University of BRISTOL

Machine Learning

Reinforcement Learning and Decisions

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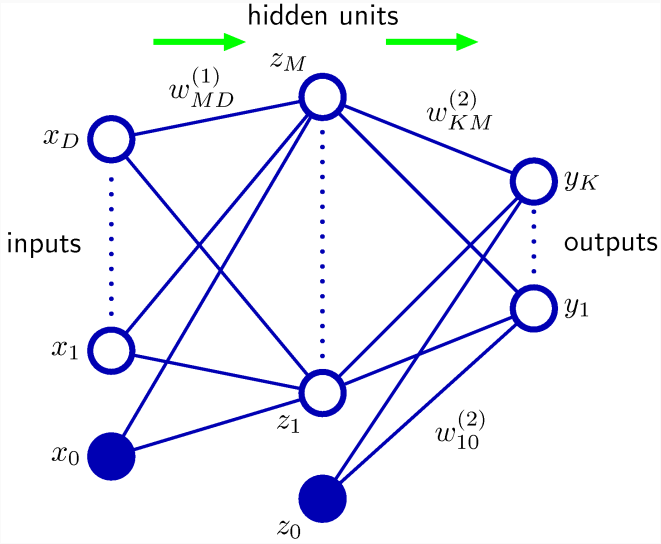
Introduction

Neural Network



$$y_k(\mathbf{x}, \mathbf{w}) = \sigma\left(\sum_j^M w_{kj}^{(2)} h\left(\sum_j^D w_{ji}^{(1)} x_i + w_{j0}^{(1)}\right) + w_{k0}^{(2)}\right)$$

Composite Functions

Why are composite functions attractive?

$$\mathbf{y} = g(\mathbf{x}) = f_K(f_{K-1}(f_{K-2}(\dots f_1(\mathbf{x})\dots)))$$

Linear Algebra

- Kernel of a function

$$\text{Kern}(f_k) = \left\{(\mathbf{x}, \mathbf{x}') | f_k(\mathbf{x}) = f_k(\mathbf{x}')\right\}$$

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- Image of a function

$$\text{Im}(f_k(\mathbf{x})) = \{\mathbf{y} \in Y | \mathbf{y} = f_k(\mathbf{x}), \mathbf{x} \in X\}$$

Linear Algebra

- Rank-Nullity Theorem

$$\dim(\mathrm{Im}(f)) = \dim(V) - \dim(\mathrm{Kern}(f))$$

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$$\mathrm{Kern}(f_1) \subseteq \mathrm{Kern}(f_{k-1} \circ \ldots \circ f_2 \circ f_1) \subseteq \mathrm{Kern}(f_k \circ f_{k-1} \circ \ldots \circ f_2 \circ f_1)$$

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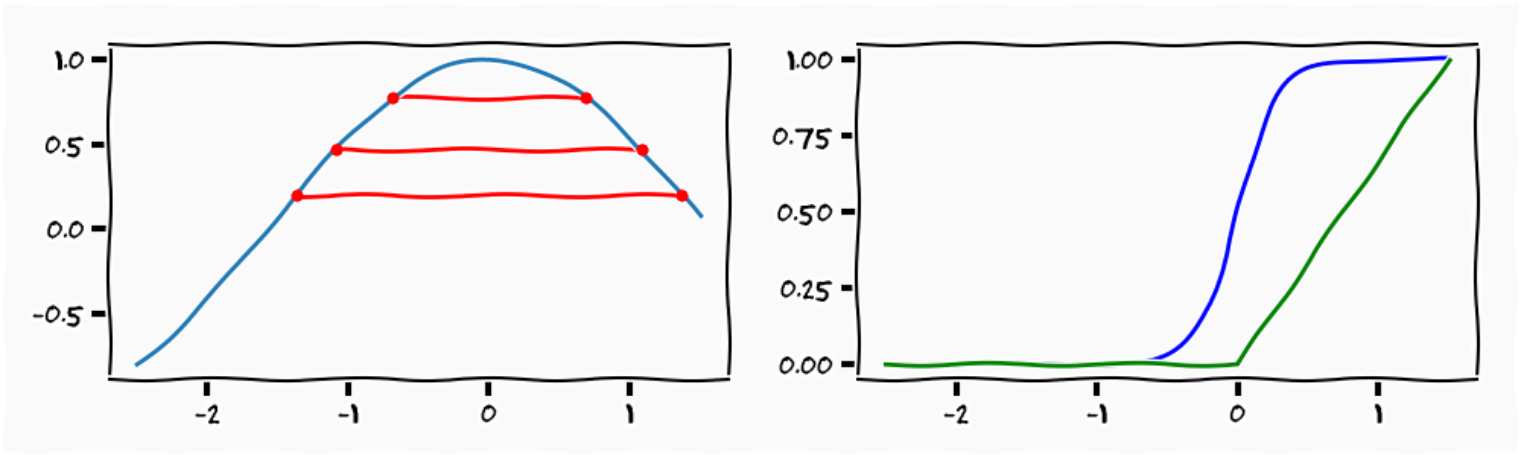
- Image of a function

$$\mathrm{Im}(f_k \circ f_{k-1} \circ \ldots \circ f_2 \circ f_1) \subseteq \mathrm{Im}(f_k \circ f_{k-1} \circ \ldots \circ f_2) \subseteq \ldots \subseteq \mathrm{Im}(f_k)$$

Composition functions



$$y = f_k(f_{k-1}(\dots f_0(x))) = f_k \circ f_{k-1} \circ \dots \circ f_1(x)$$

$$\text{Kern}(f_1) \subseteq \text{Kern}(f_{k-1} \circ \dots \circ f_2 \circ f_1) \subseteq \text{Kern}(f_k \circ f_{k-1} \circ \dots \circ f_2 \circ f_1)$$

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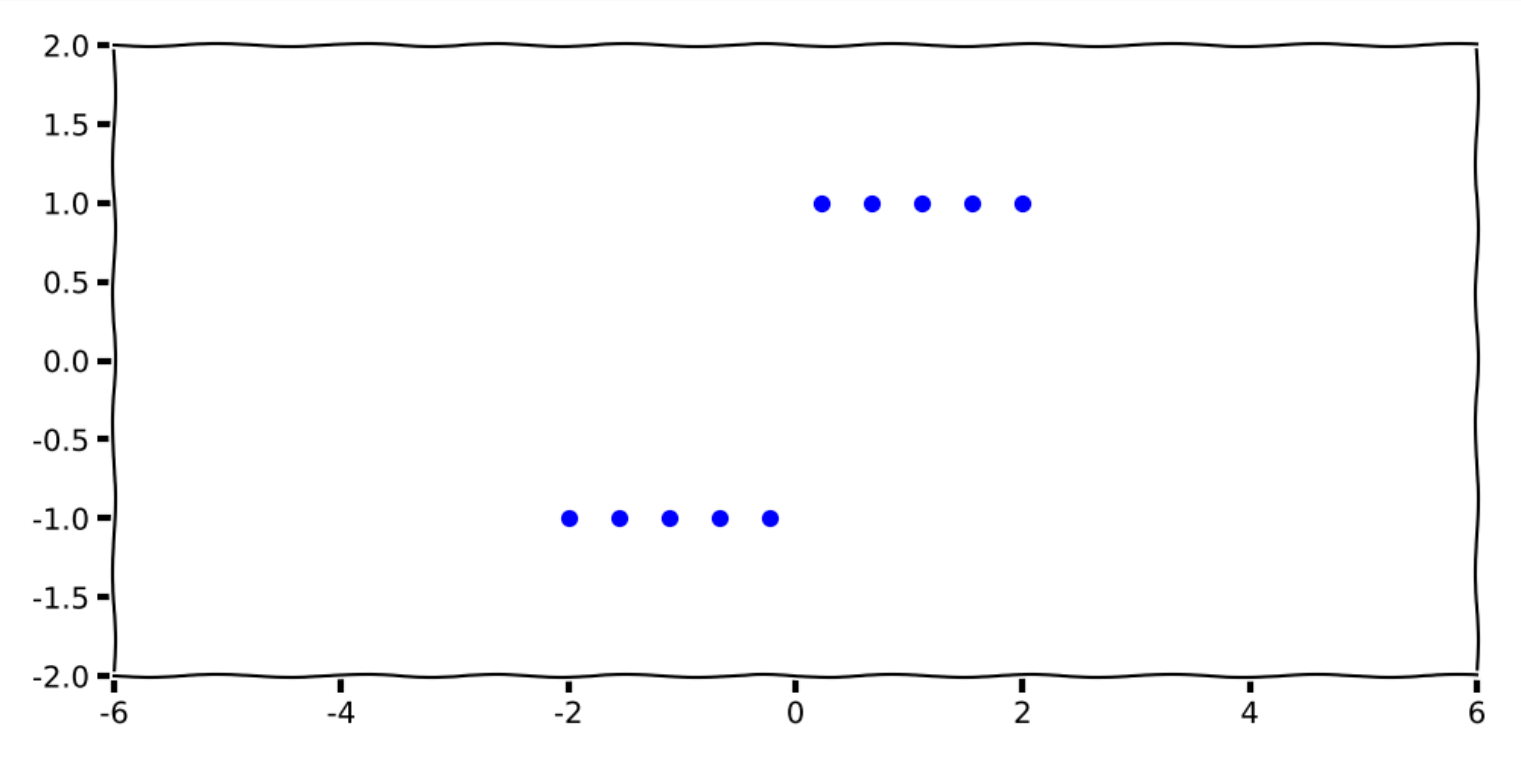

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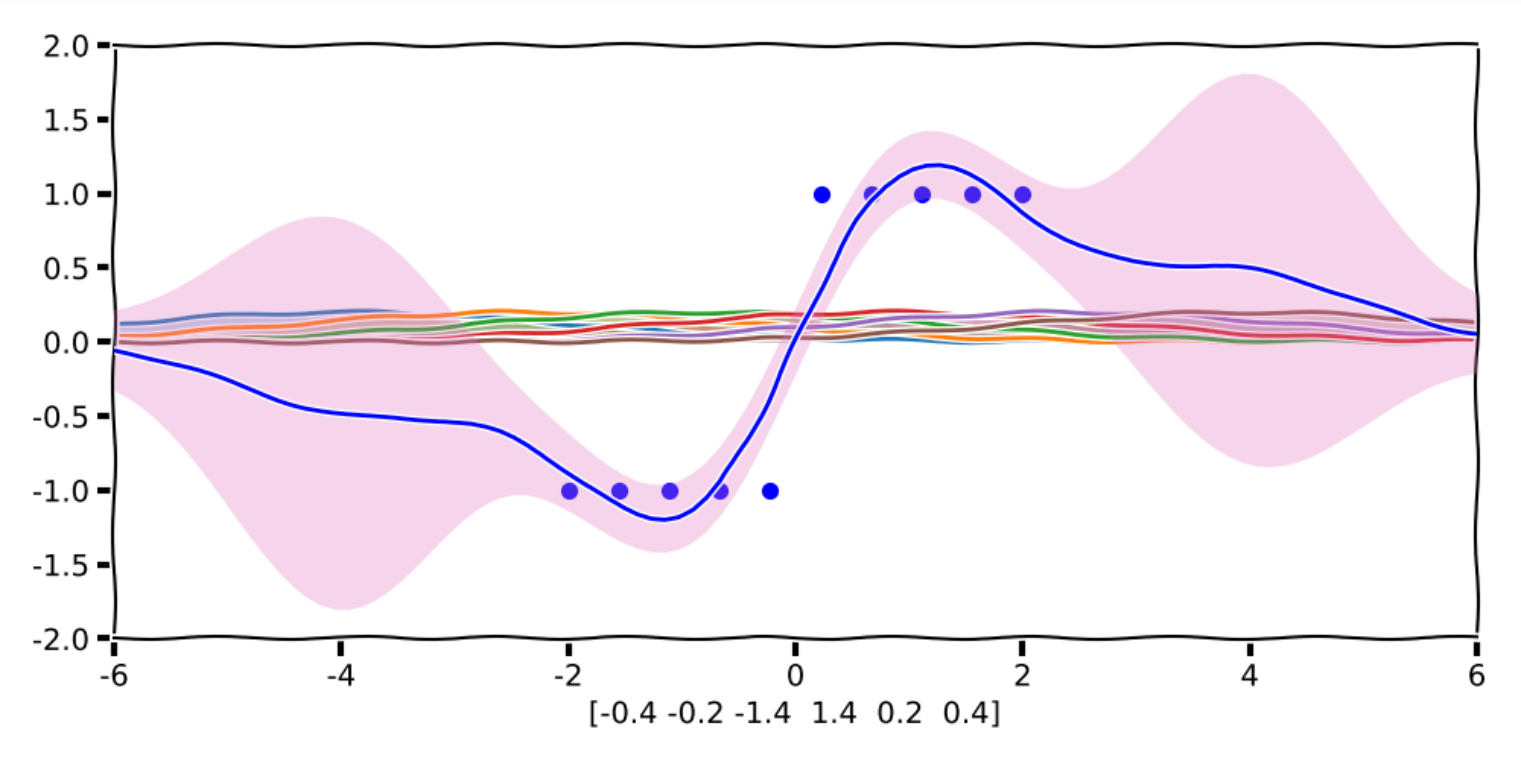

