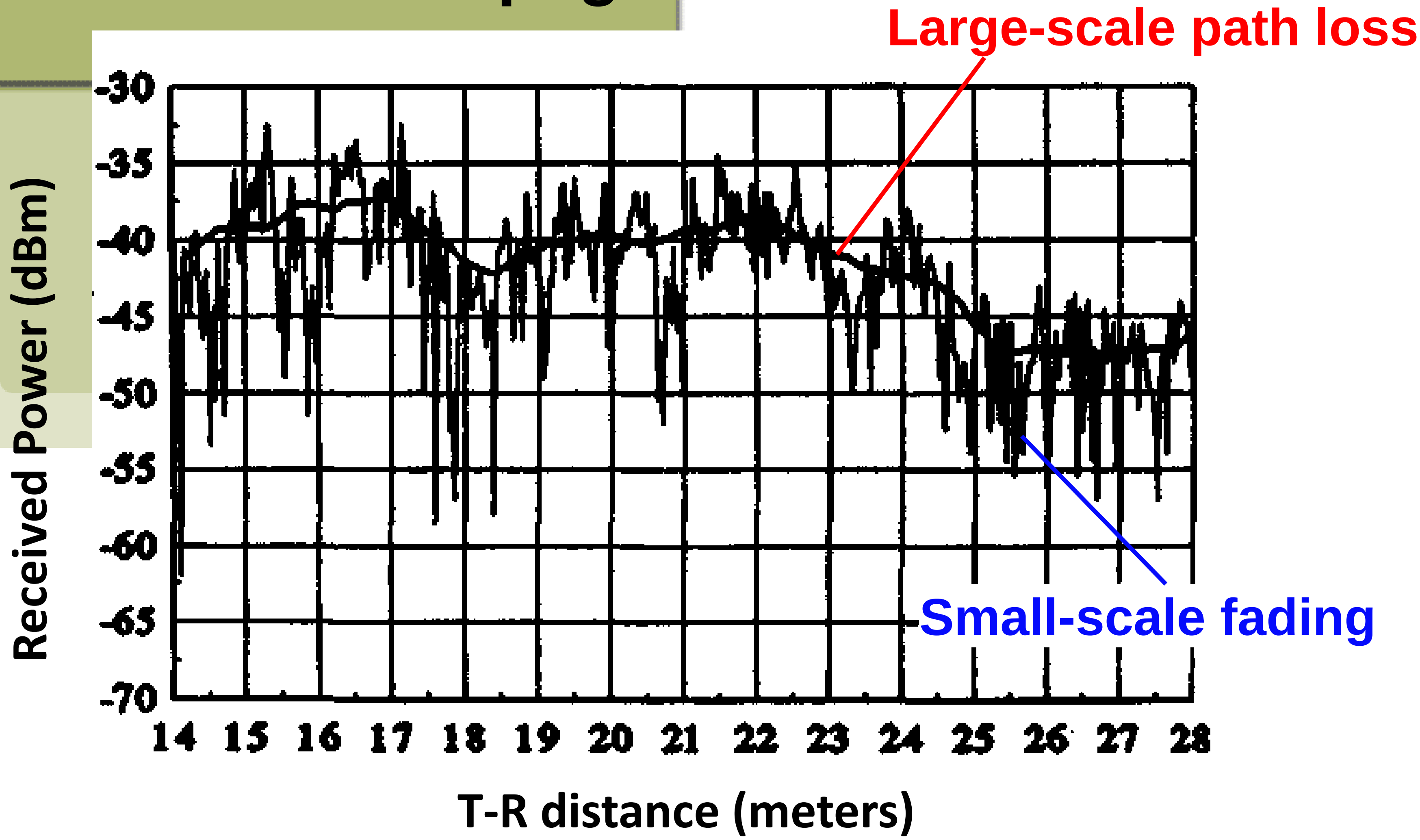
Chapter 4

Mobile Radio Propagation: Small-Scale Fading and Multipath

Mobile Radio Propagation



The received power varying with T-R distance.

Contents of Chapter 4

1. Small-Scale Multipath Propagation
2. Impulse response Model of Multipath channels
3. Parameters of Mobile Multipath Channels
4. Types of Small-Scale Fading
5. Rayleigh and Ricean Distributions

Chapter 4

1. **Small-Scale Multipath Propagation**
2. Impulse response Model of Multipath channels
3. Parameters of Mobile Multipath Channels
4. Types of Small-Scale Fading
5. Rayleigh and Ricean Distributions

Causes of Small-Scale Fading

- **Multipath:**

  Radio waves reach the receiver along multiple paths due to complex environments.

- **Time-varying:**

  Relative motion occurs between transmitter and receiver.

Small-Scale fading

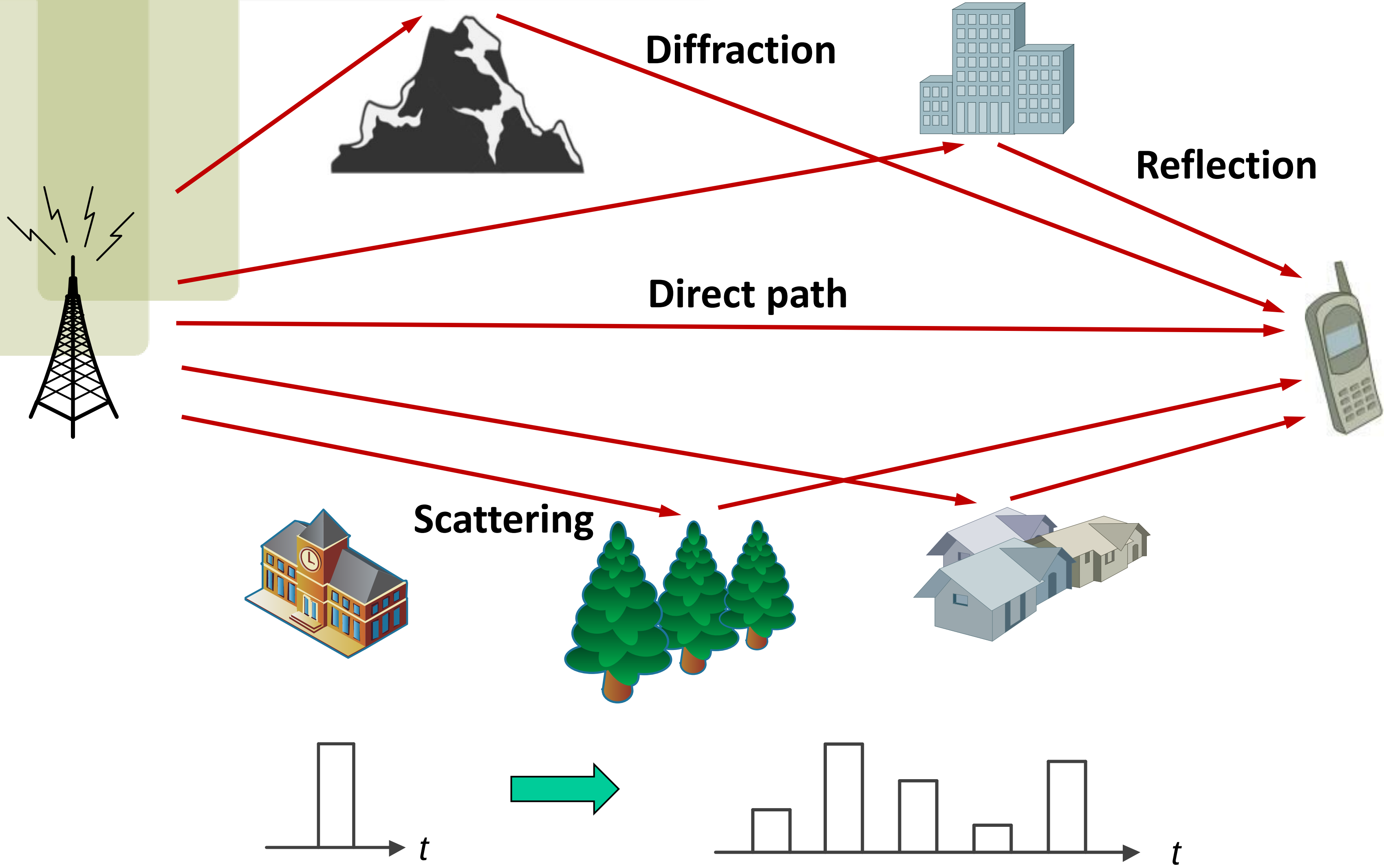
Small-scale fading is a more rapid fluctuation of signals

– caused by

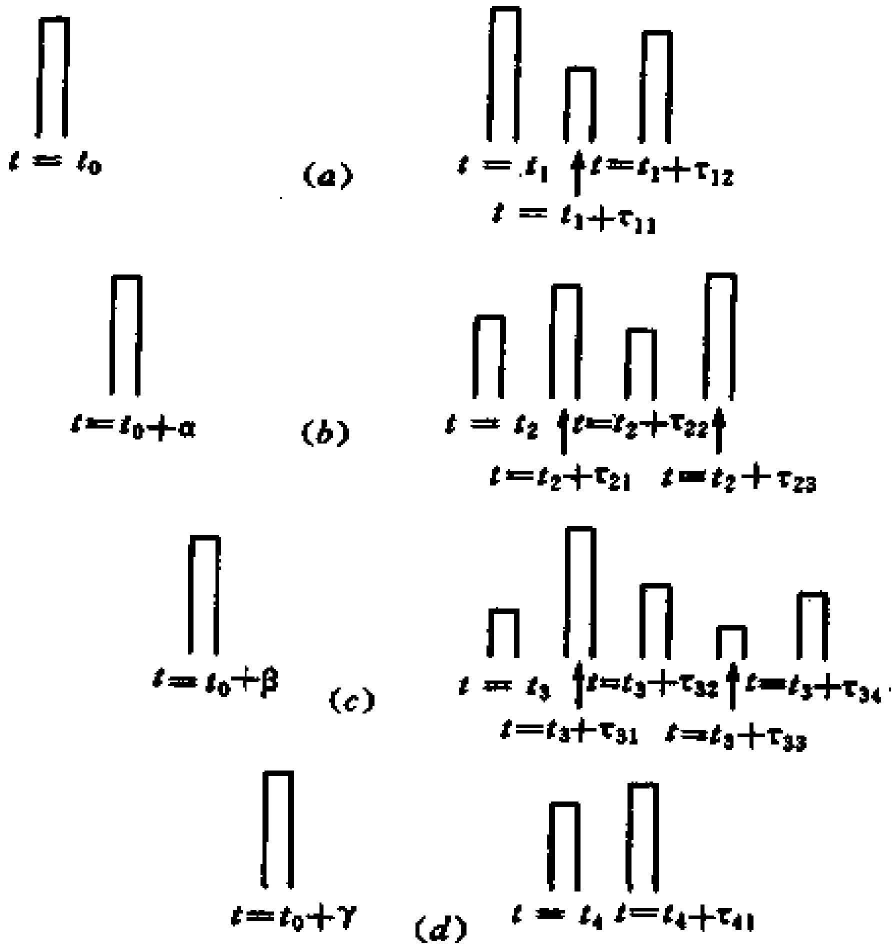
- **Multipath effect**, constructive and destructive interference between two or more versions of the same signal

