FINTECH 545 Week04 Project Report
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Problem 1:
The calculation of the expected value and standard deviation of price at time t for each of the 3 types of price returns (Classical Brownian Motion, Arithmetic Return System, and Log Return) is in the Problem 1 Section of code file (code.ipynb).

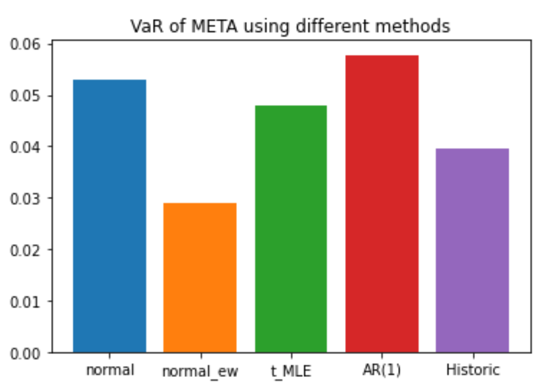
In this problem, I set the **initial price** $P_0 = 100$ (dollars), with the return rate follows the normal distribution with **mean** $\mu = 0$ and **standard deviation** $\sigma = 0.5$, and the expected values from theoretical computation and the actual values from simulation are shown below:

Method	**Mean**		**Standard Deviation**	
	Expected Value	Actual Value	Expected Value	Actual Value
Classical	P_0 (100)	100.02	σ (0.5)	0.4842
Arithmetic	P_0 (100)	102.39	σ (50)	48.42
Log	$P_0 \cdot e^{\frac{\sigma^2}{2}}$ (113.31)	115.05	$P_0 \cdot e^{\frac{\sigma^2}{2}} \cdot \sqrt{e^{\sigma^2} - 1}$ (60.39)	58.15

(Note: the expected value formulas are from Week1 Lecture Note)

According to the table above, we can figure out that the mean and standard deviation **match with the expectations**.

Problem 2:
The implementation of the "return_calculate()" function, the computation of arithmetic returns for all prices, and the calculation of VaR using all 5 methods are in the Problem 2 Section of code file (code.ipynb).



According to the graph that shows the comparison of the 5 VaR (Value at Risk) values, we can figure out that the VaR that computed using a normal distribution with an exponentially weighted variance is the lowest, and the VaR that computed using a fitted AR(1) model is the highest. All 5 values are close to 0.05 (5%), which means that these values will provide a similar estimation of risk conditions based on the time-sensitive price data provided.

Problem 3:
The computation of VaR of each portfolio using both discrete (arithmetic) return and log return are in the Problem 3 Section of code file (code.ipynb).

Firstly, I used the **Delta Normal VaR method** that assumed payoffs are linear and returns and distributed multivariate normal.

Using this method, we have the VaR for each portfolio is:
Portfolio A: $15428.34
Portfolio B: $8083.29
Portfolio C: $18164.90
And the total VaR for this portfolio is $38944.84.

The other method I have chosen is the **Historical VaR method**, and the primary reason I choose this method is that **the asset (stock) prices are not always linear and returns are not normal, and so in this case the Historical VaR method is more suitable**.

Using this method, we have the VaR for each portfolio is
Portfolio A: $15434.88
Portfolio B: $8090.33
Portfolio C: $18083.22
And the total VaR for this portfolio is $38908.30.

We can easily figure out that the **results are changed by using Historical VaR method from the original Delta Normal Method**, and from my perspective it is primarily because **Historical VaR method captures more non-linear features of the time-sensitive data, and it also include greater weights for least recent data points (prices many days ago for each stock).**