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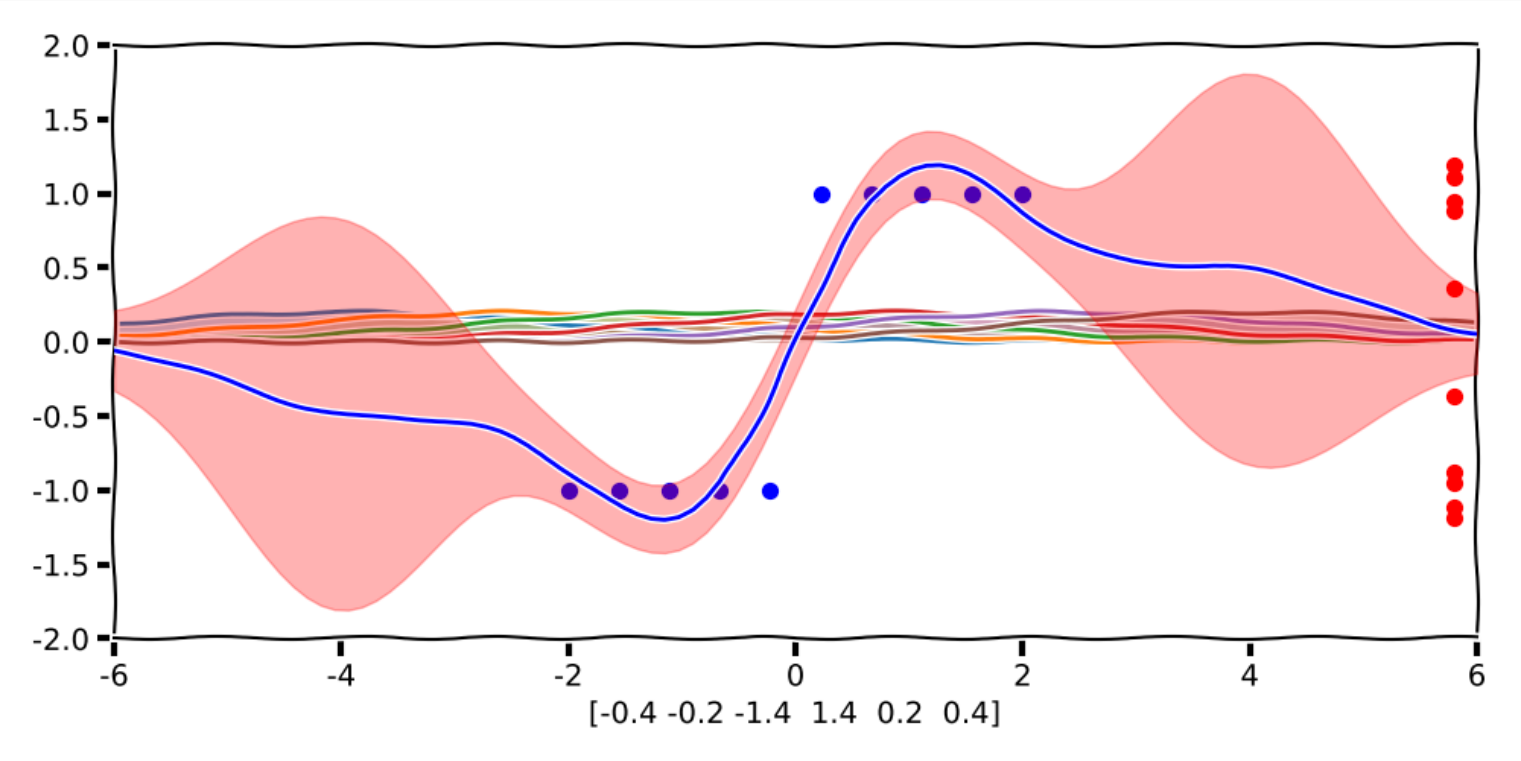

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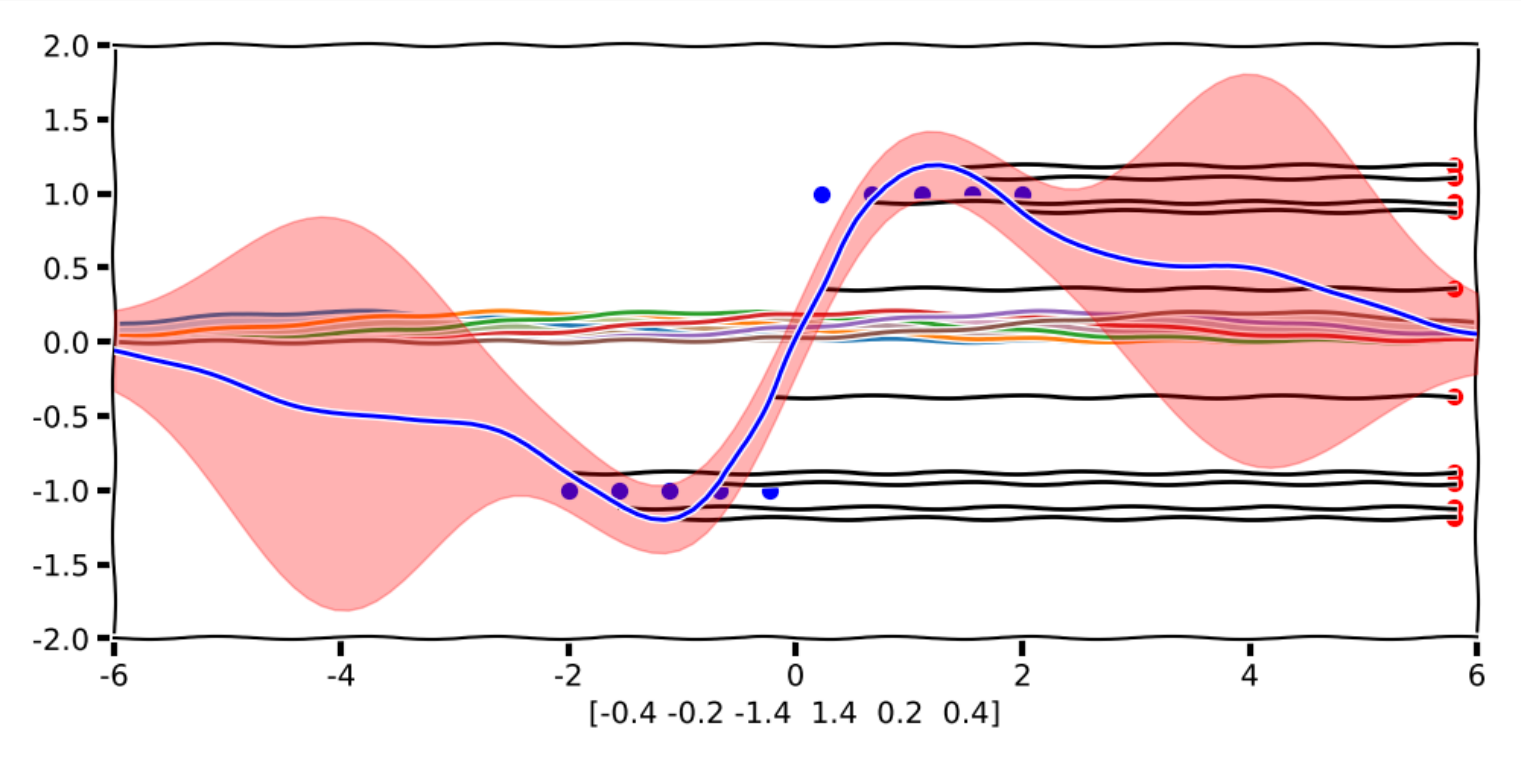

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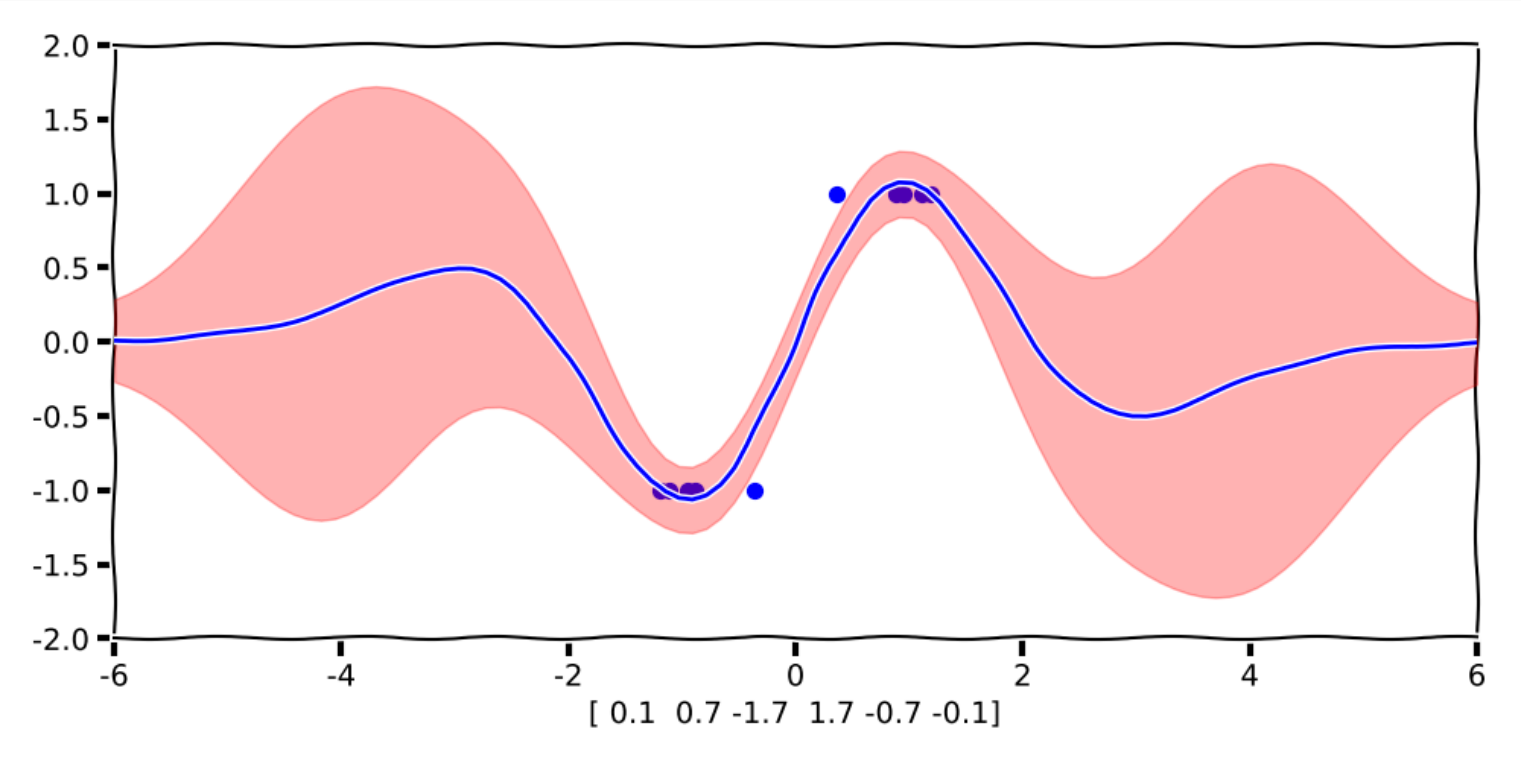

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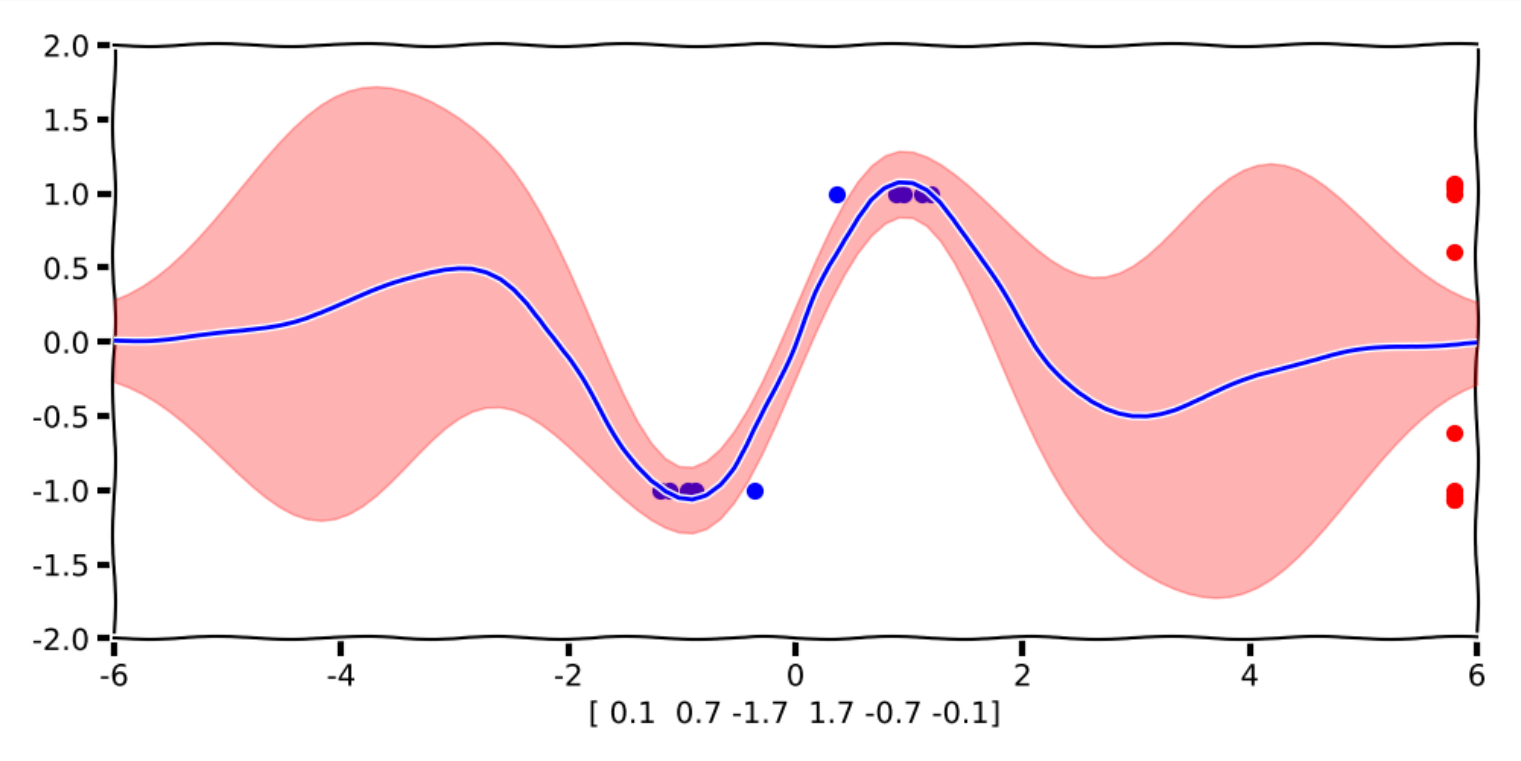