- **Doppler effect**, due to moving terminals or surroundings

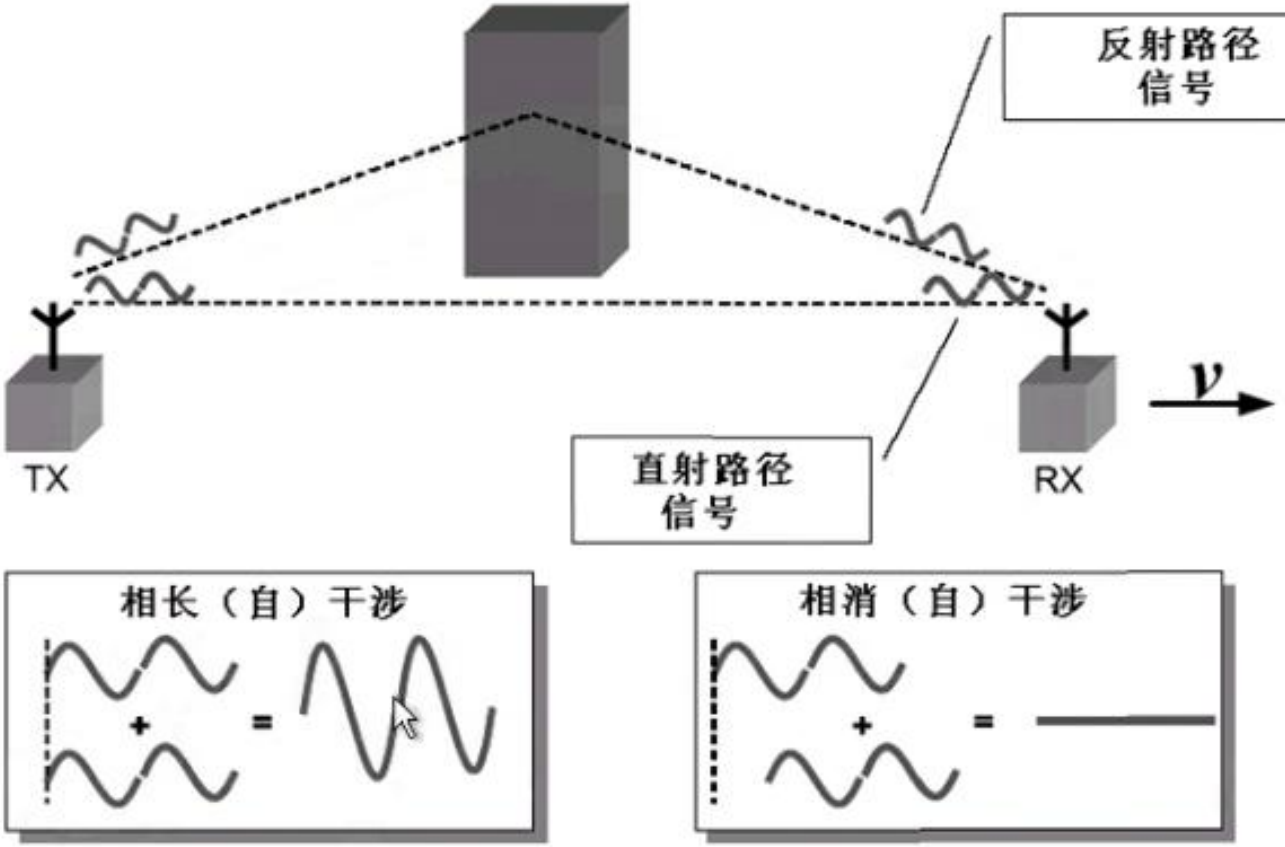
Multipath Effect



Time-variant multipath channel

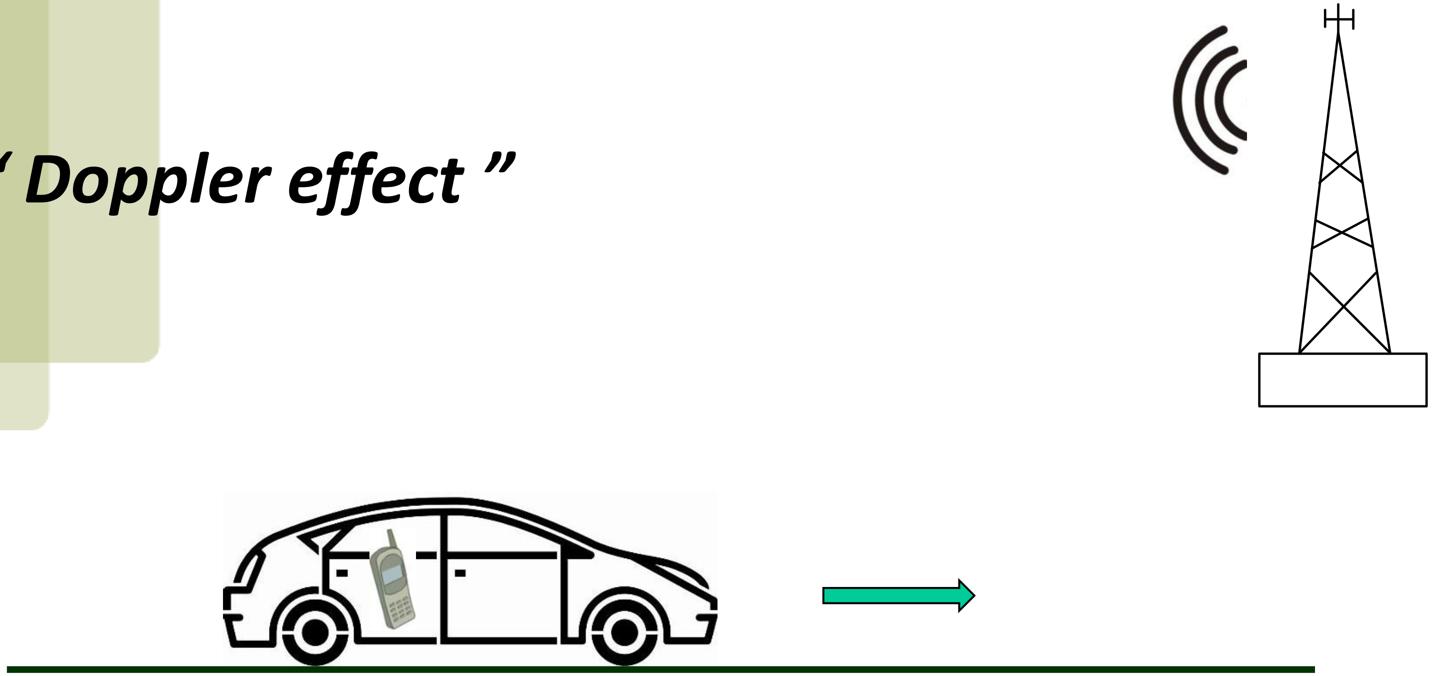


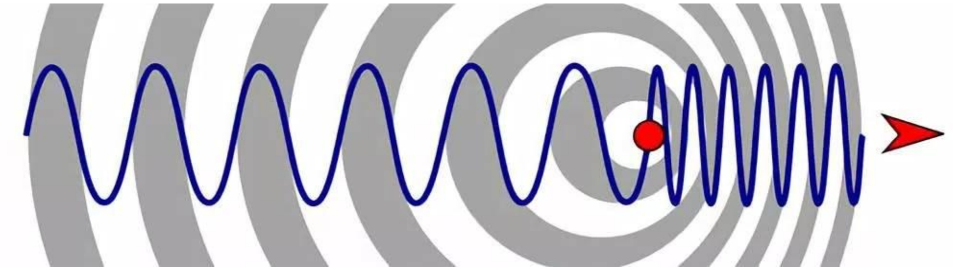
Multipath



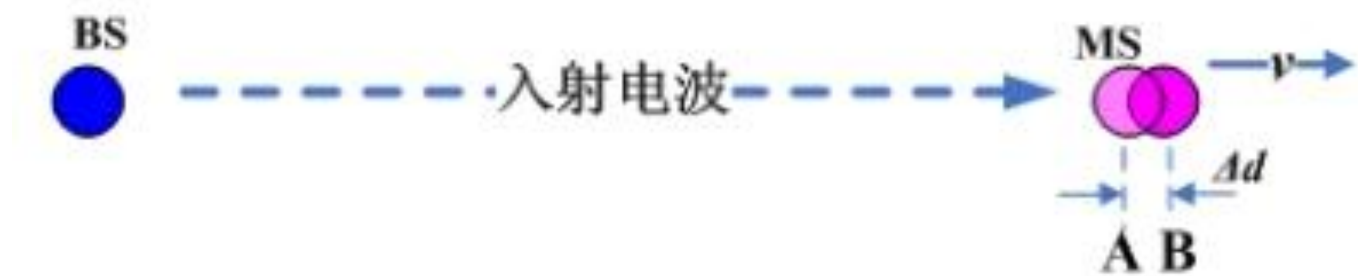
Time-varying

“ Doppler effect ”





Doppler Shift



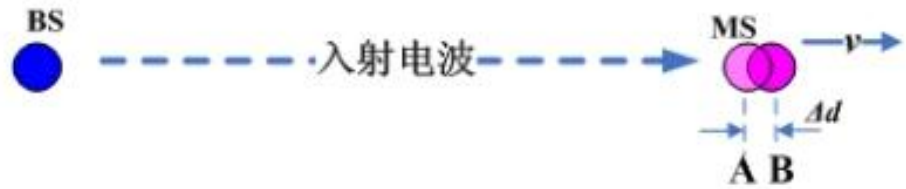
$$f_{re} = f_c + f_d$$

f_{re} : received frequency

f_c : carrier frequency

f_d : doppler shift

Doppler Shift



$$s(t) = A\cos(2\pi f_c t + \varphi_0)$$

"A"

$$s\left(t-\frac{\Delta d}{c}\right) = A\cos\left[2\pi f_c\left(t-\frac{\Delta d}{c}\right)+\varphi_0\right]$$

$$= A\cos\left(2\pi f_c t \boxed{-\frac{2\pi}{\lambda}\Delta d}+\varphi_0\right)$$

"B"

