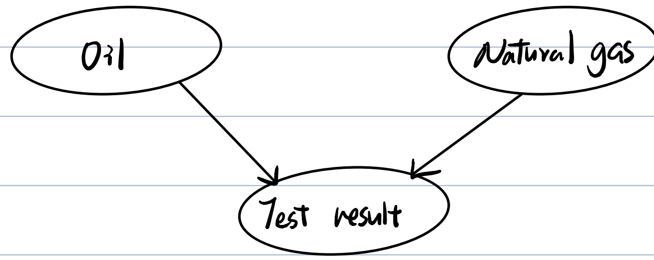
CS 161 HW 6 Rui Deng 205123245

1.(a) Bayesian network:



Oil	probability
T	0.3
F	0.7

Gas	probability
T	0.2
F	0.8

Test	Oil	Gas	Probability
T	T	T	0
T	T	F	0.8
T	F	T	0.2
T	F	F	0.1

(b) We use Baye' Rule,

$$Pr(Oil \mid Test) = \frac{P(Test \mid Oil) \cdot P(Oil)}{P(Test)}$$

$Pr(Test \mid Oil) = 0.8$, although the probability also depends on the presence of natural gas, but since oil and gas cannot be present together, $Pr(Test \mid Oil) = 0.8$

Using the law of total probability and sum over all cases regarding oil,

$$Pr(Test) = Pr(Test \mid Oil) + Pr(Test \mid \neg Oil)$$

$= Pr(Test \wedge Oil) \cdot Pr(Oil) + Pr(Test \wedge \neg Oil) \cdot Pr(\neg Oil)$

$= 0.8 \cdot 0.3 + 0.3 \cdot 0.7 = 0.45$

$P(oil \mid Test) = (0.8 \cdot 0.3) / 0.45 = 0.73$

2. (a) $Pr(A, B, C, D, E, F, G, H) = Pr(G \mid F) \cdot Pr(H \mid E, F) \cdot Pr(F \mid C, D) \cdot Pr(E \mid B) \cdot Pr(C \mid A) \cdot Pr(D \mid A, B) \cdot P(B) \cdot P(A)$

(b) $Pr(A, B, C, D, E, F, G, H) = \sum_a \sum_b \sum_c \sum_d P(A) \cdot P(B) \cdot Pr(C \mid A) \cdot Pr(D \mid A, B) \cdot Pr(E \mid B) \cdot Pr(F \mid C, D) \cdot Pr(G \mid F) \cdot Pr(H \mid E, F)$

$= \sum_a \sum_b \sum_c \sum_d f_1(A) \times f_2(B) \times f_3(A, C) \times f_4(A, B, D) \times f_5(B, E) \times f_6(C, D, F) \times f_7(E, G) \times f_8(E, F, H)$

$f_9(A, B, C, F) = \sum_d f_4(A, B, D) \times f_6(C, D, F)$

$f_{10}(A, B, F) = \sum_c f_3(A, C) \times f_9(A, B, C, F)$

$f_{11}(A, E, F) = \sum_b f_5(B, E) \times f_{10}(A, B, F)$

$f_{12}(E, F) = \sum_a f_{11}(A, E, F) \times f_1(A)$

$Pr(E, F, G, H) = f_{12}(E, F) \times f_7(E, G) \times f_8(E, F, H)$

(c) $Pr(a, \neg b, c, d, \neg e, f, \neg g, h) = Pr(\neg g \mid f) \cdot Pr(h \mid \neg e, f) \cdot Pr(f \mid c, d) \cdot 0.2 \cdot Pr(c, a) \cdot 0.6 \cdot 0.4 \cdot 0.1$

$= 0.0048 \cdot Pr(\neg g \mid f) \cdot Pr(h \mid \neg e, f) \cdot Pr(f \mid c, d) \cdot Pr(c, a)$.

(d) $Pr(\neg a, b) = Pr(\neg a) \cdot Pr(b) = 0.9 \cdot 0.6 = 0.54$, since they are independent.

$Pr(\neg e \mid a) = \frac{Pr(\neg e, a)}{Pr(a)} = \frac{Pr(\neg e) \cdot Pr(a)}{Pr(a)} = Pr(\neg e)$, since e is independent from a. Then we calculate $Pr(\neg e)$.

$$Pr(\neg e) = Pr(\neg e, b) + Pr(\neg e, \neg b) \quad \text{by the law of total probabilities}$$
$$= Pr(\neg e \mid b) \cdot Pr(b) + Pr(\neg e \mid \neg b) \cdot Pr(\neg b)$$
$$= 0.9 \cdot 0.6 + 0.2 \cdot 0.4 = 0.62$$

(e) A is independent of B. E given A's parents (none).

B is independent of A. C given B's parents (none).

C is independent of B. D. E given C's parents (A).

D is independent of C . E given D's parents (A. B).

E is independent of A. C. D. F. G given E's parents (B).

F is independent of A. B. E given F's parents (C. D).

G is independent of A. B. C. D. E. H given G's parents (F).

H is independent of A. B. C. D. G given H's parents (E. F).

(f) The markov blanket for D is {A, B, C, F}, which includes its parents, children, and other parents of its children.

(g)

A	B	D	E	Pr (D|A,B)* Pr (E|B)
0	0	0	1	**0.2*0.8=0.16**
0	0	1	1	**0.8*0.8=0.64**
0	1	1	1	**0.2*0.1=0.02**
0	1	0	1	**0.8*0.1=0.08**
1	0	0	1	**0.4*0.8=0.32**
1	0	1	1	**0.6*0.8=0.48**
1	1	1	1	**0.7*0.1=0.07**
1	1	0	1	**0.3*0.1=0.03**
0	0	0	0	**0.2*0.2=0.04**
0	0	1	0	**0.8*0.2=0.16**
0	1	1	0	**0.2*0.9=0.18**
0	1	0	0	**0.8*0.9=0.72**
1	0	0	0	**0.4*0.2=0.08**
1	0	1	0	**0.6*0.2=0.12**
1	1	1	0	**0.7*0.9=0.63**
1	1	0	0	**0.3*0.9=0.27**

(h)

Let f(A, B, D, E)=P(D | A, B) $\times$P(E | B)

f(A, B, E)=$\sum_d f(A, B, D, E)$= $f(A, B, d, E) + f(A, B, \neg d, E)$

A	B	E	f(A, B ,E)
1	1	1	0.1
1	1	0	0.9
1	0	1	0.8
1	0	0	0.2
0	1	1	0.1
0	1	0	0.9
0	0	1	0.8
0	0	0	0.2