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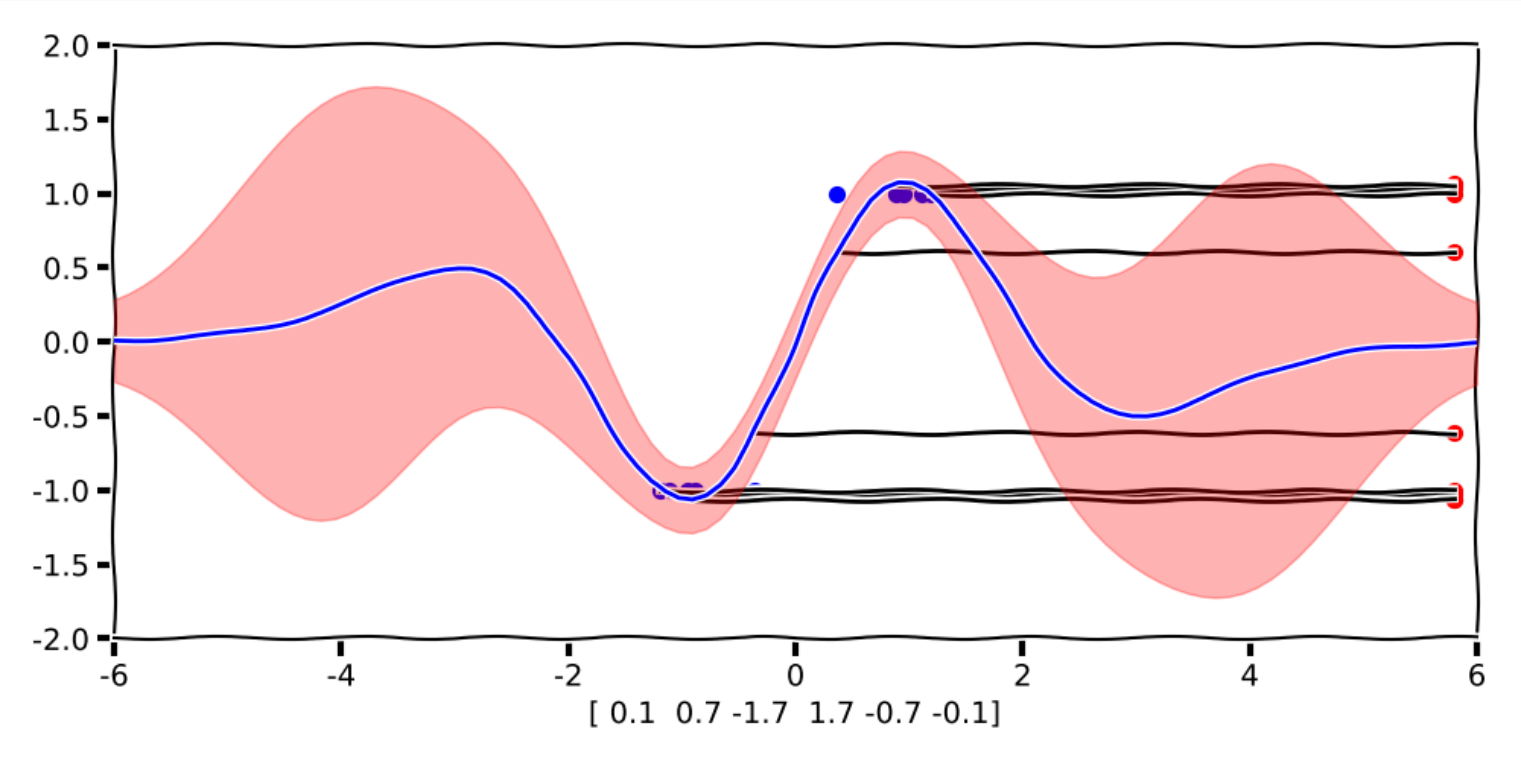

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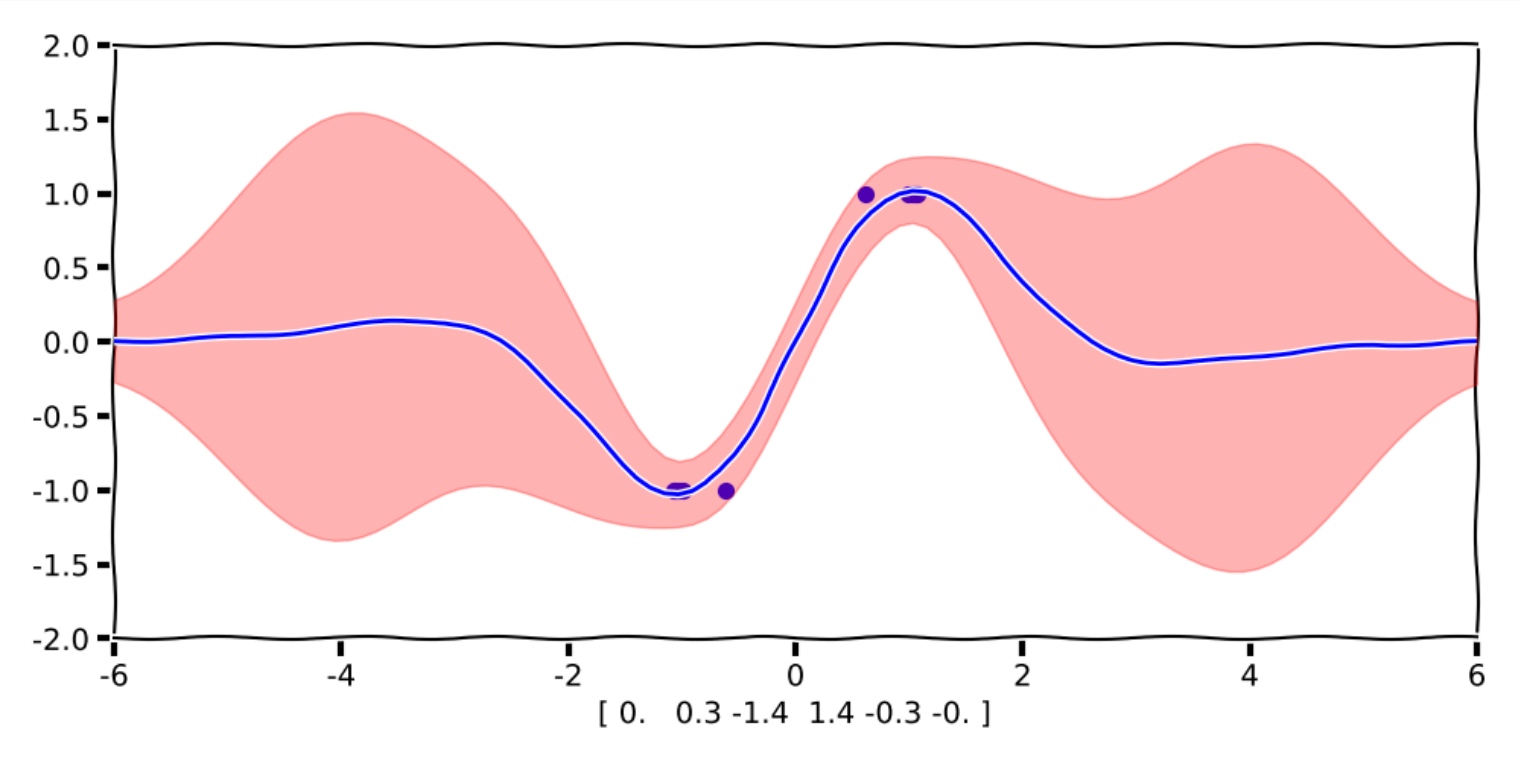

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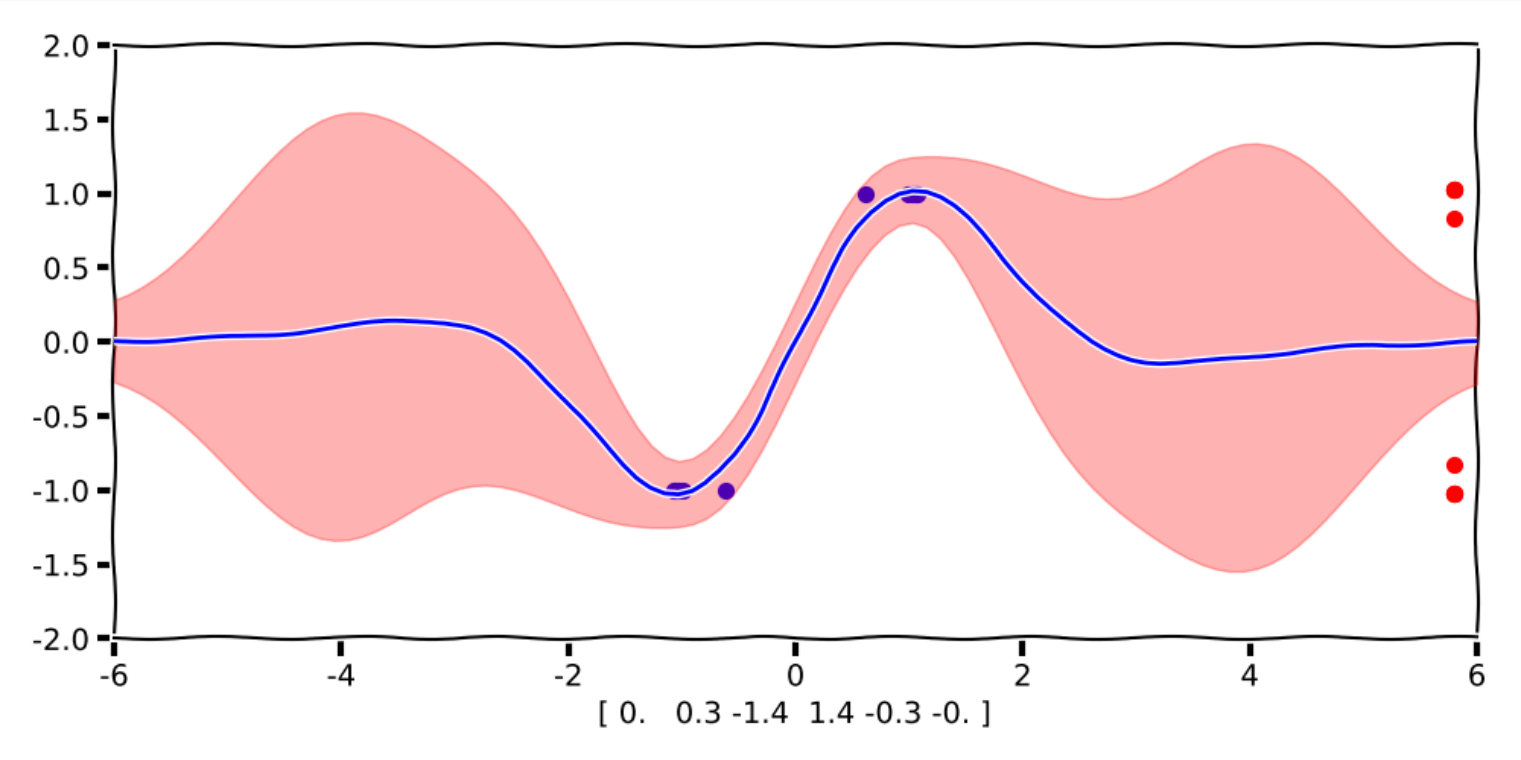

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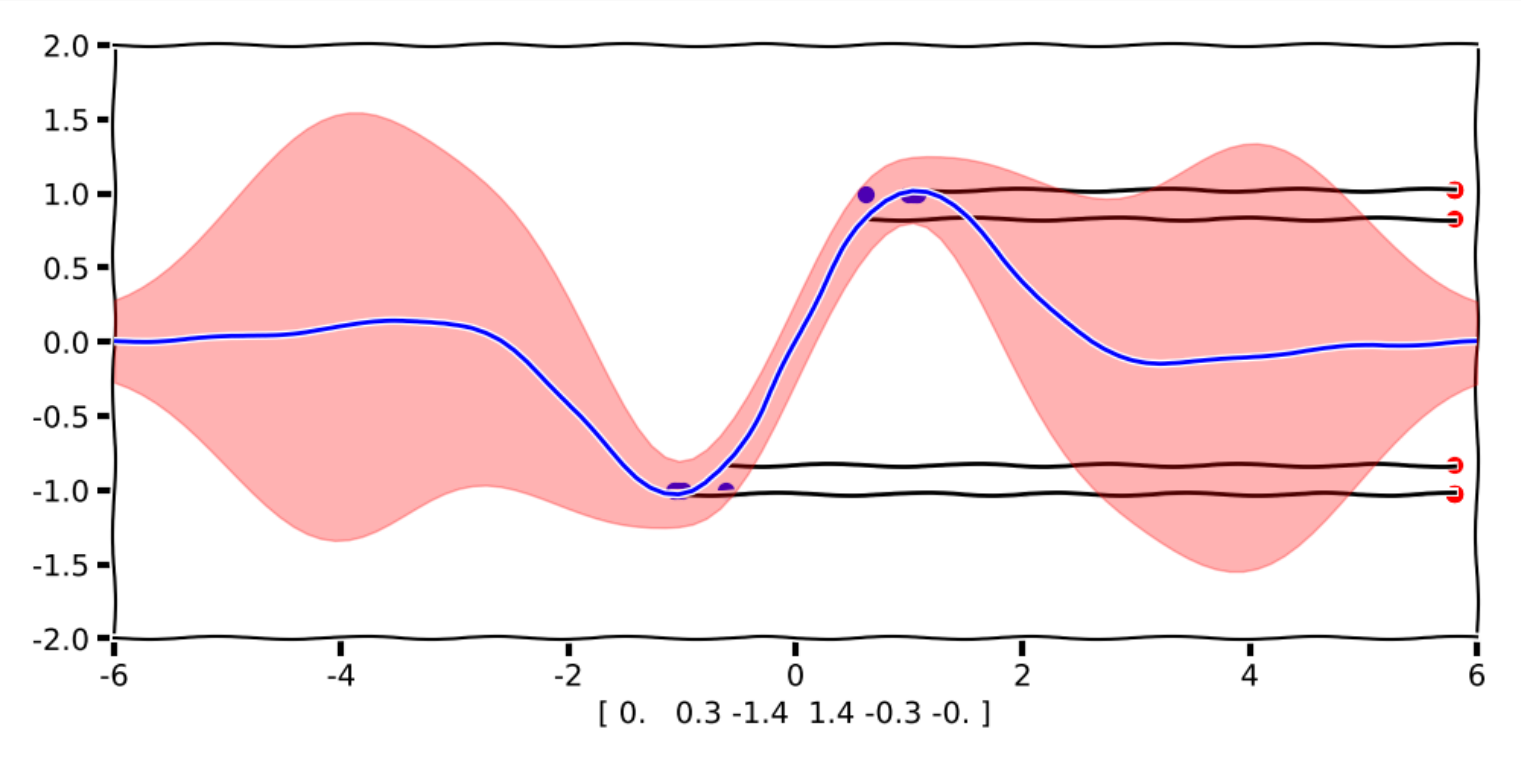