B点是波传播方向上后出现的点！！

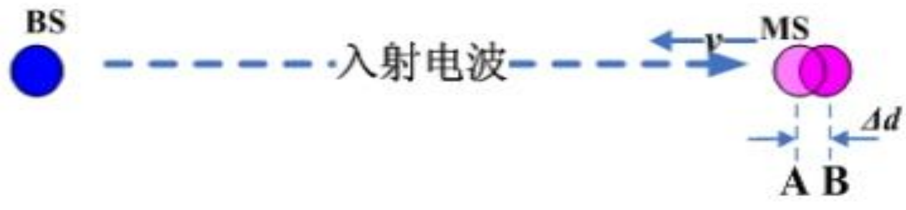
Doppler Shift

$$\Delta\varphi = -\frac{2\pi}{\lambda} \bullet \Delta d$$

$$\because \Delta d = v \bullet \Delta t$$

$$\therefore \Delta f = \frac{1}{2\pi}\frac{\Delta\varphi}{\Delta t} = -\frac{1}{2\pi}\frac{2\pi}{\lambda}v = -\frac{v}{\lambda}$$

$f_d = -v/\lambda < 0$



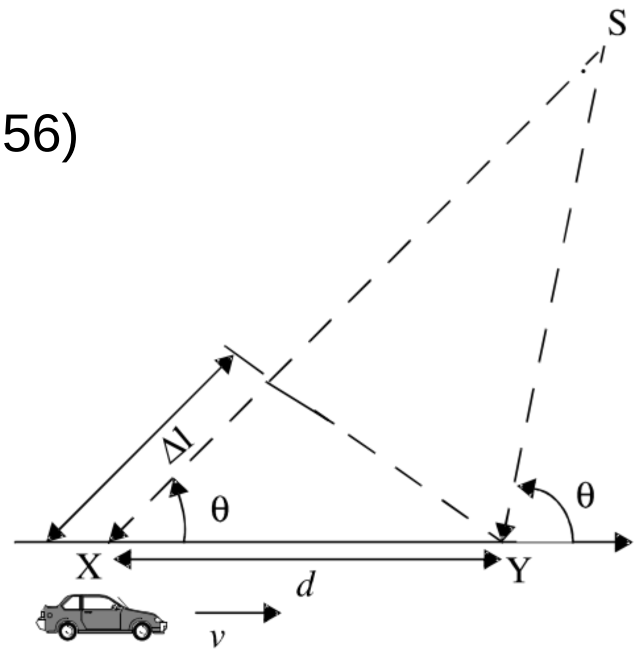
$f_d = +v/\lambda > 0$

Doppler Shift

(PP.156)

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta$$

$$\boxed{f_d = \frac{1}{2\pi}\cdot\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cdot\cos\theta}$$



<u>Example</u>

-Carrier frequency f_c = 1850 MHz (i.e. λ = 16.2 cm)

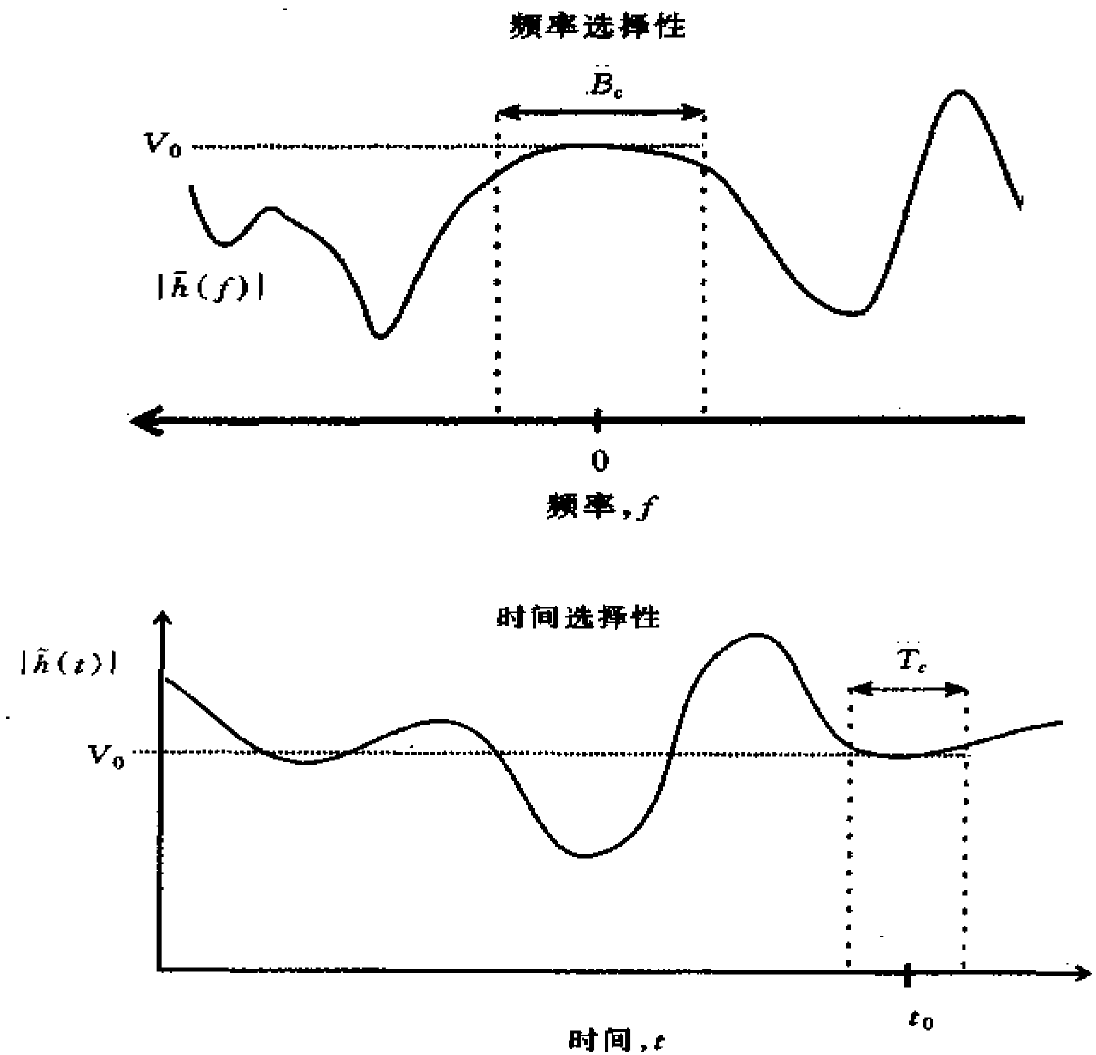
-Vehicle speed v = 60 mph = 26.82 m/s

-If the vehicle is moving directly towards the transmitter

$$\Delta f = \frac{26.82}{0.162} = 165Hz$$

-If the vehicle is moving perpendicular to the angle of arrival of the transmitted signal $\Delta f = 0$

Coherence Bandwidth & Coherence Time



Characteristics of Radio Channel

- (Large path loss)
- **Multipath** (reflection, scattering)
- **Time-variant** (time-varying, a consequence of the constantly changing physical characteristics of the media, e.g. moving of objects)

It is reasonable to characterize the time-variant multipath channels statistically.

- Time spread (multipath)
- Frequency spread (time-variant)

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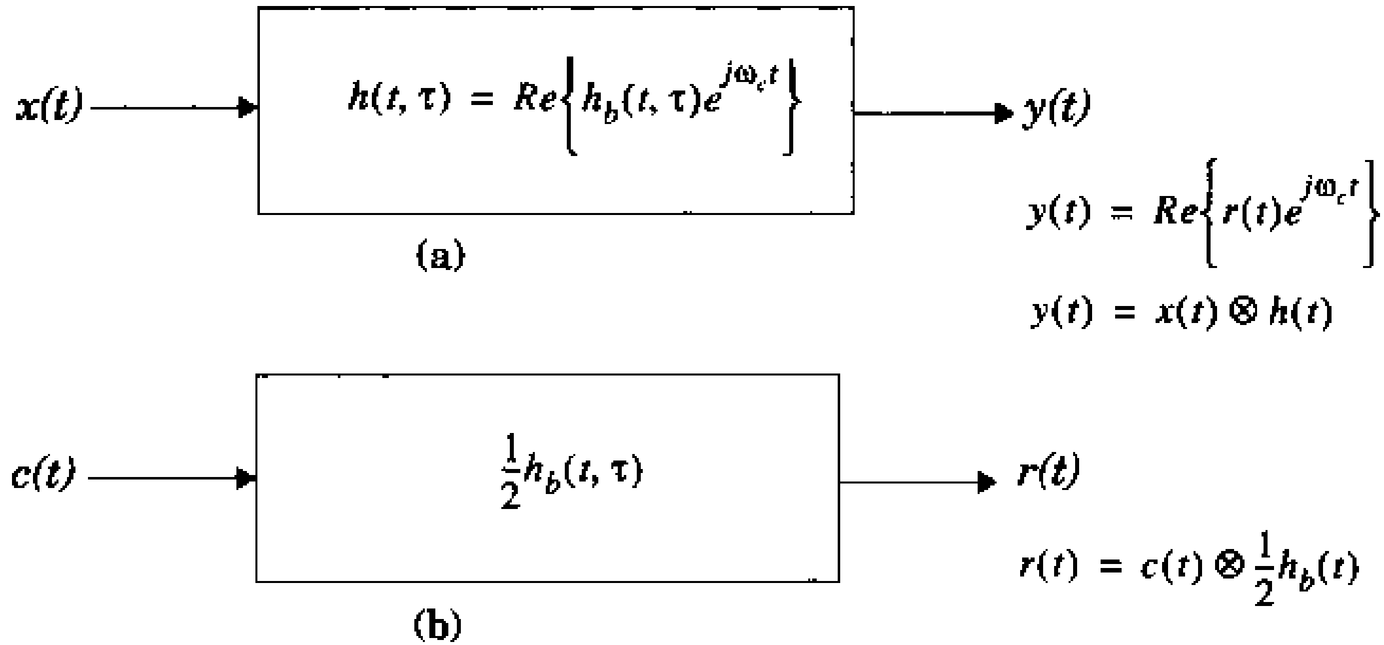
Impulse response of wireless channels

- The mobile channel can be modeled as a linear, time-varying filter with impulse response ***h* (*t*,τ),** where τ is the channel multipath delay for a fixed *t.*
- *The* ***impulse response*** $h\ (t,\tau)$ completely characterizes the channel and is a function of both t and τ
- *The received signal can be expressed as a* ***convolution*** *of the channel impulse response* $h(t,\tau)$ *with the transmitted signal x(t)*

$$y(t) = h(t,\tau) \otimes x(t) = \int_{-\infty}^{\infty} h(t,\tau)x(t-\tau)d\tau$$

$$or \quad y(t) = x(t) \otimes h(t,\tau) = \int_{-\infty}^{\infty} x(\tau)h(t,t-\tau)d\tau$$



$h(t,\tau)$ complex passband channel

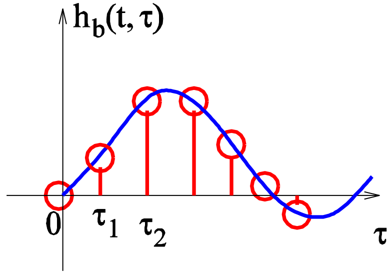
$h_b(t,\tau)$ complex baseband equivalent channel

t : time variation due to motion/Doppler shift

τ: multipath delay (time-dispersion)

