

## Conclusions

The NCC observer more closely approximates the ideal observer for the IID White texture than for the IID Binary texture.

The NCC observer generally performs close to the ideal observer.

Code and demonstrations can be found here: https://github.com/CanOluk/OptimalDisparityEstimation.

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