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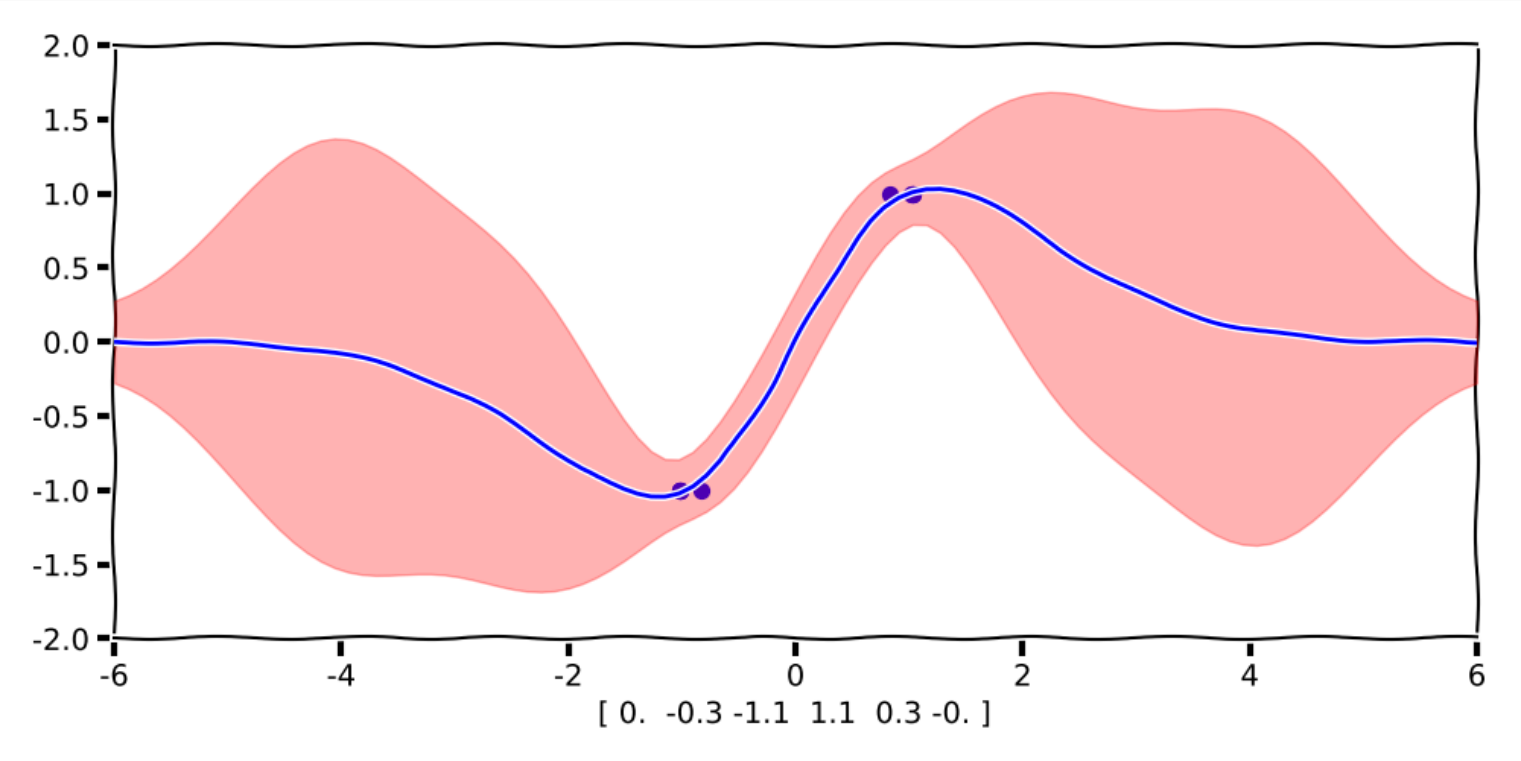
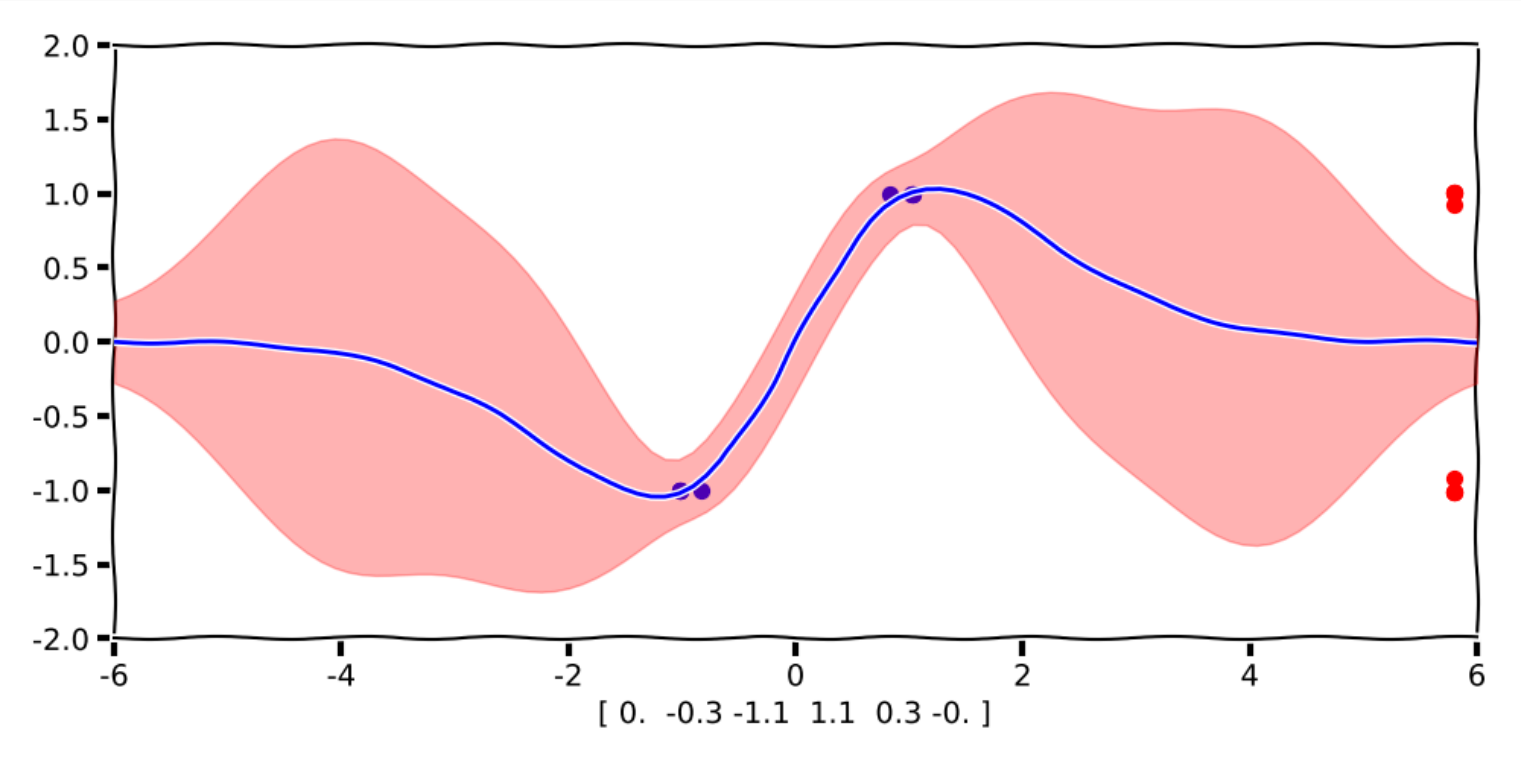


Composite Functions



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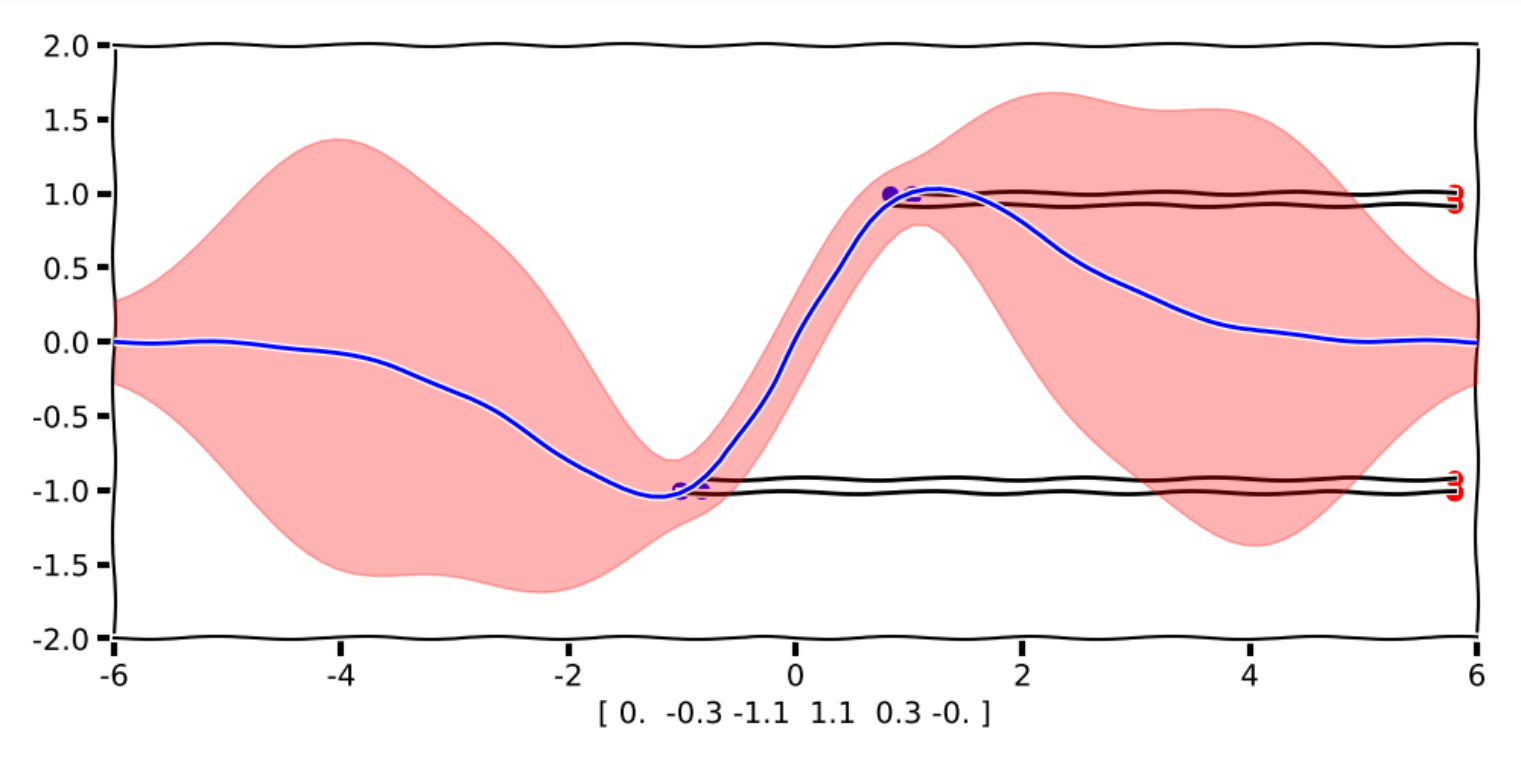
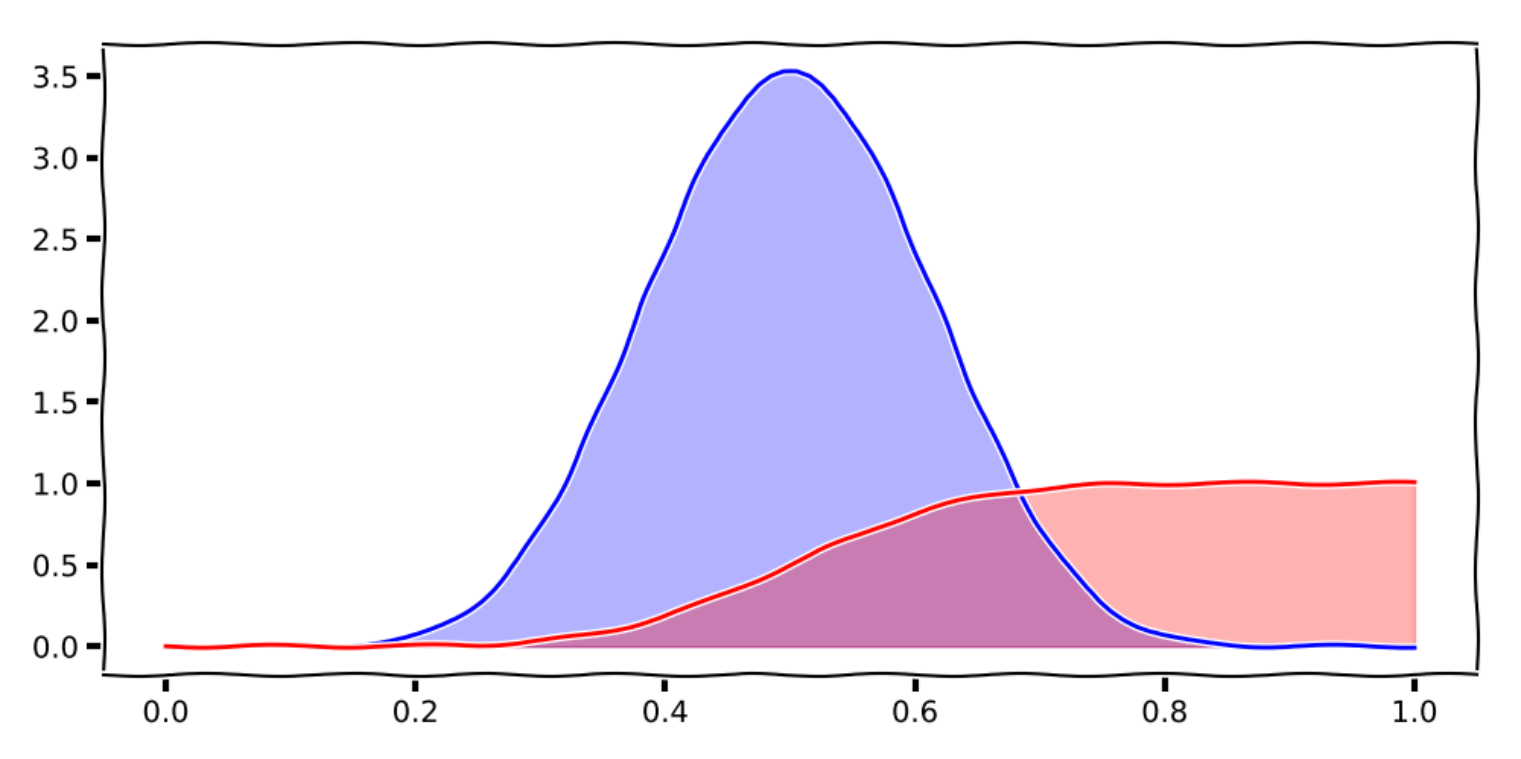