Random Characteristics of wireless channels

- The impulse response of the time-variant multipath channel is a random process
- The signal passed through a time-variant multipath channel is a random process
- Describe multipath delay as the **excess delay,** relative to the first arriving multipath component with excess delay $t_0 = 0$
- **Discretize** excess delay in ***N*** equally spaced "bins" ,such that all multipath wave components inside bin no. ***i*** are represented by **one** component with delay $\tau_i = i\Delta\tau$. (figure 5.4)

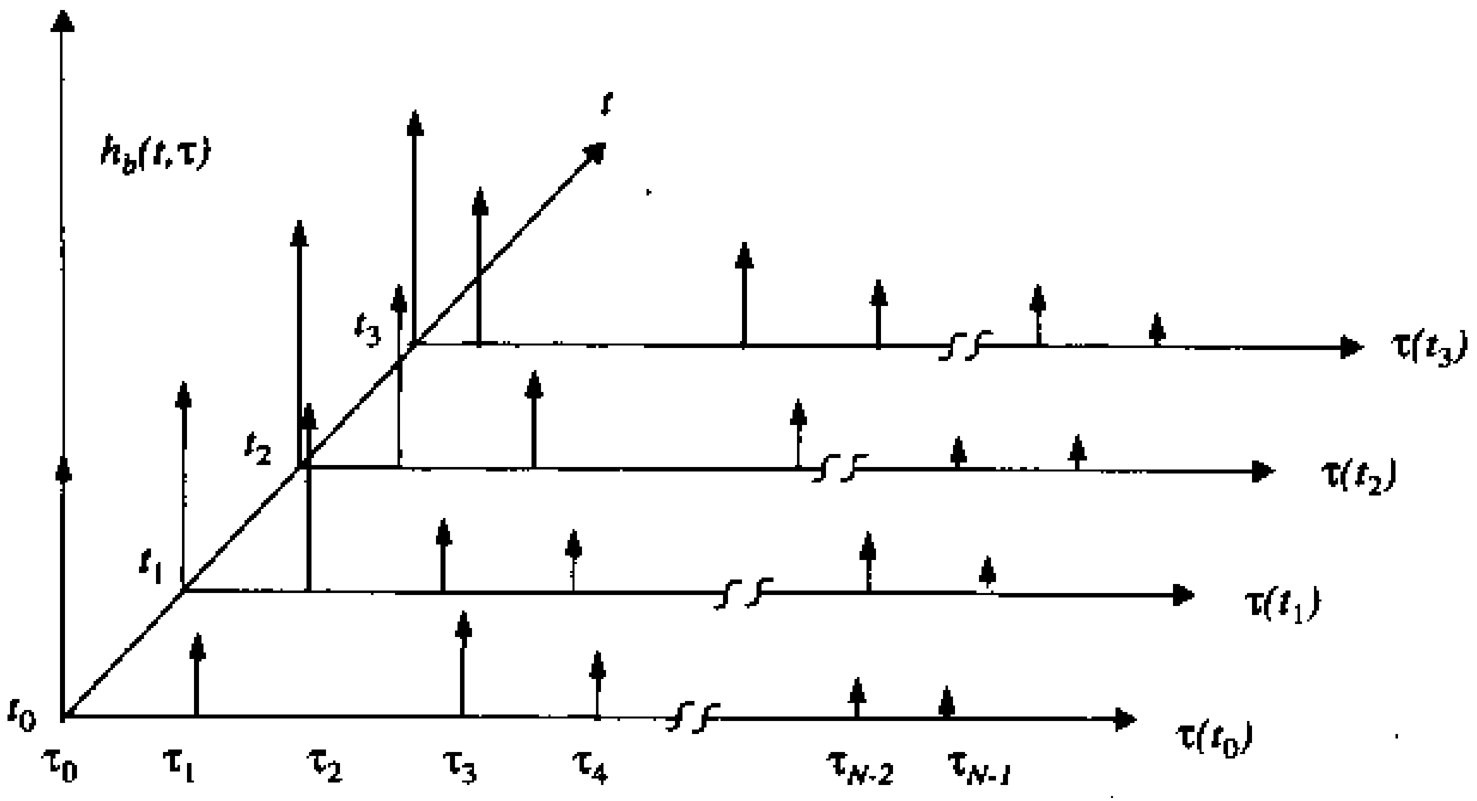


$$h_b(t,\tau)=\sum_{i=0}^{N-1} a_i(t,\tau)\exp[j\theta_i(t,\tau)]\delta(\tau-\tau_i(t))$$

$$h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp(j\theta_i)\delta(\tau-\tau_i)$$

- **So far, we have channel model**
  - **Good: it gives every detail about the channel**
  - **Bad: it is hard to see any essential characteristics of channels, such as what signal can pass, what signal can not pass**
- **We need a few major parameters for easy**
  - **Compare different channels (delay, bandwidth, spectrum, etc)**
  - **Develop design guide lines for wireless signals**

The time varying discrete-time impulse response model for a multipath radio channel



Power Delay Profile

(PP.161)

- Such parameters can be derived from channel model
  - –Specifically, from "power delay profile" of the channel
  - –Power delay profile is the spatial/time average over a local area

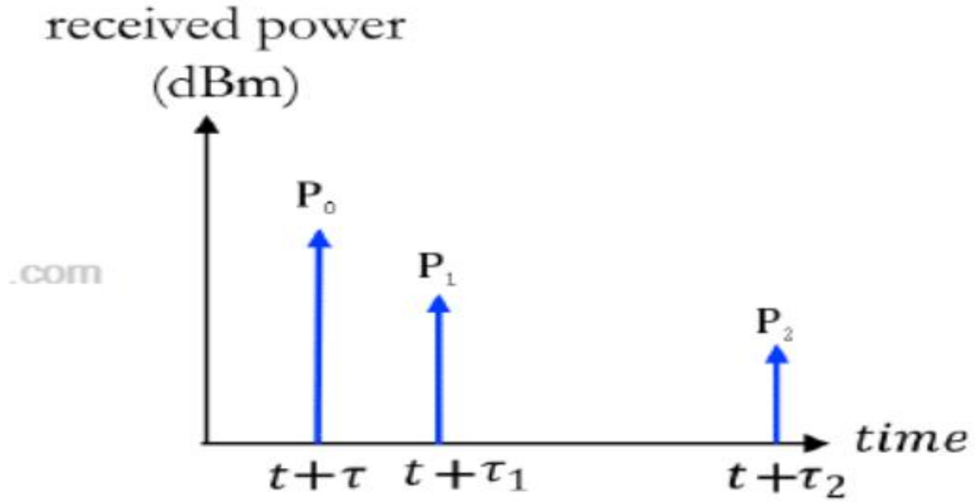
$$\overline{|h_b(\tau)|^2} = E\left\{ \left| \sum_{i=0}^{N-1} a_i e^{j\theta_i} \delta(\tau - \tau_i) \right|^2 \right\}$$

  - –It is a function: power~ delay, i.e., the average received power with some delay.

Power Delay Profile

Many major parameters can be derived from the channel models, such as power delay profile (PDP) of the channel

In a typical PDP plot, the signal power P_i of each multipath is plotted against their repective propagation delays τ_i



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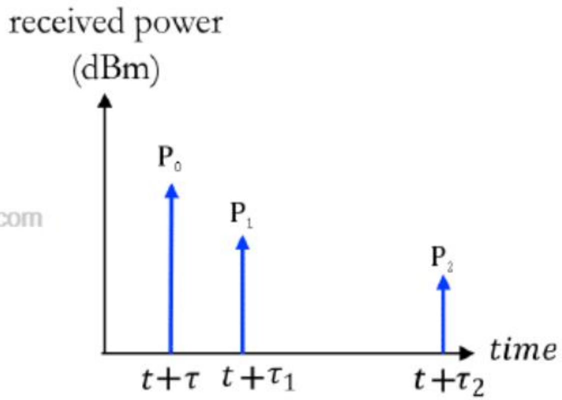
Parameters of Mobile Multipath Channels

- RMS delay spread
- Coherence bandwidth
- Doppler Spread
- Coherence Time

Multipath

Time-varying

Time Dispersion Parameters



- Mean excess delay

$$\bar{\tau} = \frac{\sum_k a_k^{\,2}\tau_k}{\sum_k a_k^{\,2}} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

- RMS delay spread

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

where

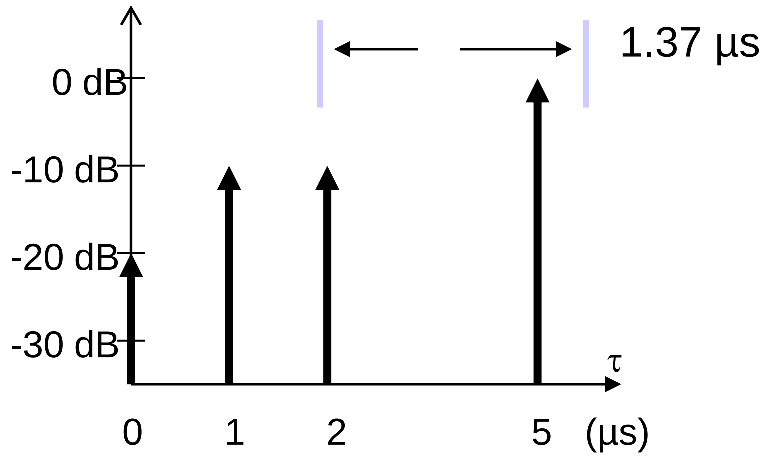
$$\overline{\tau^2} = \frac{\sum_k a_k^{\,2}\tau^2{}_k}{\sum_k a_k^{\,2}} = \frac{\sum_k P(\tau_k)\tau^2{}_k}{\sum_k P(\tau_k)}$$

Table 4.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10-25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200-310 ns	Averaged typical case	[Cox72]
Suburban	910	1960-2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10-50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70-94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

Example

(PP.176)



$$\bar{\tau} = \frac{(1)(5)+(0.1)(1)+(0.1)(2)+(0.01)(0)}{[0.01+0.1+0.1+1]} = 4.38\mu s$$

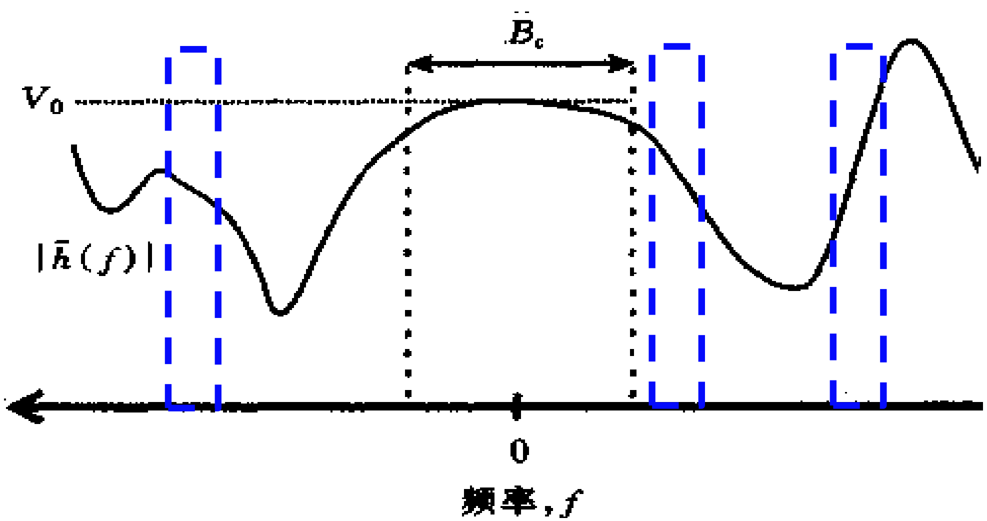
$$\bar{\tau^2} = \frac{(1)(5)^2+(0.1)(1)^2+(0.1)(2)^2+(0.01)(0)^2}{[0.01+0.1+0.1+1]} = 21.07\mu s^2$$

$$\sigma_\tau = \sqrt{21.07-(4.38)^2} = 1.37\mu s$$

Coherence Bandwidth

(PP.175)

- Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be consider “flat” (i.e. , a channel which passes all spectral components with approximately equal gain and linear phase).
- Two frequencies separated greater than B_c have different channel response.



