Incorporation of pmatrix in forward projection

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May 28, 2022

This document will discuss how to incorporate pmtrix in forward projection. The pmatrix is a 3 by 4 matrix, relating a point with coordinate (x, y, z) in the 3D reconstruction space and a point with coordinate (u, v) on the 2D detector surface:

$$\begin{bmatrix} \frac{u}{m} \\ \frac{v}{m} \\ \frac{1}{m} \end{bmatrix} = P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}, \tag{1}$$

where m is the magnification defined as follows:

$$m = \frac{\text{distance between the source and the detector element}}{\text{distance between the source and point of interest } (x, y, z)} \tag{2}$$

For back-projection, if we want to reconstruct the point at (x, y, z), we simply multiply P and $(x, y, z, 1)^T$ to get (u, v). For forward projection, we are given (u, v) and we want to find path connecting (u, v) and the position of the source. We will collect the μ values along that path and perform a integration (a summation for a digitized process). As one can tell, (u, v) is not related to a single (x, y, z). However, when the magnification m is fixed, the point (x, y, z) will be fixed. Therefore, by changing the magnification, we are able to go through all the points on the path.

The magnification is not proportional to the distance between source and (x, y, z) and it inverse is. Therefore, we define $s = \frac{1}{m}$. By linearly increasing s, one is able to go through all the points on the path with equal step distance. For example, if s to 0, (x, y, z) is at the source; if $s = 1$, x, y, z is at the detector. For given s, the pmatrix can be written as

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = P_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} P_{14} \\ P_{24} \\ P_{34} \end{bmatrix}, \tag{3}$$

where $P_{3\times3}$ is the first three columns of matrix P. Therefore,

$$P_{3\times3}^{-1} \begin{bmatrix} su \\ sv \\ s \end{bmatrix} - P_{3\times3}^{-1} \begin{bmatrix} P_{14} \\ P_{24} \\ P_{34} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \tag{4}$$

In mgfpj program, we either need to calculate $P_{3\times3}^{-1}$ in advance or need to use some libraries to calculate it in the cuda program.