Sampling



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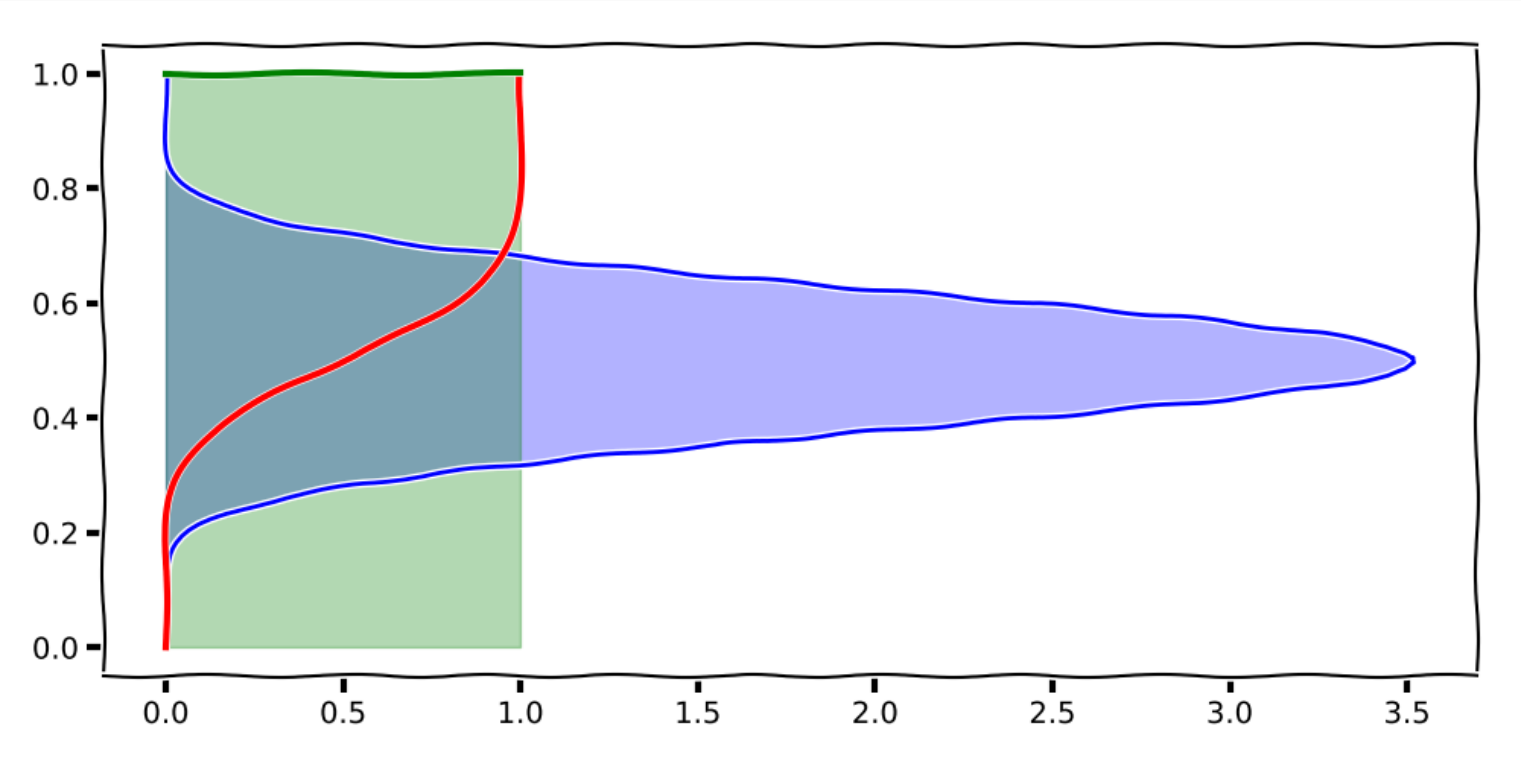
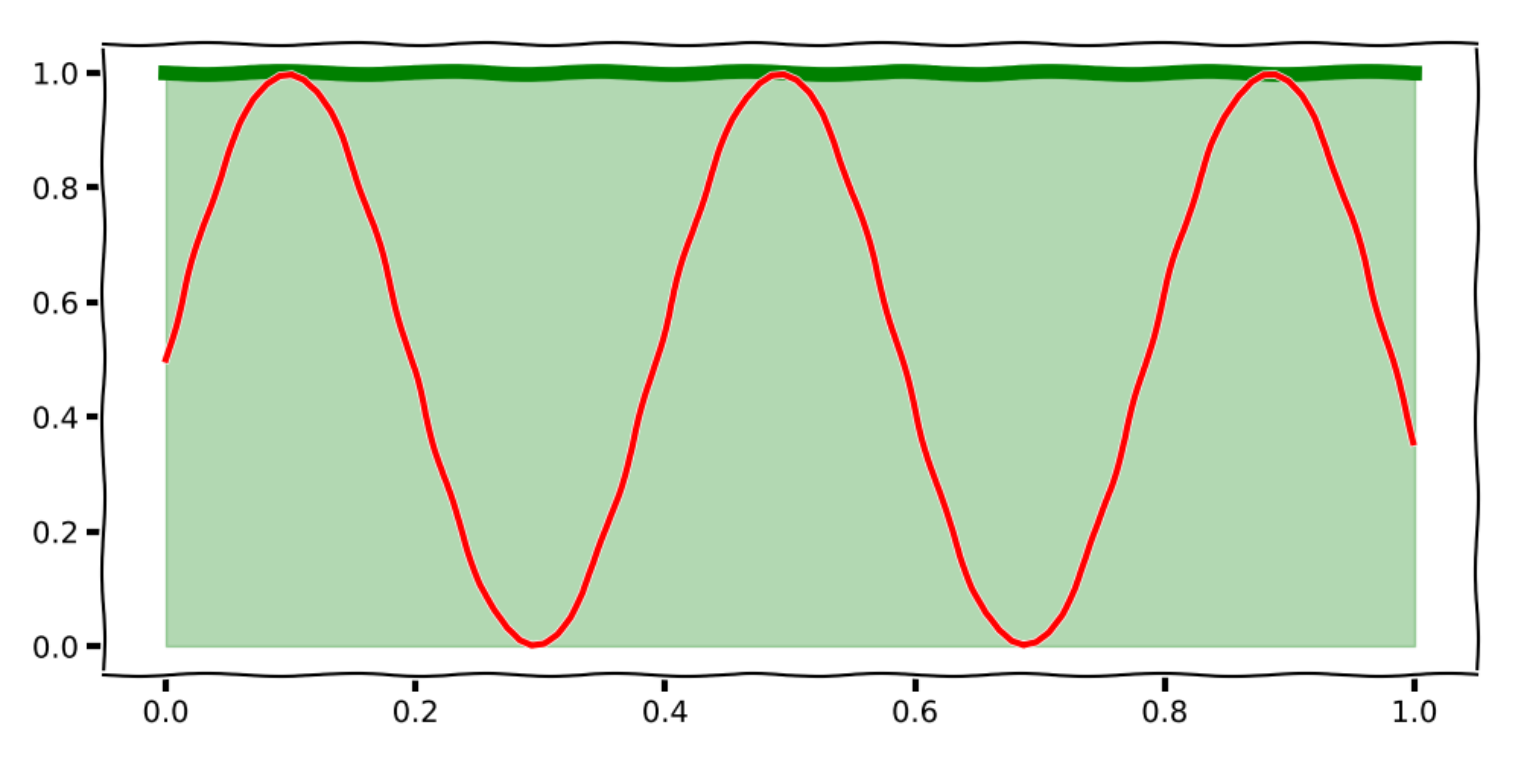
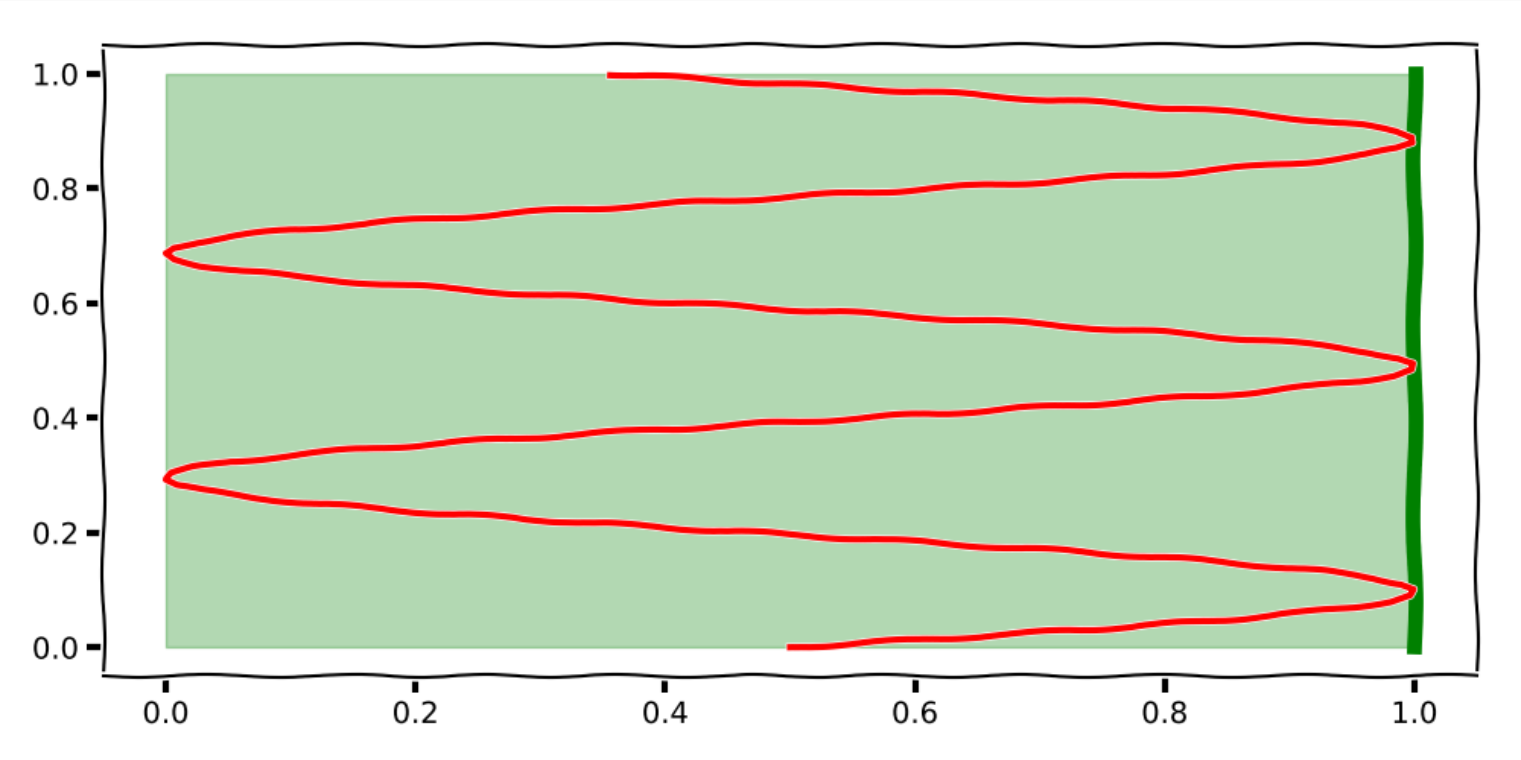