Coherence Bandwidth

- ✓ If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function R_{hh} is above 0.5, then the coherence bandwidth is approximately

$$B_c = \frac{1}{5\sigma_\tau}$$

Here σ_τ RMS delay spread.

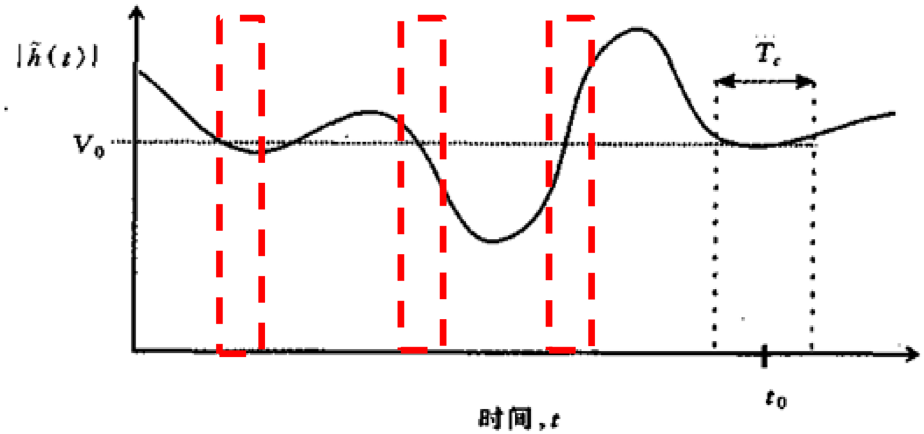
And the *Bc while* R_{hh}=0.9?

- ✓ RMS delay spread (time dispersion parameters) and coherence bandwidth are pair of parameters which describe multipath nature of the radio channel

Doppler Spread and Coherence Time (PP.177)

- Coherence time is actually a statistical measure of the time duration over which the channel impulse response is essentially invariant, and quantifies the similarity of the channel response at different times.
- A popular definition of coherence time for digital communications is

$$T_c = \frac{0.423}{f_m} \qquad f_m = \frac{v}{\lambda}$$



Doppler Spread and Coherence Time are parameters which describe the time varying nature of the channel in a small-scale region.

Parameters of Mobile Multipath Channels

Multipath:

- **rms delay spread** (time-domain) $$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$
- **coherence bandwidth** (frequency-domain) $$B_c = \frac{1}{5\sigma_\tau}$$

Time-varying:

- **doppler spread** (frequency-domain) $$f_d = \frac{v}{\lambda}\cos\theta$$
- **coherence time** (time-domain) $$T_c = \frac{0.423}{f_m}$$

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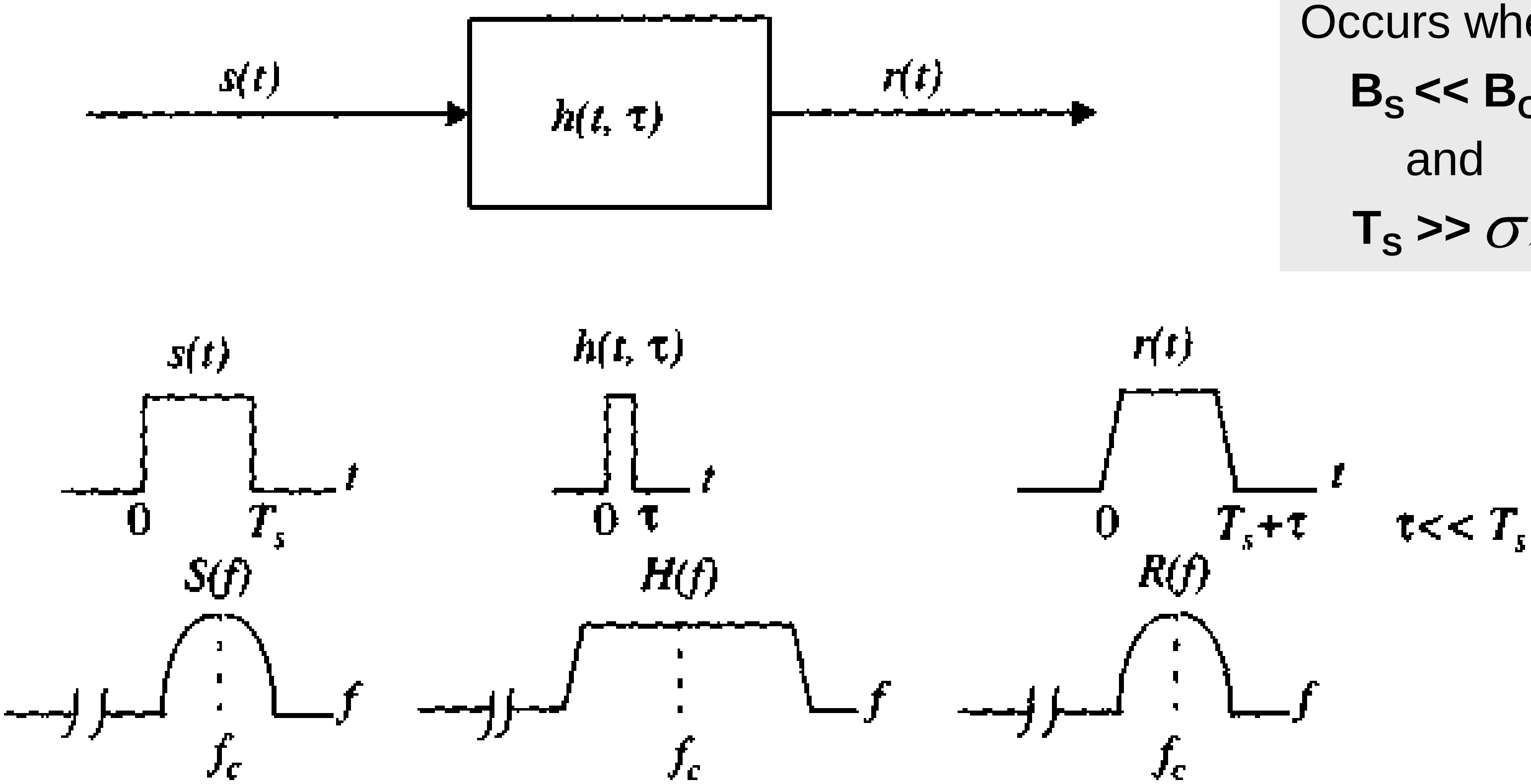
Fading Effects Due to Multipath Time Delay Spread



Occurs when:

$\mathbf{B_S << B_C}$

and

$\mathbf{T_S >>} \sigma_\tau$

Flat Fading

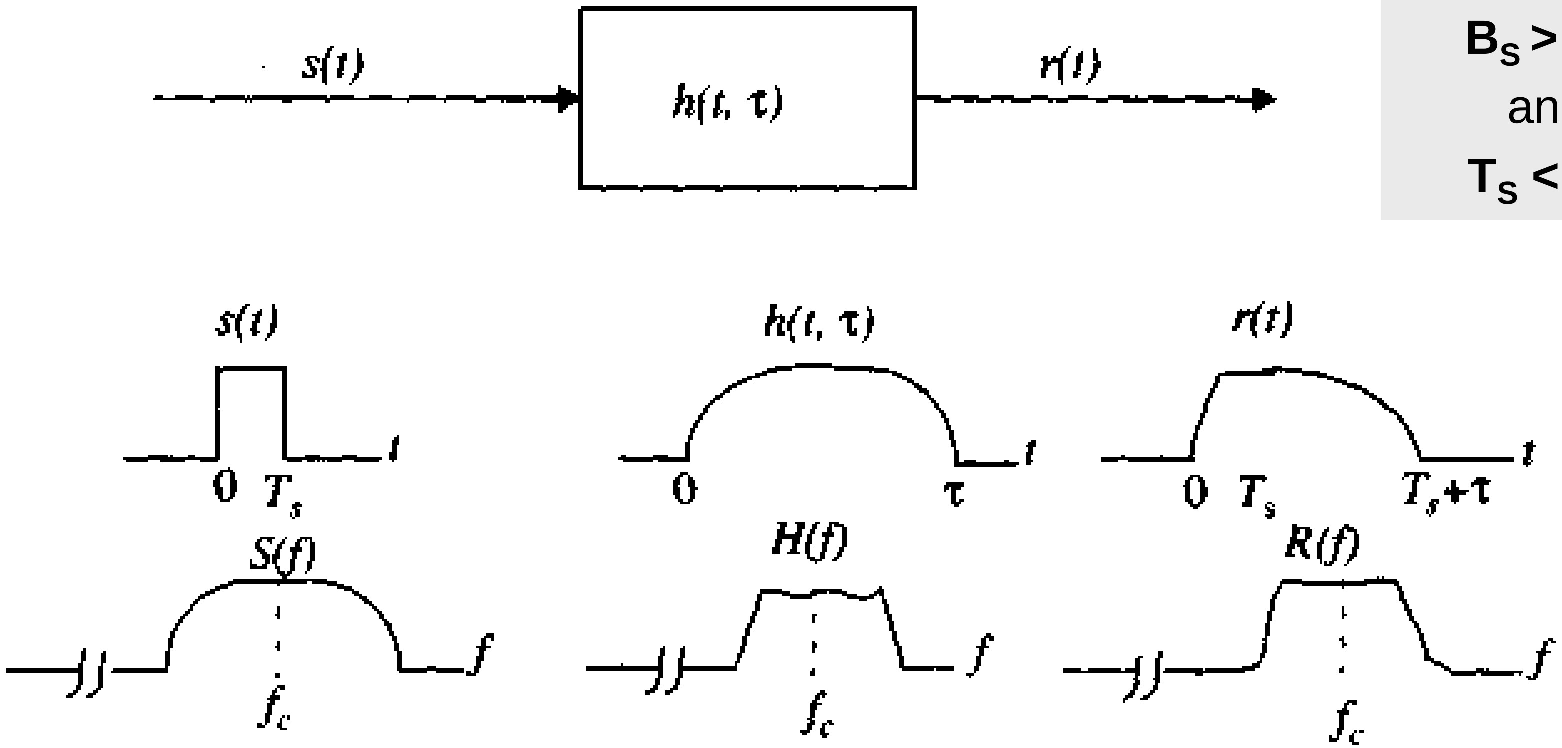
Fading Effects Due to Multipath Time Delay Spread

