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Chapter 6.2

1. Factor these two matrices into $A = S\Lambda S^{-1}$

a. $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$\lambda_1 = 1$
$\lambda_2 = 3$

$\mathbf{x}_1 = (1, 0)$
$\mathbf{x}_2 = (1, 1)$

$S\Lambda S^{-1} =$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

b. $\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$

$\lambda^2 - 4\lambda + 3 - 3$
$\lambda(\lambda - 4)$

$\lambda_1 = 0$
$\lambda_2 = 4$

$\mathbf{x}_1 = (1, -1)$
$\mathbf{x}_2 = (1, 1)$

$S^{-1} =$

$$1 / 2 \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$S\Lambda S^{-1} =$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 6 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

2. If A has $\lambda_1 = 2$ with $\mathbf{x}_1 = (1, 0)$ and $\lambda_2 = 5$ with $\mathbf{x}_2 = (1, 1)$ use $S\Lambda S^{-1}$ to find A. No other matrix has the same λ's and x's.

$S\Lambda S^{-1} =$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

A =

$$\begin{bmatrix} 2 & 5 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

=

$$\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$$

4. True or false. If the columns of S are linearly independent, then

a. A is invertible.

False. This is only true if none of the eigenvalues are 0.

b. A is diagonalizable.
True.
c. S is invertible.
True.
d. S is diagonalizable.
False. This is only true if the eigenvalues of the eigenvector matrix are themselves nonzero.

11. True or False. If a matrix has eigenvalues 2, 2, 5, then the matrix is certainly:
a. Invertible: TRUE
b. Diagonalizable: FALSE. (It’s possible, but not certain.)
c. Not diagonalizable: FALSE. (Same as above.)

12. True or False. If the only eigenvectors of A are multiples of (1, 4), then A has:
a. No inverse: FALSE. (We need to know the eigenvalues to judge this.)
b. A repeated eigenvalue. TRUE.
c. No diagonalization $S\Lambda S^{-1}$: TRUE.

18.

Chapter 10.1

9. Find the complex conjugate of each number by changing i to -i.
a. 2 + i
b. (2 + i)(1 + i)
= 2 + 2i + i + i^2
= 1 + 3i
c. $e^{i\pi/2} = -e^{i\pi/2}$
d. $e^{i\pi} = -1 + 0i$
$e^{-i\pi} = -1 - 0i$
$e^{-i\pi} = -1$
e. (1 + i) / (1 - i)
$(1 + i)^2 / (1 - i)(1 + i)$
$(1 + 2i + i^2) / (1 -i +i -i^2)$
2i / 2
i
= -i

(1 - i) / (1 + i)
$(1 - i)^2 / (1 + i)(1 - i)$
$(1 -2i +i^2) / (1 -i^2)$
-2i / 2
= -i
f. $i^{103} = i^{100} * i^3$
$= 1 * i^2 * i$
= -1 * i
= -i

12. The eigenvalues of a real 2 x 2 matrix come from the quadratic formula.

a. If a = b = d = 1, the eigenvalues are complex when c is ___.

$1 \pm \sqrt{(1 - 4(1 - c))} / 2$
$= 1 \pm \sqrt{(1 - 4 + c)} / 2$
$= 1 \pm \sqrt{(-3 + c)} / 2$

evalues are imaginary for all $c \le 2$.

b. What are the evalues when ad = bc?

$(a + d) \pm \sqrt{((a + d)2 - 4(0))} / 2$
$= (a + d) \pm (a + d) / 2$
$= 2(a + d) / 2$ AND $= 0 / 2$
$= (a + d)$ AND $= 0$

c. The two evalues are not always conjugates of each other. Why not?

Because the distribution of roots is not symmetric about 0, but rather around a + d. If a + d = 0, then the evalues will be conjugates of each other.

17. Write the numbers in Euler's form $re^{i\theta}$. Then square each number.

a. $1 + \sqrt{(3)}i$

$r = \sqrt{(1^2 + \sqrt{3}^2)}$
$= \sqrt{4}$
$= 2$
$\theta = \tan-1(\sqrt{3} / 1)$
$= \pi / 3$
$1 + \sqrt{3}i = 2e^{i(\pi/3 + 2n\pi)}$
$(1 + \sqrt{3}i)2 = 2^2e^{i2(\pi/3 + 2n\pi)}$
$= 4e^{i(2\pi/3 + 4n\pi)}$

b. $\cos(2\theta) + i\sin(2\theta)$

$r = \sqrt{(\cos2(2\theta) + \sin2(2\theta))}$
$= \sqrt{(1)}$
$= 1$

Chapter 10.2

1. Find the lengths of $\mathbf{u}$ = (1+i, 1-i, 1+2i) and $\mathbf{v}$ = (i, i, i) Also find $\mathbf{u}^H\mathbf{v}$ and $\mathbf{v}^H\mathbf{u}$.

$||\mathbf{u}|| = |1+i|2 + |1-i|2 + |1+2i|2$
$= |1 +2i +i2| + |1 -2i +i2| + |1 +4i +i2|$
$= |2i| + |-2i| + |4i|$
$= 8$
$||\mathbf{v}|| = |i2| + |i2| + |i2|$
$= 3|-1|$
$= 3$

$\mathbf{u}^H\mathbf{v} = ((1-i)(i) + (1+i)(i) + (1-2i)(i))$
$= (i -i2 + i +i2 + i -2i2)$
$= (3i -2(-1))$
$= 3i + 2$
$= 2 + 3i$

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$$
\begin{aligned}
\mathbf{v}^H\mathbf{u} &= (\ (-i)(1+i)\ +\ (-i)(1-i)\ +\ (-i)(1+2i)\ ) \\
&= (\ -i\ -i2\ -i\ -i2\ -i\ -2i2\ ) \\
&= (\ -3i\ -2(-1)\ ) \\
&= 2 - 3i
\end{aligned}
$$

2. Compute A^HA and AA^H. Those are both ____ matrices.

$$A = \begin{bmatrix} i & 1 & i \\ 1 & i & 1 \end{bmatrix}$$

$$A^H = \begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & 1 \end{bmatrix}$$

$$A^HA = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$AA^H = \begin{bmatrix} 3 & i \\ i & 3 \end{bmatrix}$$

They are both Symmetric matrices.

6. NOT REQUIRED