HW#2, Nan Deng

(1)

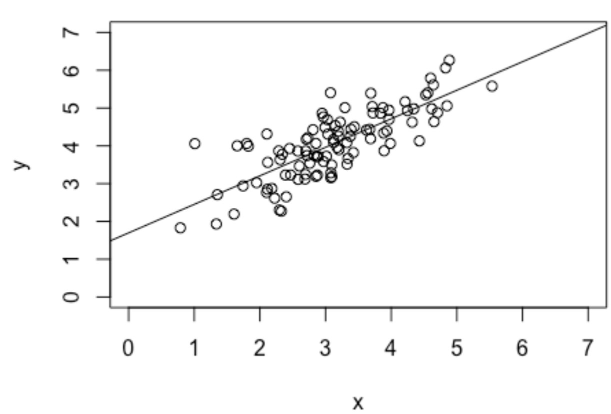
```
library(MASS)
mu <- c(3,4)
sigma <- matrix(c(1.0,0.8,0.8,1.0),nrow=2)
set.seed(123)
datam <- data.frame(mvrnorm(100,mu,sigma))
colnames(datam) <- c("x","y")
```

(a)

```
library(faraway)
fit_line <- lm(y ~ x,data=datam)
summary(fit_line)

##
## Call:
## lm(formula = y ~ x, data = datam)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.18120 -0.44633 -0.01159  0.36843  1.60020
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.69698    0.19998   8.486 2.31e-13 ***
## x            0.75479    0.06144  12.285  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5701 on 98 degrees of freedom
## Multiple R-squared:  0.6063, Adjusted R-squared:  0.6023
## F-statistic: 150.9 on 1 and 98 DF,  p-value: < 2.2e-16

plot(datam$x,datam$y,xlab="x",ylab="y",xlim=c(0,7),ylim=c(0,7))
abline(fit_line)
```




```
coefficients(fit_line)
```

```
## (Intercept)           x
##   1.6969772   0.7547914

summary(fit_line)$sigma

## [1] 0.5701041

summary(fit_line)$r.squared

## [1] 0.606287
```

(b) α=1.6969772; β=0.7547914; σ=0.5701041; R^2 =0.606287

(c)

```
t.test(datam, mu=0, alternative="greater", conf.level=0.95)

##
##  One Sample t-test
##
## data:  datam
## t = 49.314, df = 199, p-value < 2.2e-16
## alternative hypothesis: true mean is greater than 0
## 95 percent confidence interval:
##  3.465606      Inf
## sample estimates:
## mean of x
##  3.585767
```

Considering that p-value in this model is less than 2.2e-16 and definitely less than 0.05, H_0 should be rejected.

(d)

```
fit_line1 <- lm(y ~ offset(0.9*x),data=datam)
anova(fit_line,fit_line1)

## Analysis of Variance Table
##
## Model 1: y ~ x
## Model 2: y ~ offset(0.9 * x)
##   Res.Df    RSS Df Sum of Sq      F  Pr(>F)
## 1     98 31.852
## 2     99 33.667 -1   -1.8154 5.5854 0.02008 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since p-value of this model is around 0.02, which is less than 0.05, H_0 should also be rejected.

(2)

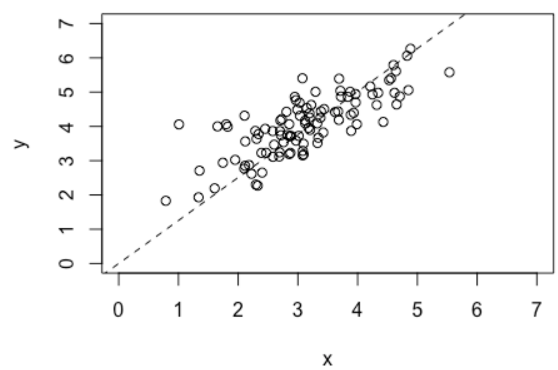
```
m0 <- lm(y ~ x -1 , data=datam)
summary(m0)

##
## Call:
## lm(formula = y ~ x - 1, data = datam)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -1.4258 -0.3656  0.1252  0.5823  2.7931
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## x  1.25453    0.02295   54.66   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7471 on 99 degrees of freedom
## Multiple R-squared:  0.9679, Adjusted R-squared:  0.9676
## F-statistic:  2987 on 1 and 99 DF,  p-value: < 2.2e-16
```

(a)

```
plot(datam$x,datam$y,xlab="x",ylab="y",xlim=c(0,7),ylim=c(0,7))
abline(m0,lty=2)
```



(b)

```
sum(summary(m0)$resid)

## [1] 13.79104
```

No, the sum of residuals is about 13.8 rather than 0.

(c)

```
sum(datam$x * residuals(m0))

## [1] -4.010681e-15

sum(fitted.values(m0) * residuals(m0))

## [1] -4.458239e-15
```

$$\frac{\partial}{\partial\beta}\Sigma(y_i-\beta x_i)^2=0 \Rightarrow \beta=\frac{\Sigma x_i y_i}{\Sigma(x_i)^2}$$

$$\Sigma x_i e_i=\Sigma x_i(y_i-\beta x_i)$$

$$=\Sigma x_i\left(y_i-\frac{\Sigma x_i y_i}{\Sigma(x_i)^2}\cdot x_i\right)$$

$$=0$$

$$\Sigma\hat{y}_i e_i=\hat\beta\,\Sigma x_i e_i=0$$

(3)

(a)

$$\hat\beta=\frac{\Sigma(x_i-\bar x)y_i}{\Sigma(x_i-\bar x)^2}=\frac{S_{xy}}{S_{xx}} \Rightarrow S_{xy}=\hat\beta\cdot S_{xx}$$

$$r=\frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}=\frac{\hat\beta\cdot S_{xx}}{\sqrt{S_{xx}S_{yy}}}=\hat\beta\cdot\frac{\sqrt{S_{xx}}}{\sqrt{S_{yy}}}$$

In this equation, r and β have the same sign.

(b)

$$SSE=\Sigma(y_i-\hat y_i)^2=\Sigma[(y_i-\bar y)-\hat\beta(x_i-\bar x)]^2$$

$$=\Sigma(y_i-\bar y)^2-2\hat\beta\,\Sigma(y_i-\bar y)(x_i-\bar x)+\hat\beta^2\,\Sigma(x_i-\bar x)^2$$

$$=S_{yy}-2\hat\beta S_{xy}+\hat\beta^2 S_{xx}$$

$$=S_{yy}-\hat\beta S_{xy}$$

$$R^2=\frac{SSR}{S_{yy}}=\frac{S_{yy}-SSE}{S_{yy}}$$

$$=\frac{S_{yy}-(S_{yy}-\hat\beta S_{xy})}{S_{yy}}$$

$$=\hat\beta\cdot\hat\beta\cdot\frac{S_{xx}}{S_{yy}}=r^2$$