Occurs when:

$B_S > B_C$

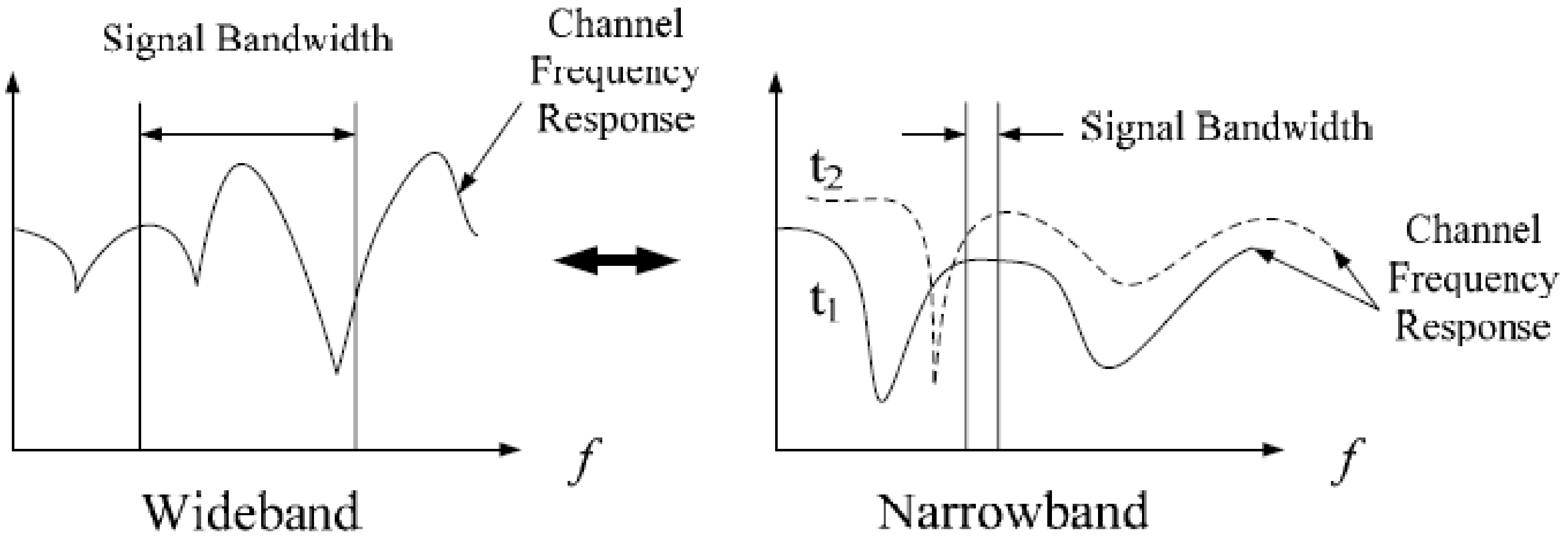
and

$T_S < \sigma_\tau$



Frequency Selective Fading

Questions



Signal Bandwidth
Channel
Frequency
Response
f
Wideband
Signal Bandwidth
t2
t1
Channel
Frequency
Response
f
Narrowband

Fading Effects Due to Doppler Spread

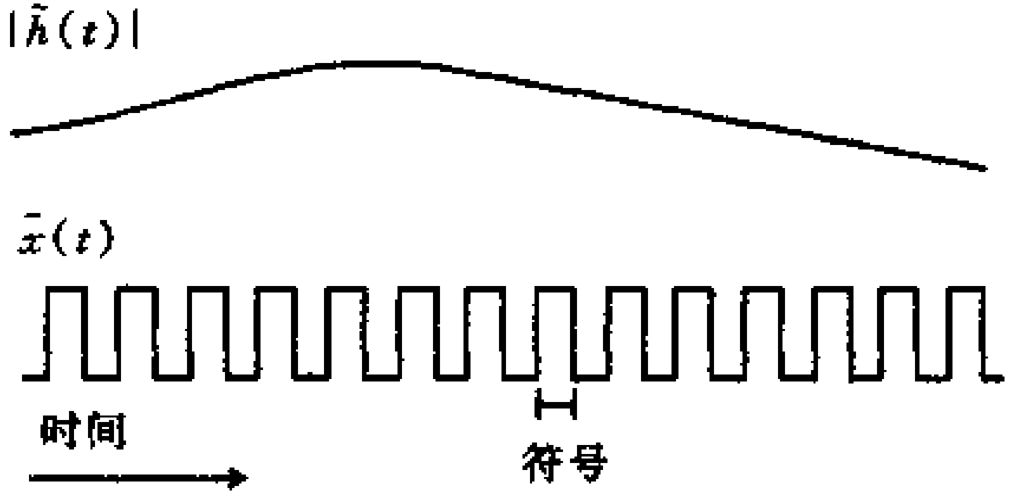
- Rate of change of the channel characteristics is **much smaller** than the rate of change of the transmitted signal

Occurs when:

$T_S << T_C$

and

$B_S >> B_D$



Slow Fading

Fading Effects Due to Doppler Spread

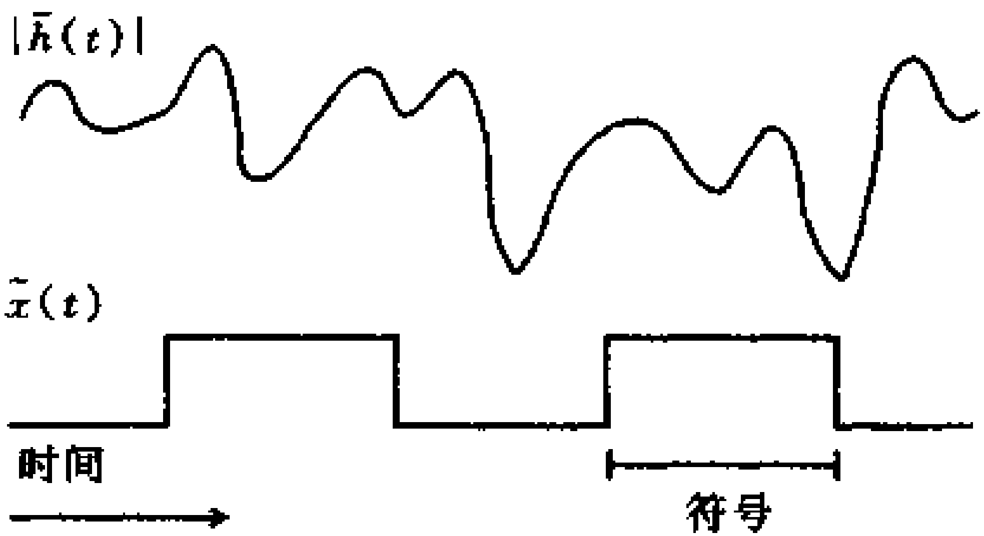
- Rate of change of the channel characteristics is **much larger** than the rate of change of the transmitted signal

Occurs when:

$T_S > T_C$

and

$B_S < B_D$



Fast Fading

	Signal		Channel	
Multipath	B_S	$\ll$	B_C	$\Rightarrow$ Flat fading ($T_S \geq 10\ \sigma_\tau$)
	T_S	$\gg$	σ_τ	
	B_S	$>$	B_C	$\Rightarrow$ Frequency selective fading ($T_S < 10\ \sigma_\tau$)
	T_S	$<$	σ_τ	
Time-varying	T_S	$\ll$	T_C	$\Rightarrow$ Slow fading
	B_S	$\gg$	B_D	
	T_S	$>$	T_C	$\Rightarrow$ Fast fading
	B_S	$<$	B_D	

Types of Small-Scale Fading

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

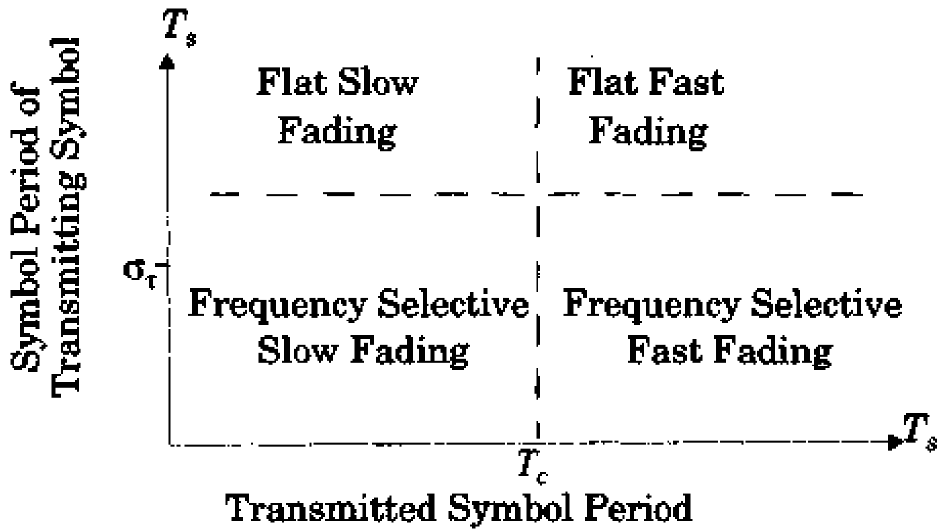
Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than base-band signal variations

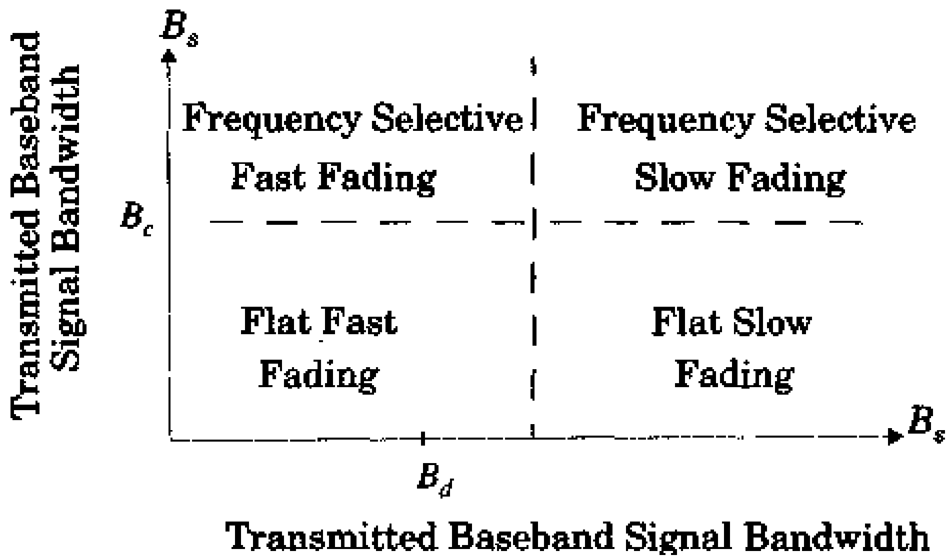
Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

