HW_5 F7406703 常定_利

5.10 In a certain fitness test for athletes, it is found that 10% of the athletes fail to complete the test. Of the next 15 athletes tested, find the probability that

(a) from 3 to 6 fail;

(b) fewer than 4 fail;

(c) more than 5 fail.

$P = 10\% = 0.1$

a) $P(3 \le X \le 6) = \sum_{i=3}^{6} P(X=i) = \sum_{i=3}^{6} b(i;15,0.1)$

$= \sum_{i=0}^{6} b(i;15,0.1) - \sum_{i=0}^{2} b(i;15,0.1)$

$= 0.9997 - 0.8159 = 0.1838$

0.1838

b) $P(X<4) = \sum_{i=0}^{3} P(X=i) = \sum_{i=0}^{3} b(i;15,0.1)$

$= 0.9444$

c) $P(5<X) = P(6 \le X) = 1 - P(5<X) = 1 - P(6 \le X)$

$= 1 - \sum_{i=0}^{6} P(X=i) = 1 \sum_{i=0}^{6} b(i;15,0.1)$

$1 - 0.9997 = 0.0003$

5.28 A manufacturer knows that on average 20% of the electric toasters produced require repairs within 1 year after they are sold. When 20 toasters are randomly selected, find appropriate numbers x and y such that

(a) the probability that at least x of them will require repairs is less than 0.5;

(b) the probability that at least y of them will *not* require repairs is greater than 0.8.

a) $P(X \geq x) < 0.5 = 1 - P(P < x) < 0.5 = 0.5 < P(X \leq x-1)$

$0.5 < P(X=0) + P(X=1) + \cdots + P(X = x-1)$

$0.5 < b(0;20,0.2) + b(1;20,0.2) + \cdots + b((x-1);20,0.2)$

$0.5 < \sum_{i=0}^{x-1} b(i;20,0.2)$

$0.4114 = \sum_{i=0}^{3} b(i;20,0.2) < 0.5 < \sum_{i=0}^{4} b(i;20,0.2) = 0.6296$

$r = x-1 = 4, \quad x = 5$

b) $P(X \leq 20-y) > 0.8$

$P(X=0) + P(X=1) + \ldots + P(X=20-y) > 0.8$

$b(0;20,0.2) + b(1;20,0.2) + \ldots + b(20-y;20,0.2) > 0.8$

$\sum_{i=0}^{20-y} b(i;20,0.2) > 0.8$

$0.6296 \sum_{i=0}^{4} b(i;20,0.2) < 0.8 < \sum_{i=0}^{5} b(i;20,0.2) = 0.8042$

$r = 5$

$r = 20 - y = 5 \quad y = 20 - 5 = 15$

5.40 It is estimated that 4000 of the 10,000 voting residents of a town are against a new sales tax. If 15 eligible voters are selected at random and asked their opinion, what is the probability that at most 7 favor the new tax?

$10000 - 4000 = 6000$

$p = \frac{6000}{10,000} = 0.6$

$P(X=x) = b(x;15,0.6) = \binom{15}{x} 0.6^x (1-0.6)^{15-x} \quad x = 0,1,\ldots,15$

$P(X \leq 7)$

$P(X \leq 7) = \sum_{i=0}^{7} P(X=i) = \sum_{i=0}^{7} b(i;15,0.6)$

$= 0.2131$

5.80 Service calls come to a maintenance center according to a Poisson process, and on average, 2.7 calls are received per minute. Find the probability that

(a) no more than 4 calls come in any minute;

(b) fewer than 2 calls come in any minute;

(c) more than 10 calls come in a 5-minute period.

$\lambda = 2.7$

$\mu = \lambda t = 2.7t$

$$P(X=x) = e^{-\lambda t}\frac{(\lambda t)^x}{x!} \quad x = 0, 1, 2 \ldots$$

a) $P(X \le 4) = \sum_{x=0}^{4} e^{-2.7}\frac{(2.7)^x}{x!} = \mathbf{0.863}$

b) $t = 1 \quad \mu = 2.7$

$$P(X<2) = \sum_{x=0}^{1} e^{-2.7}\frac{(2.7)^x}{x!} = \mathbf{0.2487}$$

c) $t = 5, \ \mu = 2.7 \times 5 = 13.5$

$$P(X > 10) = 1 - P(X \le 10)$$
$$= 1 - \sum_{x=0}^{10} e^{-13.5}\frac{(13.5)^x}{x!}$$

$$P(X>10) = (1 - 0.2112) = \mathbf{0.789}$$