Two-Echelon Multicommodity Location Model

Sets

- I = set of production plants; (*index* i)
- J = set of potential Distribution Centers (DCs); (*index* j)
- R = set of demand nodes; (*index* r)
- K = set of homogeneous commodities; (*index* k)

Parameters

- p = maximum number of DCs that can be opened;
- c^k_{ijr} = unit transportation cost of commodity $k \in K$ from plant node $i \in I$ to demand node $r \in R$ across DC $j \in J$;
- d^k_r = demand of commodity $k \in K$ from demand node $r \in R$;
- p^k_i = maximum quantity of commodity $k \in K$ that can be manufactured by plant $i \in I$;
- q^-_j = minimum activity level of potential DC $j \in J$;
- q^+_j = maximum activity level of potential DC $j \in J$;
- f_j = fixed cost of potential DC $j \in J$;
- g_j = marginal cost of potential DC $j \in J$;

Variables

$$z_j = \begin{cases} 1 & \text{if DC } j \in J \text{ is opened} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jr} = \begin{cases} 1 & \text{if demand node } r \in R \text{ is assigned to DC } j \in J \\ 0 & \text{otherwise} \end{cases}$$

- s^k_{ijr} = amount of commodity $k \in K$ transported from plant node $i \in I$ to demand node $r \in R$ across DC $j \in J$;

The following feasibility condition must hold:

$$\sum_{i \in I} p^k_i \geq \sum_{r \in R} d^k_r \quad k \in K$$

TEMC Mathematical Formulation

$$\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\sum_{k\in K} c^k_{ijr}\cdot s^k_{ijr} + \sum_{j\in J}\left(f_j\cdot z_j + g_j\cdot \sum_{r\in R}\sum_{k\in K} d^k_r\cdot y_{jr}\right)$$

$$\sum_{j\in J}\sum_{r\in R} s^k_{ijr} \le p^k_i \quad i\in I,\, k\in K \tag{1}$$

$$\sum_{i\in I} s^k_{ijr} = d^k_r\cdot y_{jr} \quad j\in J,\, r\in R,\, k\in K \tag{2}$$

$$\sum_{j\in J} y_{jr} = 1 \quad r\in R \tag{3}$$

$$q^-_j\cdot z_j \le \sum_{r\in R}\sum_{k\in K} d^k_r\cdot y_{jr} \le q^+_j\cdot z_j \quad j\in J \tag{4}$$

$$\sum_{j\in J} z_j = p \tag{5}$$

$$z_j\in\{0,1\} \quad j\in J \tag{6}$$

$$y_{jr}\in\{0,1\} \quad j\in J,\, r\in R \tag{7}$$

$$s^k_{ijr}\ge 0 \quad i\in I,\, j\in J,\, r\in R,\, k\in K \tag{8}$$

Demand Allocation Problem

If a set $\overline{z}\ j\in J$ and $\overline{y}_{jr},\ j\in J,\ r\in R$ of feasible values is available, you just have to solve the following LP problem in order to determine the optimal demand allocation:

$$\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\sum_{k\in K} c^k_{ijr}\cdot s^k_{ijr}$$

$$\sum_{j\in J}\sum_{r\in R} s^k_{ijr} \le p^k_i \quad i\in I,\, k\in K$$

$$\sum_{i\in I} s^k_{ijr} = d^k_r\cdot \overline{y}_{jr} \quad j\in J,\, r\in R,\, k\in K$$

$$s^k_{ijr}\ge 0 \quad i\in I,\, j\in J,\, r\in R,\, k\in K$$