(PP.182)

(a)


T_s
Symbol Period of Transmitting Symbol
Flat Slow Fading
Flat Fast Fading
σ_τ
Frequency Selective Slow Fading
Frequency Selective Fast Fading
T_s
T_c
Transmitted Symbol Period

(b)


B_s
Transmitted Baseband Signal Bandwidth
Frequency Selective Fast Fading
Frequency Selective Slow Fading
B_c
Flat Fast Fading
Flat Slow Fading
B_s
B_d
Transmitted Baseband Signal Bandwidth

- Small scale fading and multipath
  - Impulse response model of channel
    - Discrete channel model
  - Parameters of channels
    - rms delay spread
    - Coherence bandwidth
    - Doppler Spread
    - Coherence Time

Techniques to mitigate fading (PP. 304-318)

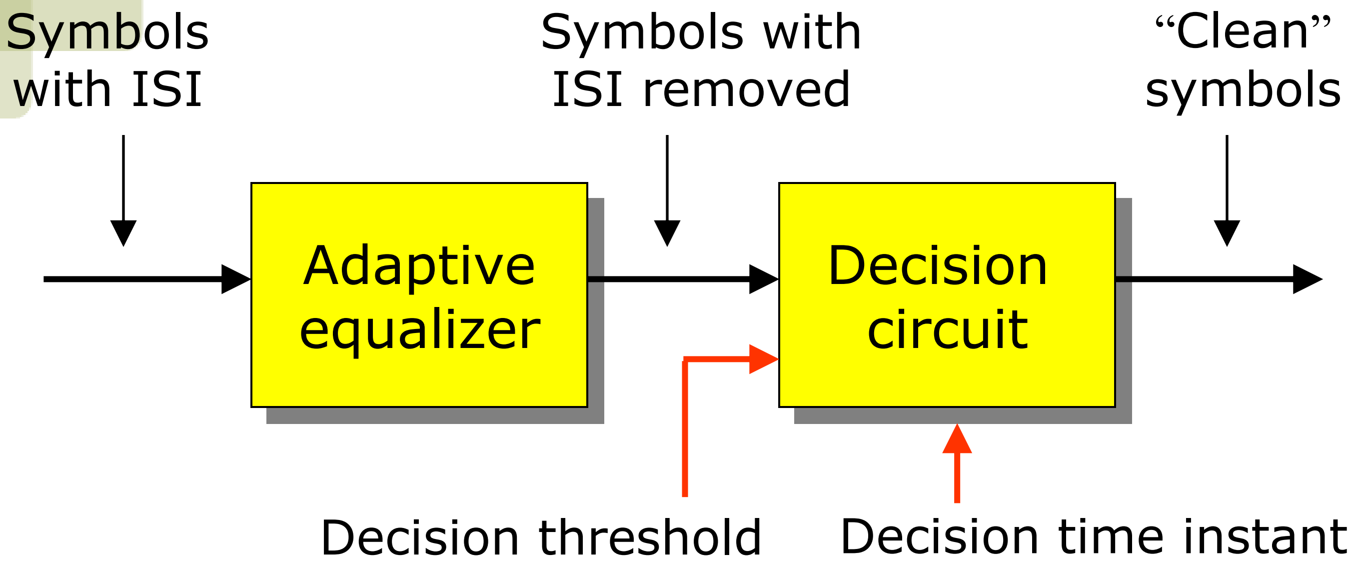
- **Equalization**
- **Diversity**
- **Interleaving**
- **Channel Coding**

(PP.304)

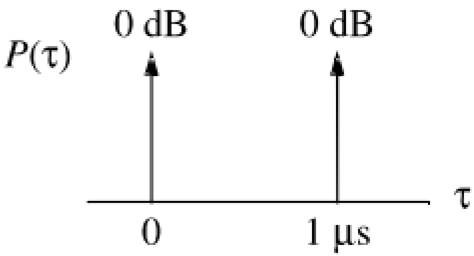
Equalization

- Compensates for **intersymbol interference (ISI)** created by multipath within time dispersive channels. ($\sigma_T > 0.1\, T_S$)
- An equalizer within a receiver compensates for average range of expected channel amplitude and delay characteristics.
- Equalizers are generally adaptive since channel is unknown and time varying, for example **GSM**.
- Channel must be learned through training and tracked during data transmission.

The intersymbol interference of received symbols (bits) must be removed before decision making (the case is illustrated below for a binary signal, where symbol = bit):



Example



(PP.175)

A local spatial average of a power delay profile measured at 900 MHz is shown in figure. (referred to Example 5.4)

(a) Determine the rms delay spread and mean excess delay for the channel.

(b) If the channel is to be used with BPSK modulation that requires an equalizer whenever the symbol duration T_s is less than $10\ \sigma_\tau$, determine the maximum symbol rate R_s that can be supported without requiring an equalizer.

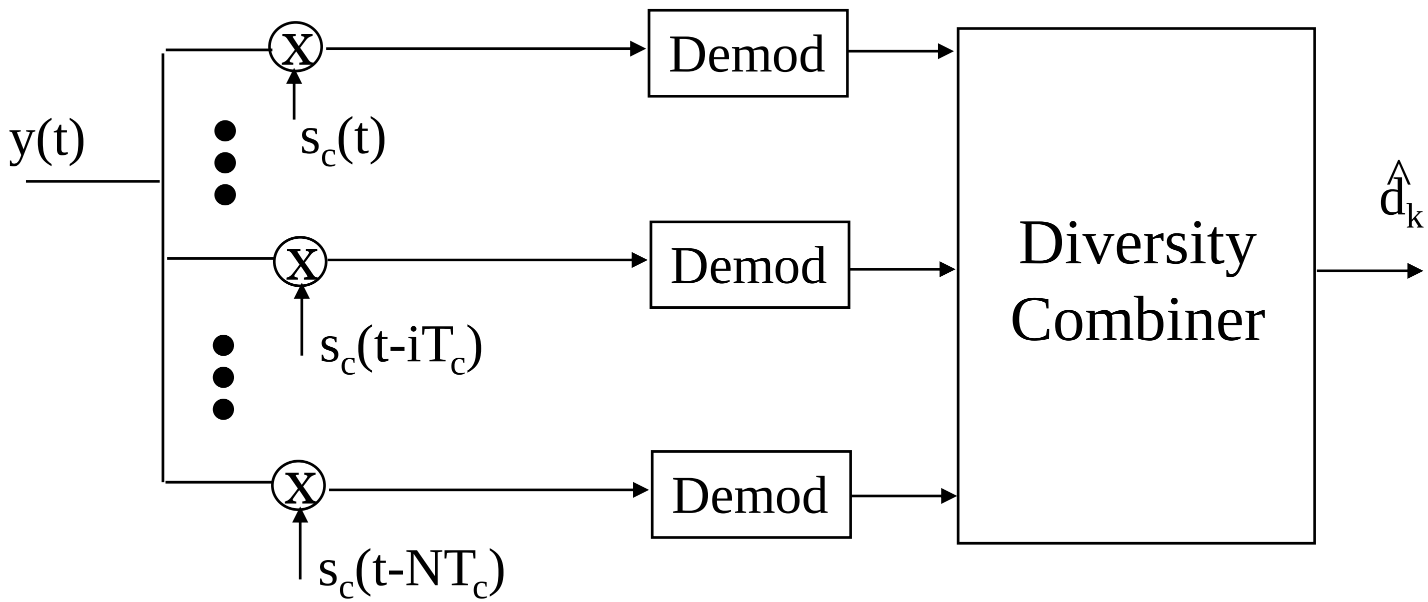
(c) If a mobile traveling at 30 km/hr receives a signal through the channel , determine the time over which the channel appears stationary (or at least highly correlated).

比特速率(bit rate) = 符号速率(symbol rate) × $\log_2$ 进制数

$$R_b = R_s \times \log_2 M$$

CDMA RAKE Receiver

(PP.316)



Diversity Techniques

(PP.308)

$$p(\gamma_i) = \frac{1}{\Gamma} e^{\frac{-\gamma_i}{\Gamma}} \qquad \gamma_i \geq 0$$

"Chi-square distribustion"
(PP.296 Equation 6.155)

γ_i **: Instantaneous SNR**

$$SNR = \Gamma = \frac{E_b}{N_0}$$

Γ **: Average SNR**

$$Pr[\gamma_i \leq \gamma] = \int_0^{\gamma} p(\gamma_i)\, d\gamma_i = \int_0^{\gamma} \frac{1}{\Gamma} e^{\frac{-\gamma_i}{\Gamma}}\, d\gamma_i = (1 - e^{-\gamma/\Gamma})$$

$$Pr[\gamma_1, \ldots, \gamma_M \leq \gamma] = \boxed{(1 - e^{-\gamma/\Gamma})^M = P_M(\gamma)}$$

Pr : **"Outage probability"**

Example:
Outage probability for M selective diversity branches

Assume that average SNR is 20dB, the SNR threshold is 10dB and M=4. Determine outage probability for M selective diversity branches.

