6.18 The weights of 1000 students are normally distributed with a mean of 62.5 kilograms and a standard deviation of 2.7 kilograms. Assuming that the weights are recorded to the nearest half-kilograms, how many of these students would you expect to have weights

(a) less than 55 kilograms?

(b) between 60 and 65 kilograms, both inclusive?

(c) equal to 63 kilograms?

(d) greater than 61 kilograms?

(1) $(X \le 55) = P\left(\frac{X-\mu}{\sigma} \le \frac{55-62.5}{2.7}\right) = P\left(Z \le \frac{-7.5}{2.7}\right) =$

$P(Z \le -2.777\ldots) = 0.0028$

$E(X) = nP = 1000 \times 0.0028 = 2.8 \approx \mathbf{3}$

(2) $(X \le 65) = P\left(\frac{X-\mu}{\sigma} \le \frac{65-62.5}{2.7}\right) = P\left(Z \le \frac{2.5}{2.7}\right) = P(Z \le 0.9259)$
$= 0.8283$

$(X \le 60) = P\left(\frac{X-\mu}{\sigma} \le \frac{60-62.5}{2.7}\right) = P\left(Z \le \frac{-2.5}{2.7}\right) = P(Z \le -0.9259$
$= 0.1762$

$P(60 \le X \le 65) = P(X \le 65) - P(X \le 60)$
$= 0.8283 - 0.1762 = 0.652\,1$

$E(x) = nP = 1000 \times 0.6521 = 652.1 \approx \mathbf{652}$

(3) $P(X \le 63.5) = P\left(\frac{X-\mu}{\sigma} \le \frac{63.5-62.5}{2.7}\right) = P(Z \le 0.3703) = 0.6443$

$P(X \le 62.5) = P\left(\frac{X-\mu}{\sigma} \le \frac{62.5-62.5}{2.7}\right) = P(Z \le 0) = 0.5$

$P(63 - 0.5 \le X \le 63 + 0.5) = P(62.5 \le X \le 63.5)$
$= P(X \le 63.5) - P(X \le 62.5)$
$= 0.6443 - 0.5 = 0.1443$

$E(x) = nP = 1000 \times 0.1443 = 144.3 \approx \mathbf{144}$

(4) $P(X \ge 61) = 1 - P(X \ge 61) = 1 - P\left(\frac{X-\mu}{\sigma} \le \frac{61-62.5}{2.7}\right) = 1 - P(Z \le 0.550)$
$= 1 - 0.2877 = 0.7123$

$E(X) = nP = 1000 \times 0.7123 = 712.3 \approx \mathbf{712}$

6.24 The probability of success of 600 students appearing in a test independently is 0.40. Using normal approximation, find the probability that

(a) between 215 and 250 students pass, both inclusive;

(b) exactly 270 students pass;

(c) fewer than 205 and more than 260 students pass.

$\mu = np$
$= 600 \times 0.4 = 240$

$\sigma = \sqrt{np(1-p)}$
$= \sqrt{240(1-0.4)} = 12$

a) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{215 - 240}{12} = -2.08333$

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{250 - 240}{12} = 0.8333$

$P(215 \le X \le 250) = \sum_{210}^{250} b(x, 600, 0.4)$
$= P(215 \le X \le 250)$
$= P(-2.0833 \le X \le 0.833)$
$= 0.7967 - 0.0188$
$= \mathbf{0.7779}$

b) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{269.5 - 240}{12} = 2.45833$

$z_2 = \dfrac{x - \mu}{\sigma} = \dfrac{270.5 - 240}{12} = 2.54166$

$P(x = 270) = b(270, 600, 0.4) = P(2.45833 \le z \le 2.5466)$
$= P(z \le 2.54466) - P(z \le 2.45833)$
$= 0.9445 - 0.9929$
$= \mathbf{1.6 \times 10^{-3}}$

c) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{205 - 240}{12} = -2.916666$

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{260 - 240}{12} = 1.666$

$P(205 \le X \le 260) = \sum_{205}^{260} b(x, 600, 0.4) = P(-2.916 \le z \le 1.666)$
$= P(z \le 1.666) - P(z \le -2.916) = 0.9515 - 0.0018$
$= 0.9497$

$P(X < 205) + P(X > 260) = 1 - P(205 \le X \le 260)$
$= 1 - 0.9497$
$= \mathbf{0.0503}$

6.40 In a certain city, the daily consumption of water (in millions of liters) follows approximately a gamma distribution with $\alpha = 2$ and $\beta = 3$. If the daily capacity of that city is 9 million liters of water, what is the probability that on any given day the water supply is inadequate?

$$f(x;2,3) = \begin{cases} \frac{1}{(3)^2\,\Gamma(2)} x^{2-1} e^{-x/3} & x>0 \\ 0 & \text{Othewise} \end{cases}$$

$$P(X>9) = 1 - P(X \le 9)$$

$$= 1 - \int_0^9 f(x;2,3)\,dx = 1 - \int_0^9 \frac{1}{9} x e^{-x/3}$$

$$= 1 - \frac{1}{9}\left[x(-3e^{-x/3}) - \int(-3e^{-x/3})\,dx\right]_0^9$$

$$= 1 - \left(-\frac{1}{3}xe^{-x/3} - e^{-x/3}\right)_0^9$$

$$= 1 - \left(\left(-\frac{1}{3}(9)e^{-9/3} - e^{-9/3}\right) - \left(-\frac{1}{3}(0)e^{-0/3} - e^{-0/3}\right)\right)$$

$$= 1 - \left(\left(-3e^{-9/3} - e^{9/3}\right) - (0-1)\right)$$

$$= 1 - (1 - 4e^{-3})$$

$$= 1 - (1 - 0.1991)$$

$$= 1 - 0.8009$$

$$= 0.1991$$