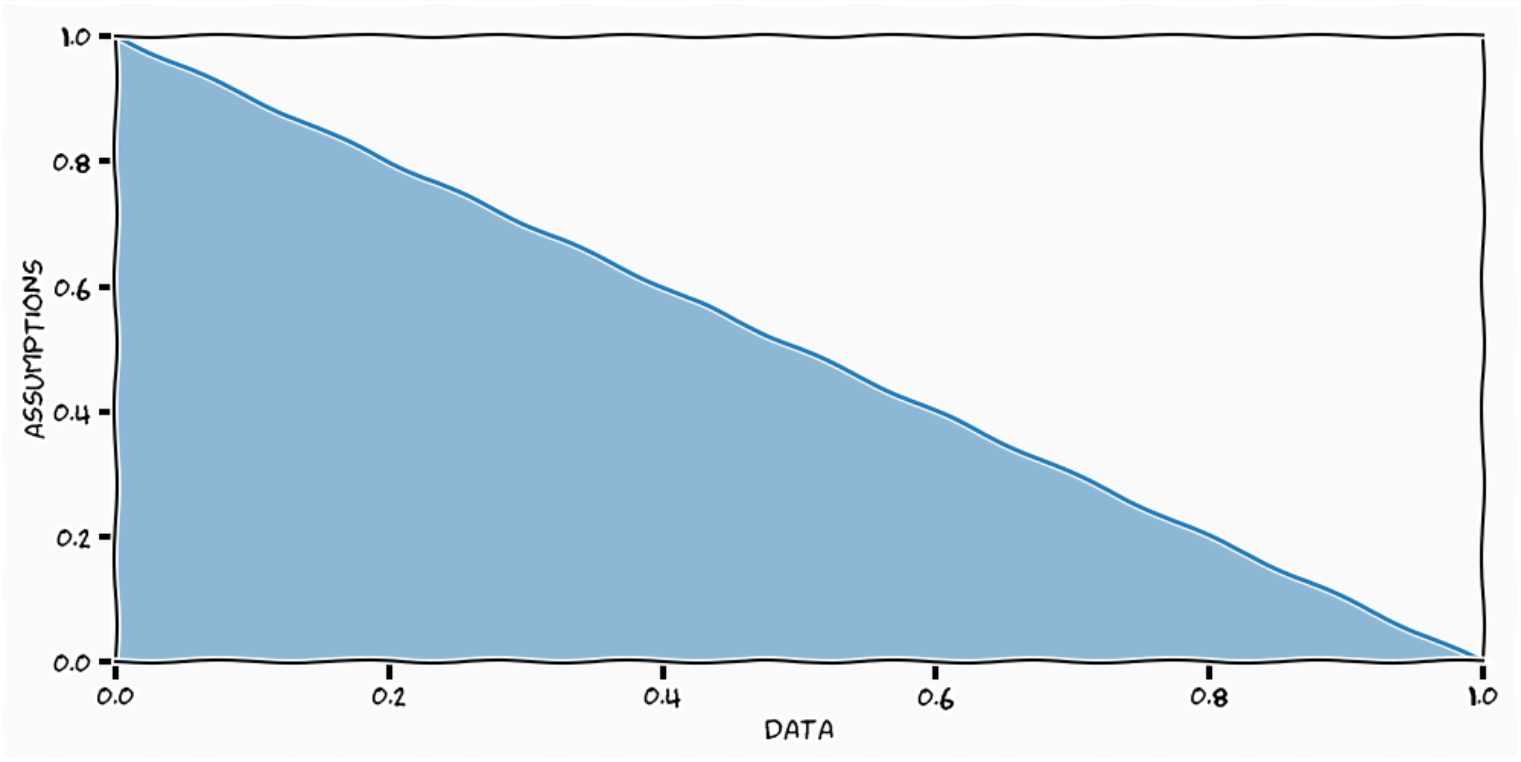
Change of Variables



Change of Variables



Challenge



- We can build a larger range of "functions" by combining simpler ones
- But we reduce our prior assumptions
- However, because its simple we can trade-off with data

Reinforcement Learning

Machine Learning

Supervised Learning predict output from input

$$\mathcal{D} = \{y_i, x_i\}_{i=1}^{N}$$

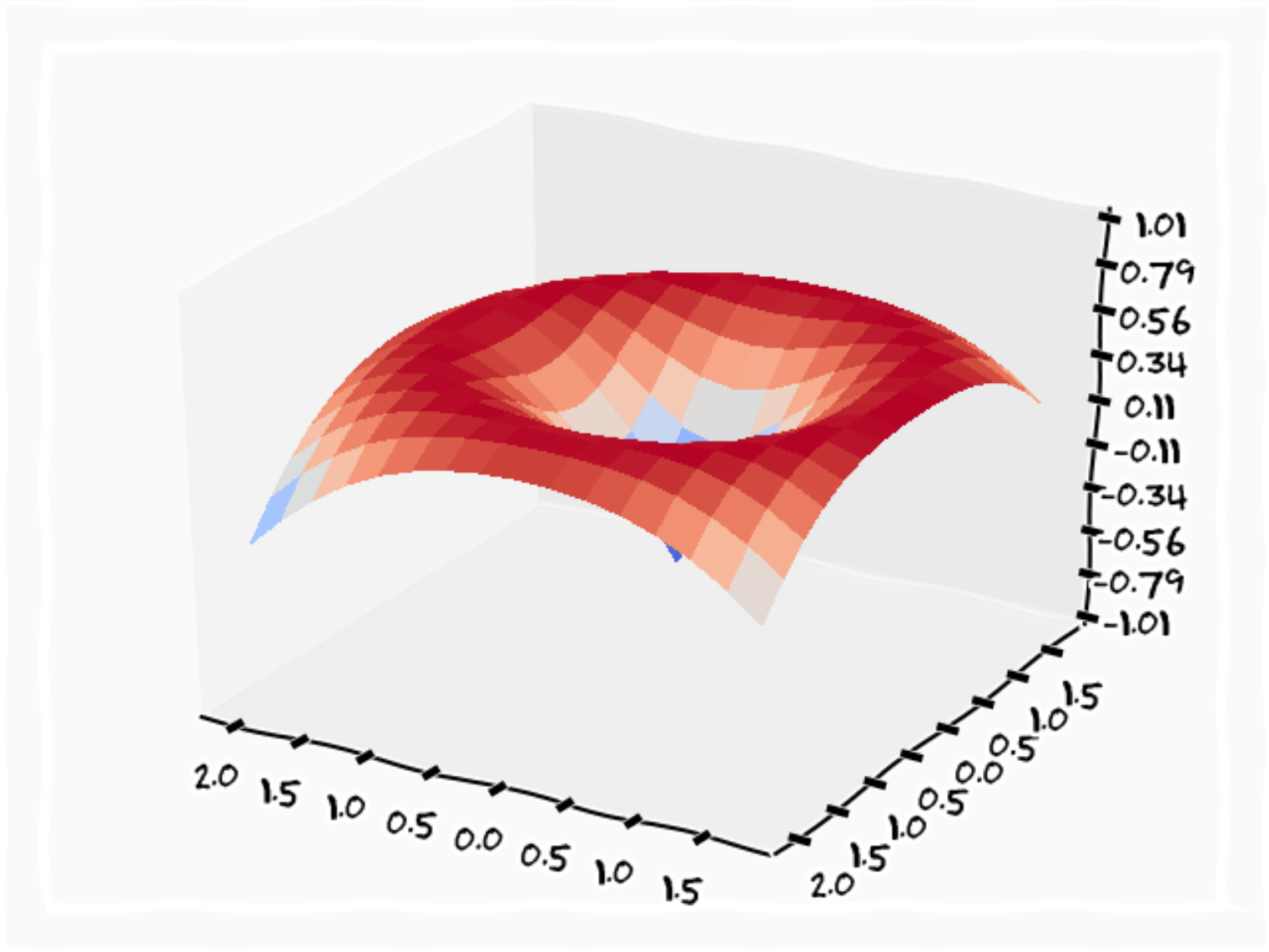
$$p(\mathbf{y}|\mathbf{x}, \theta)$$

Unsupervised Learning model the data

$$\mathcal{D} = \{y_i\}_{i=1}^{N}$$

$$p(\mathbf{y}|\theta)$$

Structure

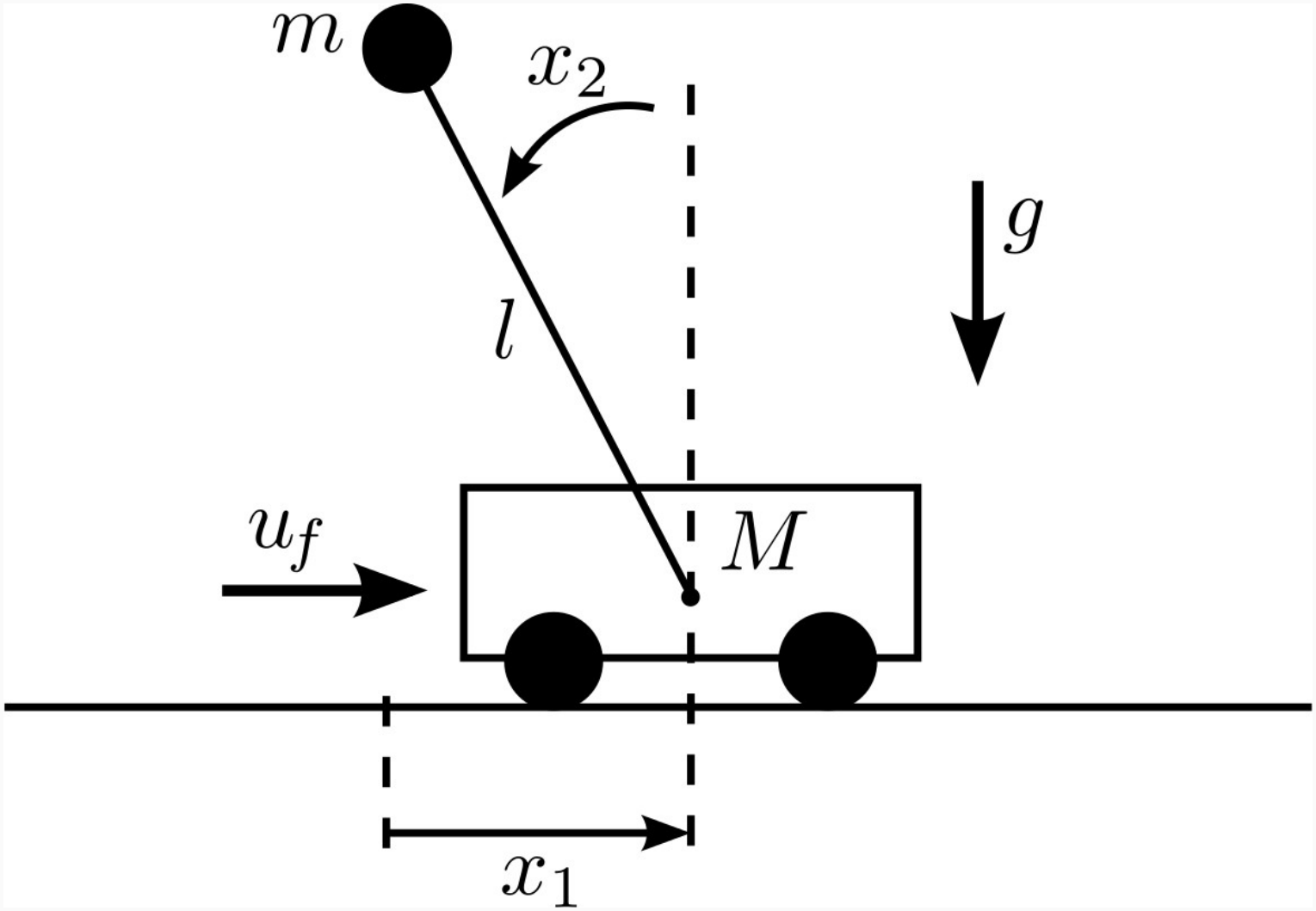

1.01
0.79
0.56
0.34
0.11
-0.11
-0.34
-0.56
-0.79
-1.01
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5

Reinforcement

Reinforcement Learning

Can we learn without specifying how the task should be achieved by providing, rewards (positive) and punishment (negative)?

Inverted Pendlum[1]