Ans:

$$r/\Gamma = 10/100 = 0.1$$

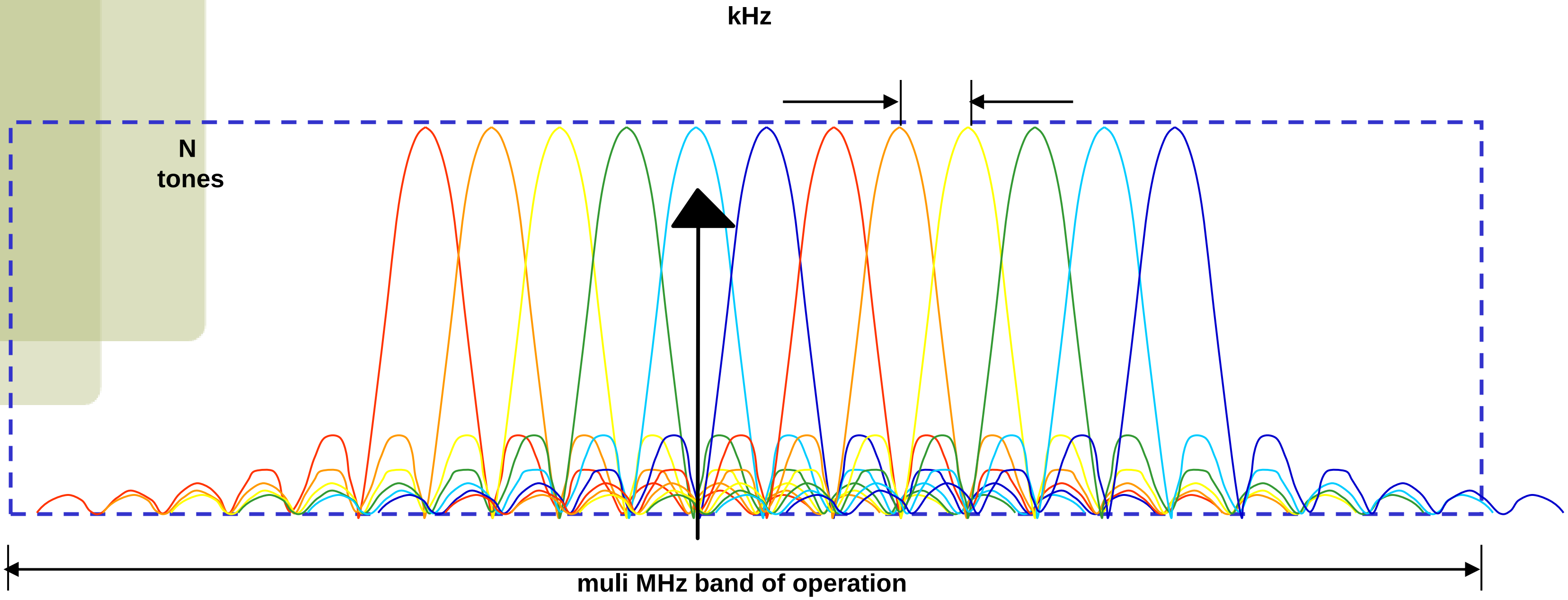
With the selective diversity，

$$\mathrm{P_M}(r) = \left[1 - e^{\left(-r/\Gamma\right)}\right]^M = \left[1 - e^{(-0.1)}\right]^4 \approx 0.0001$$

Without the selective diversity

$$\mathrm{P_M}(r) = \left[1 - e^{\left(-r/\Gamma\right)}\right] \approx 0.1$$

OFDM



- OFDM is made up of carriers that are orthogonal frequencies.
- Each carrier modulated separate data.
- OFDM provides a symbol rate that is long and offer immunity to ISI.

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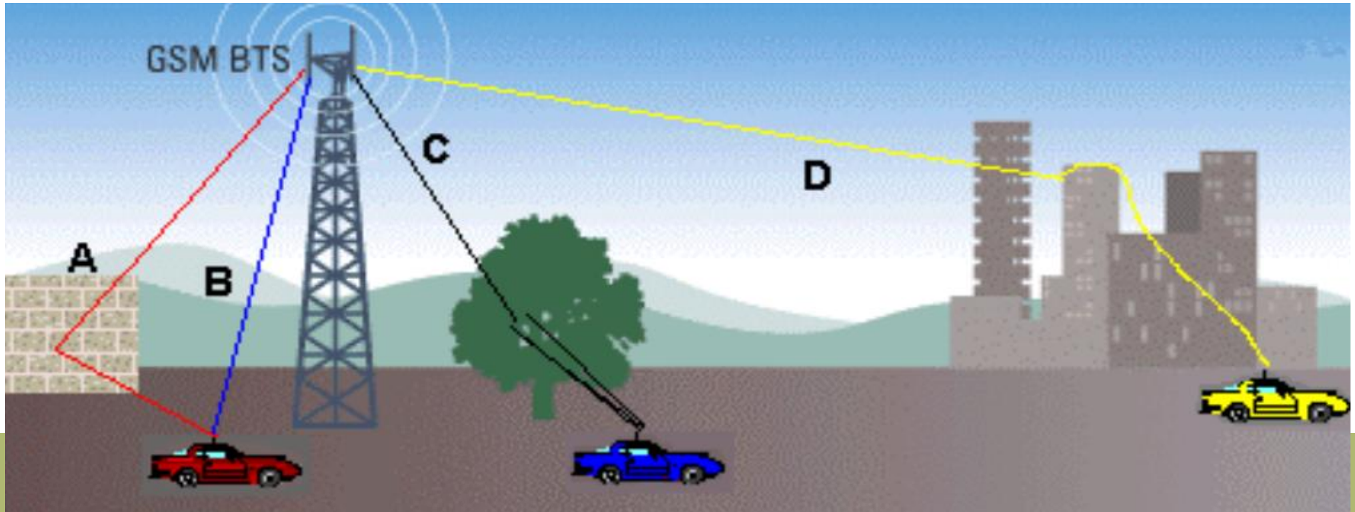
Fading Distributions

- Describes how the received signal amplitude changes with time.
  - Remember that the received signal is combination of multiple signals arriving from different directions, phases and amplitudes.
  - With the received signal we mean the baseband signal, namely the envelope of the received signal (i.e. r(t)).
- It is a statistical characterization of the multipath fading.
- Two distributions
  - Rayleigh Fading
  - Ricean Fading

Rayleigh fading

In the radio propagation for path C, there is no direct ray component. The one ray to the mobile station is scattered near the receiver resulting in a large number of rays arriving at the receiver from all directions. Thus the statistical model used to describe the amplitude variations is Rayleigh



Rayleigh fading

$$p_{Ra}(u) = \begin{cases} \dfrac{u}{\sigma^2} e^{-\frac{u^2}{2\sigma^2}} & ; \quad u \geq 0 \\ 0 & \quad elsewhere \end{cases}$$

$$p_{Ra}(\varphi) = \begin{cases} \dfrac{1}{2\pi} & ; \quad 0 \leq \varphi \leq 2\pi \\ 0 & \quad elsewhere \end{cases}$$

Ricean fading

The Ricean (Rician) model adds a line-of-sight (LOS) component to the Rayleigh model.

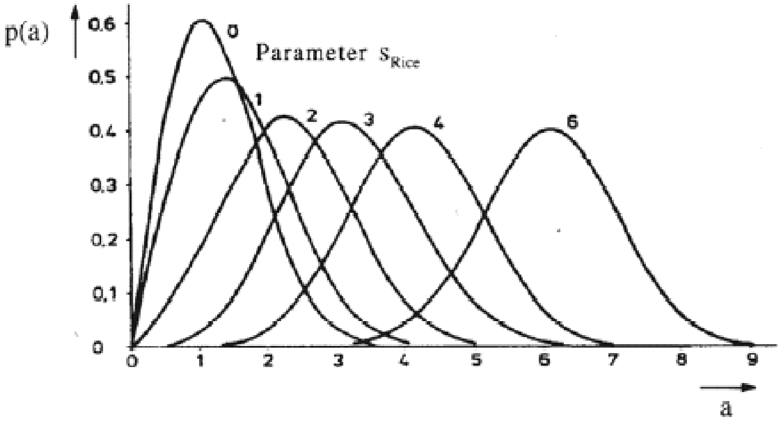
The ratio of signal power in the LOS component to the (local-mean) scattered power of the Rayleigh modulated component is defined as the K-factor.

Ricean fading

$$p_{Ri}(u) = \begin{cases} \dfrac{u}{\sigma^2} e^{-\left(\frac{u^2}{2\sigma^2}+c\right)} I_0\left(\dfrac{\sqrt{2cu}}{\sigma}\right) & ; \quad u \geq 0 \\ 0 & elsewhere \end{cases}$$

I_0 = Besselfunction 0'th order
c = ratio of direct and scattered signal

When there is a dominant stationary (nonfading) signal component present, such as a line-of–sight propagation path, the small-scale fading envelope distribution is Ricean.



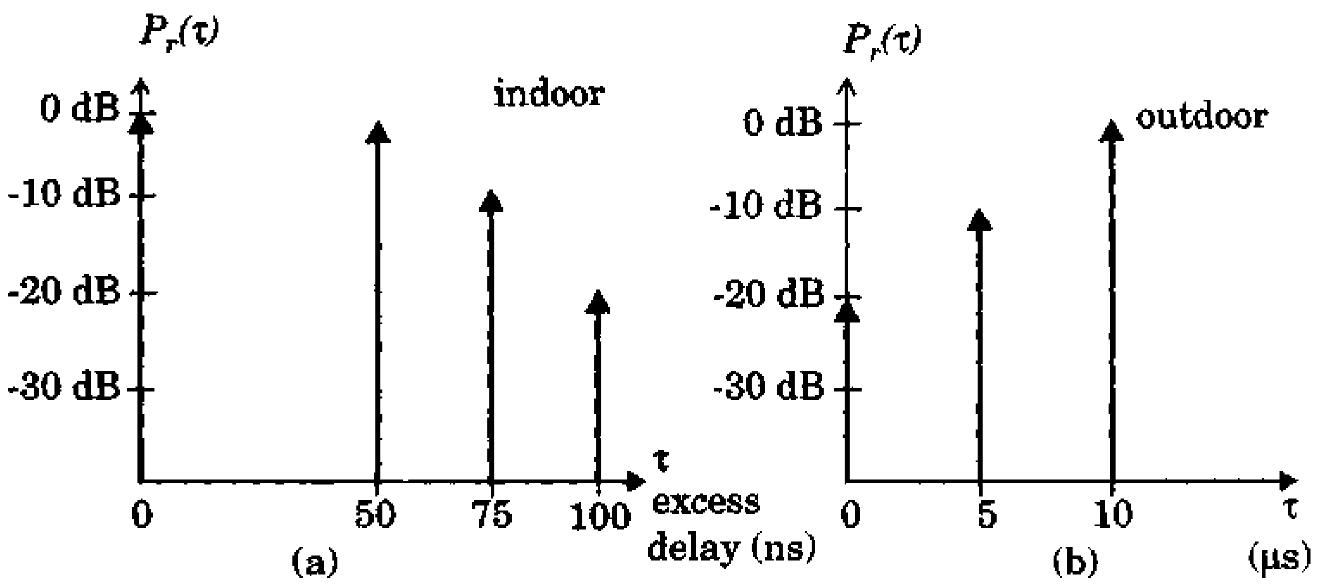
The Ricean distribution degenerates to a Rayleigh distribution when the dominant component fades away.

Summary

1. **Small-Scale Multipath Propagation**
2. **Impulse response Model of Multipath channels**
3. **Parameters of Mobile Multipath Channels**
4. **Types of Small-Scale Fading**
5. **Rayleigh and Ricean Distributions**

Example

1. For the power delay profiles in Figure, estimate the 90% correlation and 50% correlation coherence bandwidths.



Try to explain the meaning of the following :

(i) the channel is frequency-nonselective;

(ii) the channel is slow fading;

(iii) the channel is frequency-selective.

(1). Small scale fading based on multipath time delay can be divided into which two types of fading?

(2). When the rms delay spread increases, the coherence bandwidth of the channel Bc decreases or increases?

(3). When the signal bandwidth Bs>Bc, viewed in the frequency domain, certain frequency components in the received signal spectrum have greater gains than others. This results in serious ISI.