# IE432 - Project Stage 1

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For this project's first stage, we started conducting an SSLP model. Parameters and decision variables as well as the model are visible and explained below:

parameters

$c_j$ : fixed of locating a server at location $j$ , $j \in J$

$u$ : maximum resource capacity to serve clients

$q_{j0}$ : shortage cost per unit of unmet demand among the clients that are assigned to server $j$.

$q_{ij}$ : revenue gained from serving $i^{th}$ client assigned to server $j$

$d_{ij}$ : units of resources used by client $i$, at server $j$.

$p_s$ : probability of scenario $s$ happening

$$h_{is} = \begin{cases} 1 & \text{if client } i \text{ exists at scenario } s. \\ 0 & \text{otherwise} \end{cases}$$

## Decision Variables

1st stage decision variable : decide the location of the servers

$$X_J = \begin{cases} 1 & \text{if there is a server at location } J \\ 0 & \text{otherwise} \end{cases}$$

2nd stage decision variable : decide client server assignments:

$$Y_{iJs} = \begin{cases} 1 & \text{if } i^{\text{th}} \text{ client is served at server } J \text{ at Scenario } s. \\ 0 & \text{otherwise} \end{cases}$$

**Explanations of Decision Variables:**

1. $x_j$: If there exists a server at location j, this binary variable takes value 1; otherwise 0.
2. $y_{ijs}$: If client i is served at server j at scenario s, this binary variable takes value 1; otherwise 0.

Model:

$$\min \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{s=1}^{S} c_j X_j + p_s \left[ \left( h_{is} \cdot X_J - Y_{iJs} \right) d_{ij} \cdot q_{J0} - \left( Y_{iJs} \cdot q_{iJ} \right) \right]$$

$$\text{s.t.} \quad X_J \cdot h_{is} \geq Y_{iJs} \qquad \forall s \in S, \ \forall i \in I, \ \forall J \in J \qquad (1)$$

$$\sum_{j=1}^{J} Y_{iJs} = 1 \qquad \forall i \in I, \ \forall s \in S \qquad (2)$$

$$\sum_{i=1}^{I} Y_{iJs} \cdot d_{iJ} \leq u \qquad \forall J \in J, \ \forall s \in S \qquad (3)$$

**Explanations of the Constraints:**

1. Constraint (1) ensures that the number of customers present in a specific location are consistent with the number of customers that are being served. It forces each customer that is being served is actually present. This is a logical constraint for the validity of the model.
2. Constraint (2) ensures that each person in each scenario is served a single server. We did nott include the variations of these because each server can serve more than 1 customer and scenarios do not have such a limitation.
3. The constraint (3) is the capacity constraint. It ensures the served capacity to each customer (that has been served) is lower than the maximum capacity of each server denoted as u.

**Explanation of the Objective Function:**

Objective is to minimize the cost incurred by locating the servers and by the unmet demand of served customers. For every client i, location j and scenario s, there is a cost of locating the server ($c_j$) if the server is located in that location or not ($x_j$). Unmet demand is found by the multiplication of resources used by clients ($d_{ij}$), shortage cost incurred ($q_{j0}$), and the binary outcome. The difference between the client being present in scenario s ($h_{is}$=1) and served at location j ($x_j$=1), and the client being served by the server j at scenario s ($y_{ijs}$=1) gives us the binary outcome to be multiplied. The revenue generated ($q_{ij}$) by serving the client is subtracted. This is multiplied with the probabilities of each scenario s happening ($p_s$).

Assumptions:

1) We did not create an assignment variable and directly created a service variable $y_{ijs}$. We skipped the assignment variable because if a customer is assigned to some location j at some scenario s, this directly implies the customer is served. Assigned <-> served.

2) The customer is served either in full or none at all. In other words, if $y_{ijs}$=*1 then full service, if* $y_{ijs}$*=0 no service (no assignment either, as the first assumption implies).*