[1] https://www.youtube.com/watch?v=XiigTGKZfks

Formalism

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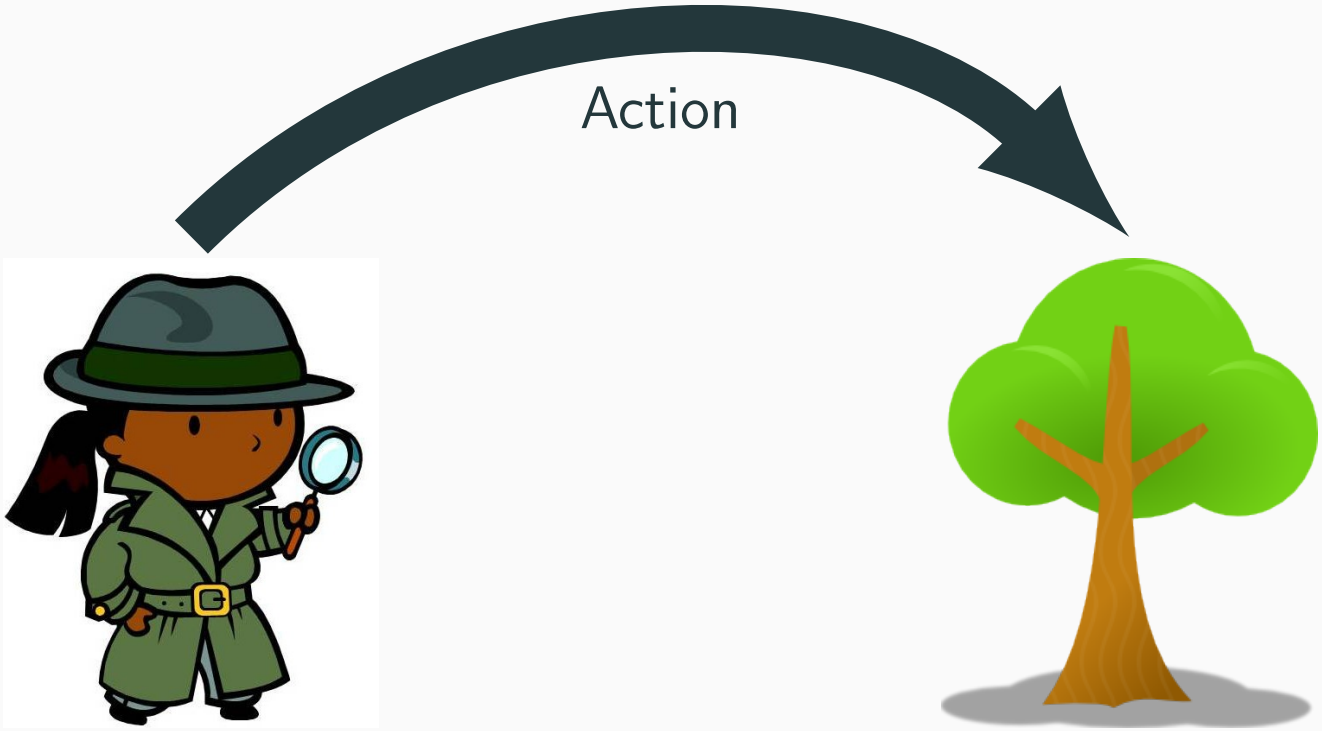
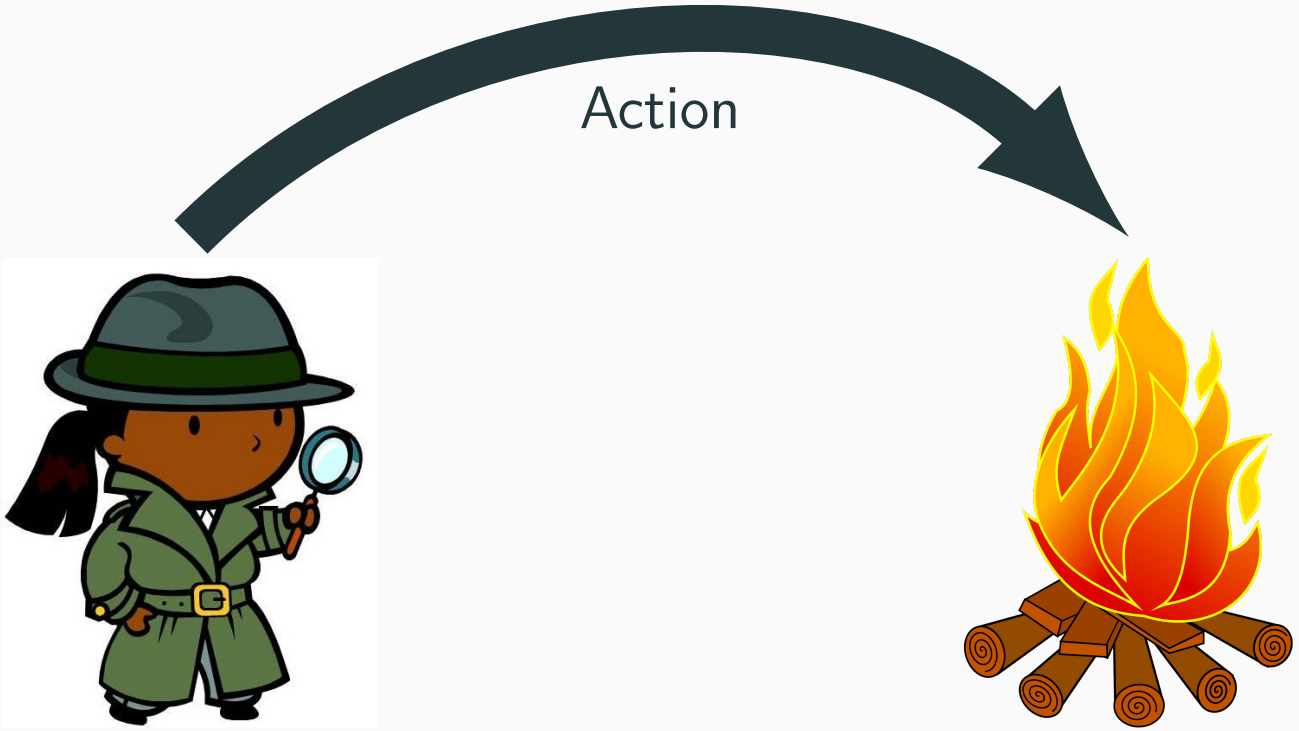
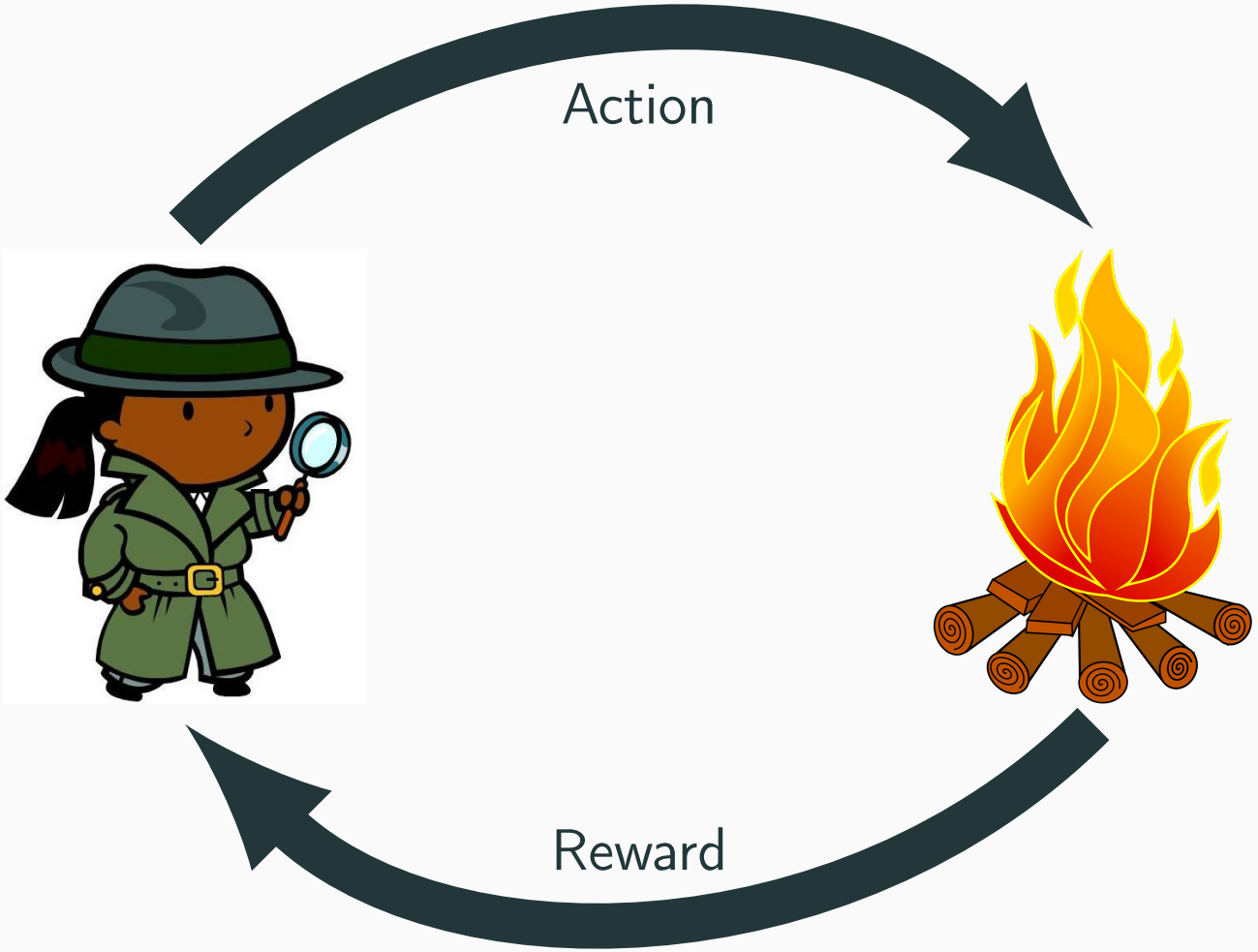


Formalism



Formalism



Example: Rollout

Environment you are in state 65 you have 4 possible actions

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Agent I take action 2

Example: Rollout

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Environment you recieved reinforcement of 7 units, you are now in state 15 you have 2 possible actions

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Environment you recieved reinforcement of 7 units, you are now in state 15 you have 2 possible actions

Agent I take action 1

Environment you recieved reinforcement of -4 units, you are now in state 65 you have 4 possible actions

Example: Rollout

Environment you are in state 65 you have 4 possible actions

Agent I take action 2

Environment you recieved reinforcement of 7 units, you are now in state 15 you have 2 possible actions

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Agent I take action 2

Example: Rollout

Environment you are in state 65 you have 4 possible actions

Agent I take action 2

Environment you recieved reinforcement of 7 units, you are now in state 15 you have 2 possible actions

Agent I take action 1

Environment you recieved reinforcement of -4 units, you are now in state 65 you have 4 possible actions

Agent I take action 2

Environment you recieved reinforcement of 5 units, you are now in state 44 you have 5 possible actions

Optimal Behaviour

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- Finite time horizon

$$E\left[\sum_{t=0}^{h} r_t\right]$$

Optimal Behaviour

- Finite time horizon

$$E\left[\sum_{t=0}^{h} r_t\right]$$

- Average reward

$$\lim_{h\to\infty} E\left[\frac{1}{h}\sum_{t=0}^{h} r_t\right]$$

Optimal Behaviour

- Finite time horizon

$$E\left[\sum_{t=0}^{h} r_t\right]$$

- Average reward

$$\lim_{h\to\infty} E\left[\frac{1}{h}\sum_{t=0}^{h} r_t\right]$$

- Infinite horizon (discounted reward)

$$E\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

$$0 \leq \gamma \leq 1$$

Markov Decision Process

Fully observable system

- Transition matrix

$$T(s, a, s') = p\left(s(t+1) = s' | s(t) = s, a(t) = a\right)$$

- Reward matrix

$$R(s, a, s') = E\left(s(t) = s, a(t) = a, s(t+1) = s'\right)$$

- For this set-up the optimal policy can be computed using Dynamic Programming

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- For this set-up the optimal policy can be computed using Dynamic Programming
- *We need to be able to enumerate all possible states and actions*

Reinforcement Learning

- Dynamic model

$$p(\mathbf{s}_t|\mathbf{a}_{t-1}, \mathbf{s}_{t-1}, f)$$

- Policy

$$p(\mathbf{a}_t|\mathbf{s}_t, \pi)$$

- Reward

$$p(r_t|\mathbf{s}_t)$$

Reinforcement Learning

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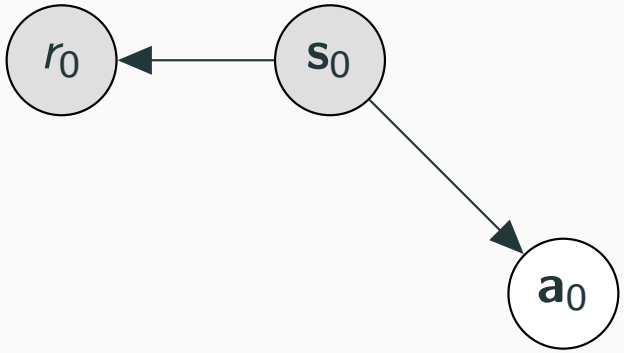
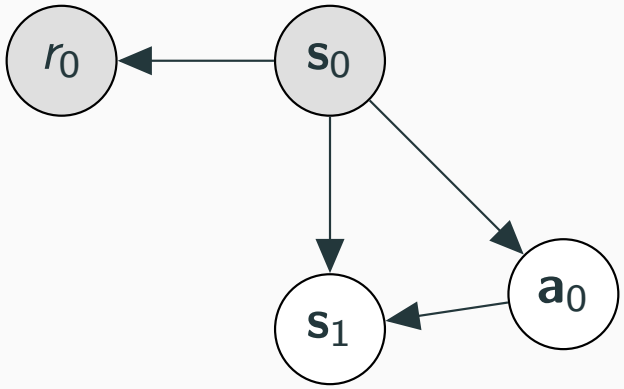


Reinforcement Learning



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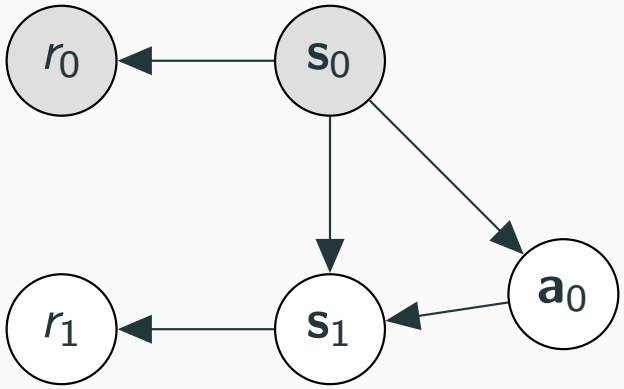

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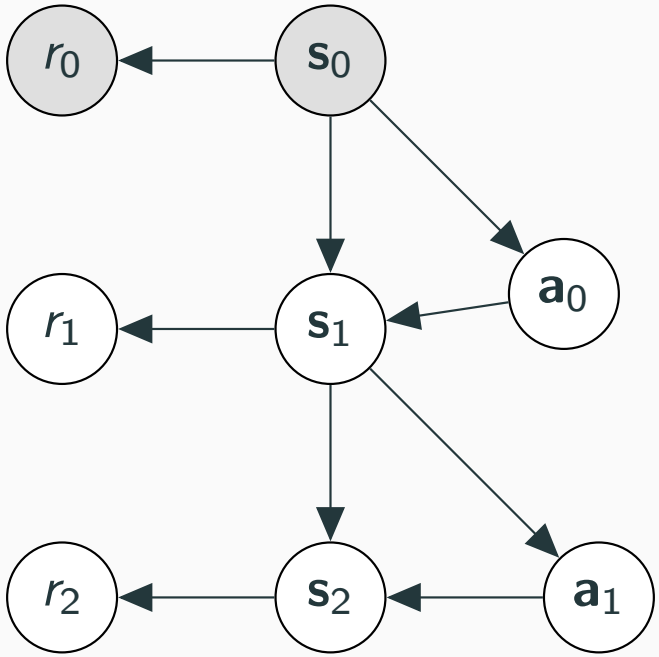

