6.18

(a) $Z = \dfrac{54.75-62.5}{2.7} \approx -2.87$

$P(X<54.75) = P(Z<-2.87) = 0.0021$

$1000 \times 0.0021 = 2.1 \approx 2$ students #

(b) $Z_1 = \dfrac{59.75-62.5}{2.7} \approx -1.02$

$Z_2 = \dfrac{65.25-62.5}{2.7} \approx 1.02$

$P(59.75 < X < 65.25) = P(-1.02 < Z < 1.02)$

$= 0.8461 - 0.1539$

$= 0.6922$

$1000 \times 0.6922 = 692.2 \approx 692$ students #

(c) $Z_1 = \dfrac{62.75-62.5}{2.7} \approx 0.09$

$Z_2 = \dfrac{63.25-62.5}{2.7} \approx 0.28$

$P(62.75 < X < 63.25) = P(0.09 < Z < 0.28)$

$= 0.6103 - 0.5359$

$= 0.0744$

$1000 \times 0.0744 = 74.4 \approx 74$ students #

(d) $Z = \dfrac{61.25-62.5}{2.7} \approx -0.46$

$P(X > 61.25) = P(Z > -0.46) = 1 - P(Z < -0.46)$

$= 1 - 0.3228 = 0.6772$

$1000 \times 0.6772 = 677.2 \approx 677$ students #

6.24

$\mu = np = 600 \times 0.4 = 240$

$\sigma = \sqrt{npq} = \sqrt{600 \times 0.4 \times 0.6} = 12$

(a) $z_1 = \frac{214.5-240}{12} \approx -2.13$

$z_2 = \frac{250.5-240}{12} \approx 0.88$

$$P(214.5 < X < 250.5) = P(-2.13 < z < 0.88)$$
$$= 0.8106 - 0.0166$$
$$= 0.794 \;\#$$

(b) $z_1 = \frac{269.5-240}{12} \approx 2.46$

$z_2 = \frac{270.5-240}{12} \approx 2.54$

$$P(269.5 < X < 270.5) = P(2.46 < z < 2.54)$$
$$= 0.9945 - 0.9931$$
$$= 0.0014 \;\#$$

(c) $z_1 = \frac{204.5-240}{12} \approx -2.96$

$z_2 = \frac{260.5-240}{12} \approx 1.71$

$P(X < 204.5) = P(z < -2.96) = 0.0015$

$$P(X > 260.5) = P(z > 1.71) = 1 - P(z < 1.71)$$
$$= 1 - 0.9564$$
$$= 0.0436$$

$0.0015 + 0.0436 = 0.0451 \;\#$

6.40

$$\gamma(2) = \int_0^{\infty} x^{2-1} e^{-x}\, dx$$

$$= \int_0^{\infty} x e^{-x}\, dx$$

$$= (-xe^{-x} - e^{-x})\Big|_0^{\infty}$$

$$= 0-(0-1)$$

$$= 1$$

$$P(x>9) = \int \frac{1}{3^2\gamma(2)} x^{2-1} e^{-\frac{x}{3}}\, dx$$

$$= \frac{1}{9}\int x e^{-\frac{x}{3}}\, dx$$

$$= \frac{1}{9}\left[-3xe^{-\frac{x}{3}} - 9e^{-\frac{x}{3}}\right]_9^{\infty}$$

$$= \frac{1}{9}(27e^{-3} + 9e^{-3})$$

$$= 4e^{-3} \approx 0.199$$

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