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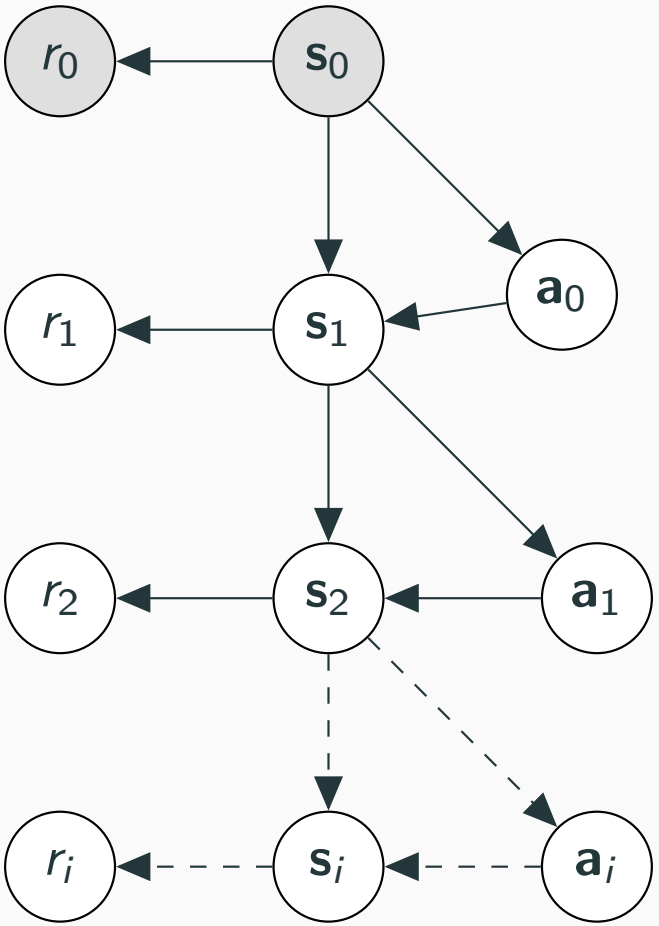

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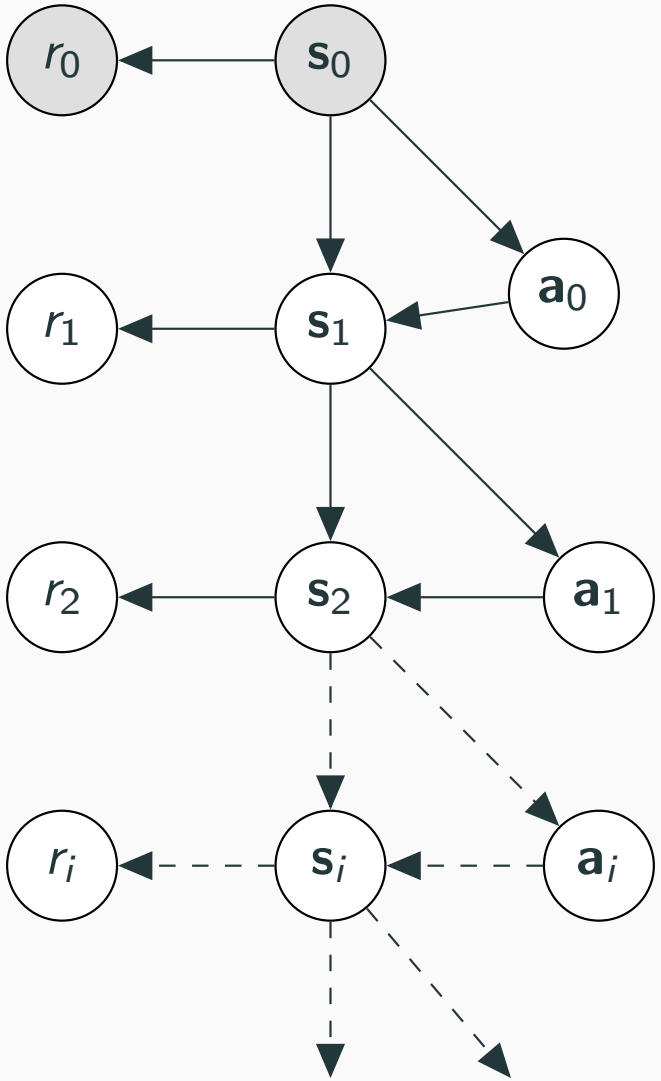

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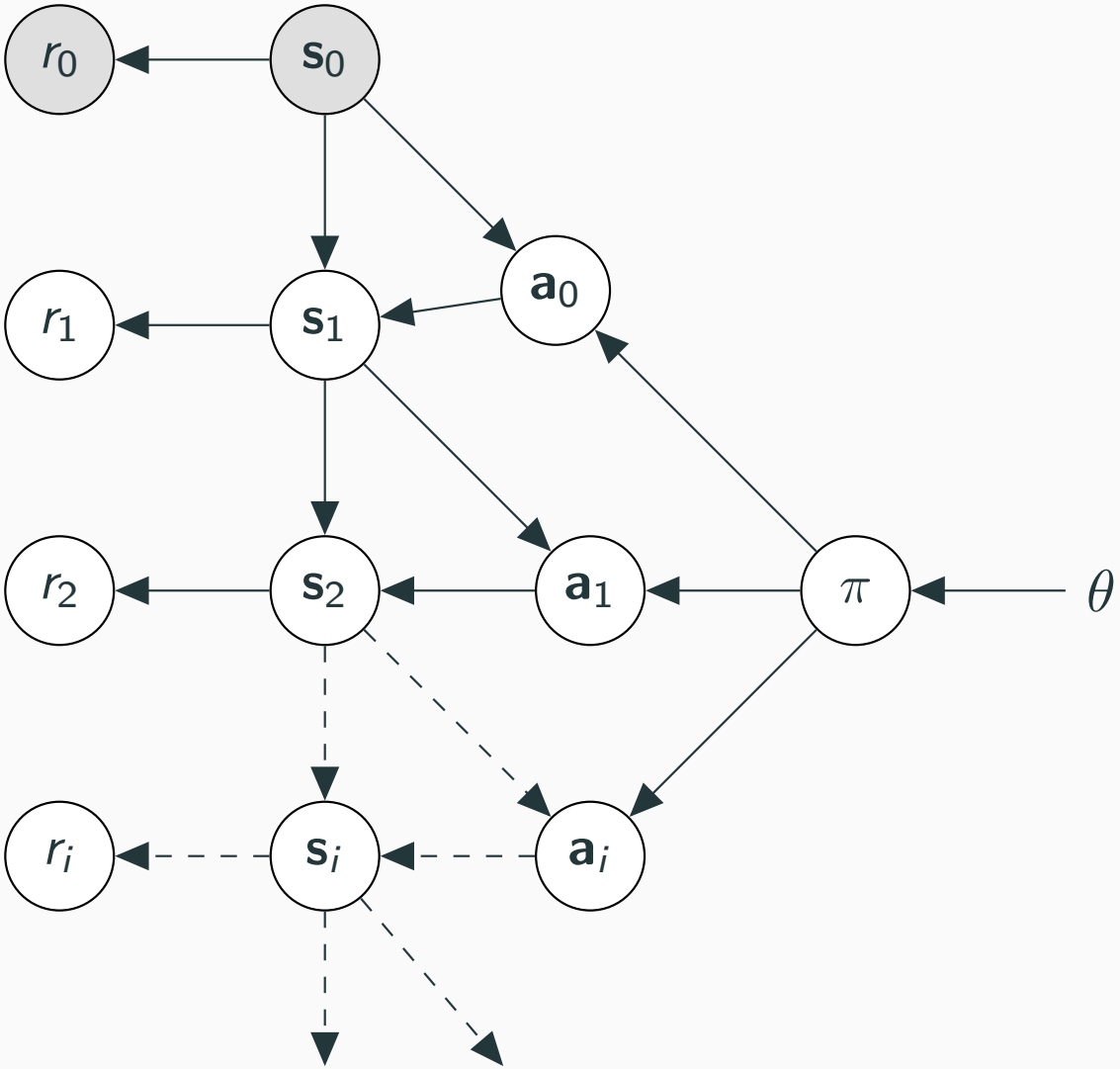

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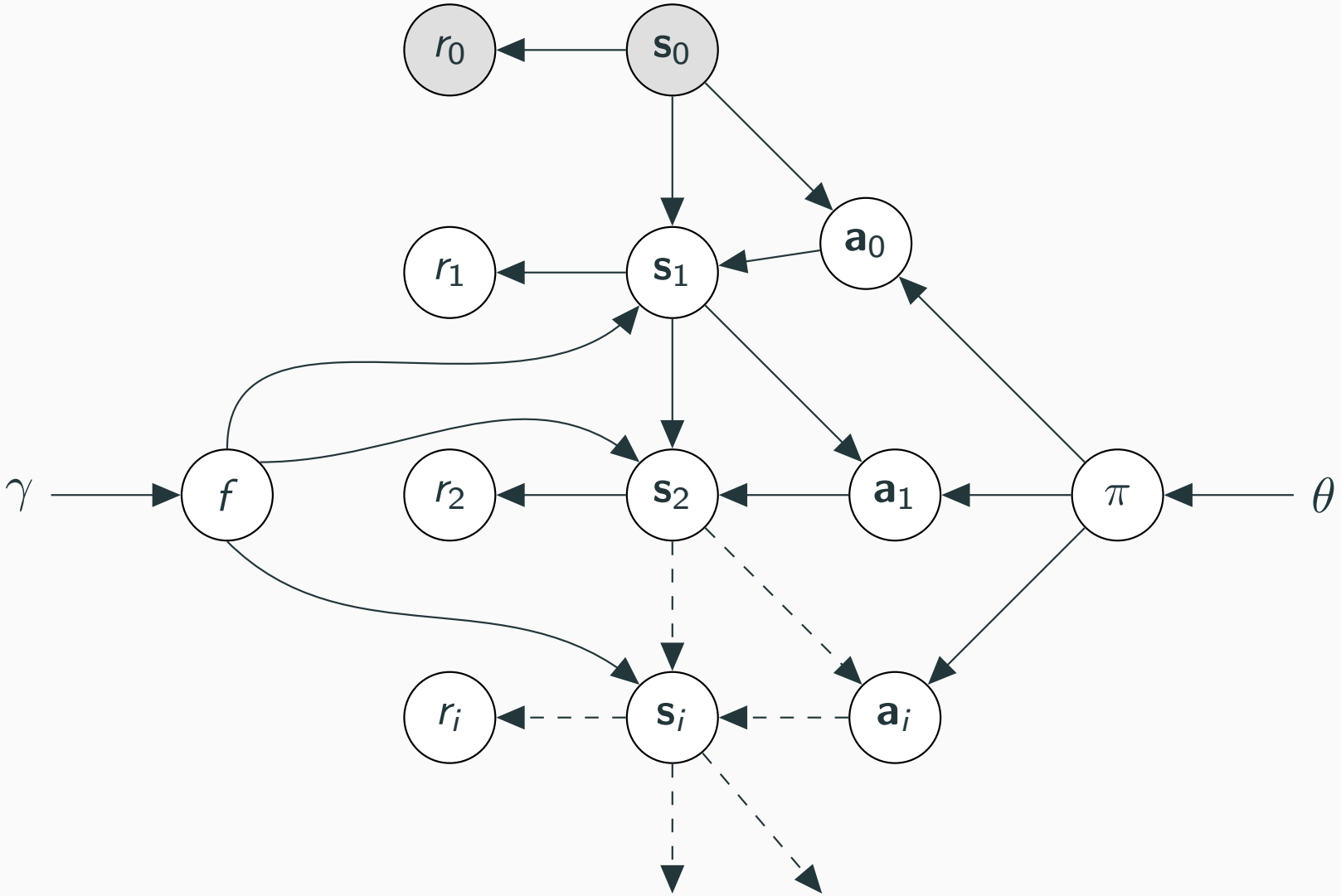

Reinforcment Learning : Rollout

$$
\begin{aligned}
p(\mathbf{s}_{0,\ldots,T}, \mathbf{a}_{0,\ldots,T-1}, r_{0,\ldots,T}, f, \pi, \gamma, \theta | \mathbf{s}_0) = \\
p(\mathbf{s}_T | a_{T-1}, s_{T-1}, f) p(a_{T-1} | s_{T-1}, \pi) \ldots \\
\vdots \\
p(\mathbf{s}_2 | a_1, s_1, f) p(a_1 | s_1, \pi) \\
p(\mathbf{s}_1 | a_0, s_0, f) p(a_0 | s_0, \pi) \\
p(f | \gamma) p(\pi | \theta) p(\gamma) p(\theta)
\end{aligned}
$$

- if we want to learn dynamics and policy we need to marginalise out f, π, θ and γ
- very very hard problem as uncertainty propagated a long way

Reinforcement Learning

$$p(\mathbf{y}_t|\mathbf{s}_t)$$

- We might not be able to observe the state

Model Free

$$Q : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$$

- Learning a model is very hard due to uncertainty propagation
- Directly learn a function from state action sets to reward

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$$Q : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$$

- Learning a model is very hard due to uncertainty propagation
- Directly learn a function from state action sets to reward
- *Do not build a model, rather a decision system*

Reinforcement Learning

- We can use any availible machine learning method to learn these functions
- Gaussian processes
- Linear regression
- Composite functions (neural networks)
- The tricky thing is how to get the data?

Data

Bandits

Bandit problems

- You are in a room with k slot machines
- Each have a different (unknown) probability of pay-off
- You are permitted h different executions
- Whats the optimal strategy?

Exploration vs. Exploration

- When thinking of a strategy to come up with a policy we need to balance
  - exploring the environment
  - exploiting what we know

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- Re-inforcement learning can be thought of in terms of bandit problems
  - sequential bandits
  - delayed reward
  - betting in poker (I think)

Exploration vs. Exploration

- When thinking of a strategy to come up with a policy we need to balance
  - exploring the environment
  - exploiting what we know
- Re-inforcement learning can be thought of in terms of bandit problems
  - sequential bandits
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  - betting in poker (I think)
- Designing this strategy is the main challenge

Exploration vs. Exploration

- When thinking of a strategy to come up with a policy we need to balance
  - exploring the environment
  - exploiting what we know
- Re-inforcement learning can be thought of in terms of bandit problems
  - sequential bandits
  - delayed reward
  - betting in poker (I think)
- Designing this strategy is the main challenge
- Is Reinforcement Learning Bayesian Optimisation?

Deep Mind

https://www.youtube.com/watch?v=faDKMMwOS2Q

Models vs. Decisions

https://youtu.be/QHcAlAprFxA

Summary

Summary

- Reinforcement learning
  - its nothing different at all, same ideas of modelling
  - how can we learn when we do not know how to do something
  - exploration exploitation
  - if we knew all states, all rewards, all actions it can be done exact

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References