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Chapter 5.3

1. Solve these linear equations by Cramer's Rule x_j = Det(B_j) / Det(A):

```
 a.   2x1 + 5x2 = 1
       x1 + 4x2 = 2

      [ 2 5 ][ x1 ] = [ 1 ]
      [ 1 4 ][ x2 ]   [ 2 ]

      det(A)      =  3
      det(B_1) = -6
      det(B_2) =  3

      x1 = -6 / 3 = -2
      x2 =  3 / 3 =  1

 b.   2x1 +  x2       = 1
       x1 + 2x2 +  x3 = 0
             x2 + 2x3 = 0

      [ 2 1 0 ][ x1 ]   [ 1 ]
      [ 1 2 1 ][ x2 ] = [ 0 ]
      [ 0 1 2 ][ x3 ]   [ 0 ]

      det(A)   = 2
      det(B_1) = 4 - 1 = 3
      det(B_2) = 0 - 2 = -2
      det(B_3) = 1 - 0 = 1

      x1 =  3 / 2 = 1.5
      x2 = -2 / 2 = -1
      x3 =  1 / 2 = 0.5
```

2. Use Cramer's rule to solve for y (only). Call the 3 x 3 determinant D.

```
 a.   ax + by = 1
      cx + dy = 0

      [ a b ][ x ] = [ 1 ]
      [ c d ][ y ]   [ 0 ]

      det(A)   = ad - cb
      det(B_y) = 0a - 1c = c

      y = c / ( ad - cb )

 b.   ax + by + cz = 1
      dx + ey + fz = 0
      gx + hy + iz = 0

      [ a b c ][ x ]   [ 1 ]
      [ d e f ][ y ] = [ 0 ]
      [ g h i ][ z ]   [ 0 ]
```

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```
D = det(A) = aei + bfg + cdh - gec - hfa - idb
  det(B_y) = a0i + 1fg + cd0 - g0c - 0fa - id1
             = 0 + fg + 0 - 0 - 0 - id
             = fg - id

y = ( fg - id ) / D
```

16. Find the area of the shapes defined by the following points.

a. A parallelogram with edges v = (3, 2) and w = (1, 4)

```
p1 = (3, 2)
p2 = (1, 4)
p3 = (0, 0)

[ 3 2 ]
[ 1 4 ]

det(A) = 12 - 2 = 10
```

b. A triangle with sides v, w, and v + w. Draw it.

```
p1 = (3, 2)
p2 = (4, 6)
p3 = (0, 0)

[ 3 2 ]
[ 4 6 ]

det(A) = 18 - 8 = 10
Area = 1/2 det(A) = 5
```

19. A triangle with sides v, w, and w - v. Draw it.

```
p1 = (3, 2)
p2 = (1, 4)
p3 = (0, 0)

det(A) = 10
Area = 1/2 det(a) = 5
```

Chapter 6.1

NOTE: Lambda's are a lot of work, I'll just use l instead.

2. Find the evalues and evectors of A and A + I

```
A = [ 1 4 ]
     [ 2 3 ]

evalues(A) = l^2 - 4l - 5
             = (l + 1)(l - 5)
l1 = -1
```

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```
l2 = 5

l = -1:
        [ 2 4 | 0 ]
        [ 2 4 | 0 ]

        [ 1 2 | 0 ]
        [ 0 0 | 0 ]

        [ x ] = [  2 ]
        [ y ]   [ -1 ]

l = 5:
        [ -4  4 | 0 ]
        [  2 -2 | 0 ]

        [ -4  4 | 0 ]
        [  0  0 | 0 ]

        (x, y) = (1, 1)

A + I = [ 2 4 ]
            [ 2 4 ]

0 = l^2 - 6l + 0
  = l(l - 6)

l1 = 0
l2 = 6

l = 0:
        [ 2 4 | 0 ]
        [ 2 4 | 0 ]

        (x, y) = (2, -1)

l = 6:
        [ -4  4 | 0 ]
        [  2 -2 | 0 ]

        [ -4  4 | 0 ]
        [  0  0 | 0 ]

        (x, y) = (1, 1)
```

A + I has the same eigenvectors as A. It's eigenvalues are increased by 1.

3. Find the evalues and evectors of A and A^-1. Check the trace!

```
A = [ 0 2 ]
    [ 1 1 ]

0 = (l + 1)(l - 2)
```

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```
l1 = -1
l2 = 2

trace = 0 + 1 = 1
l1 + l2 = 2 + -1 = 1

l = -1:
      [ 1 2 | 0 ]
      [ 1 2 | 0 ]

      [ 1 2 | 0 ]
      [ 0 0 | 0 ]

      (x, y) = (2, -1)

l = 2:
      [ -2  2 | 0 ]
      [  1 -1 | 0 ]

      [ -1 1 | 0 ]
      [  0 0 | 0 ]

      (x, y) = (1, 1)

A^-1 = [ -1/2 1 ]
       [  1/2 0 ]

0 = (-1/2 - l)(0 - l) - 1/2
  = l^2 +1/2l -1/2
  = (l + 1)(l - 1/2)

l1 = -1
l2 = 1/2

l = -1:
      [ 1/2 1 | 0 ]
      [ 1/2 1 | 0 ]

      (x, y) = (2, -1)

l = 1/2
      [  -1    1 | 0 ]
      [ 1/2 -1/2 | 0 ]

      (x, y) = (1, 1)
```

A^-1 has the same eigenvectors as A. When A has eigenvalues l1 and l2, its inverse has eigenvalues 1/l1 and 1/l2.

4. Find the evalues and evectors of A and A^2.

```
A = [ -1 3 ]
    [  2 0 ]
```

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```
    0 = (l - 2)(l + 3)

    l1 = 2
    l2 = -3

    l = 2:
          [ -3  3 | 0 ]
          [  2 -2 | 0 ]

          (x, y) = (1, 1)

    l = -3:
          [ 2 3 | 0 ]
          [ 2 3 | 0 ]

          (x, y) = (3, -2)

    A^2 = [  7 -3 ]
            [ -2  6 ]

    Trace = 13
    Det = 36
    Factors = 4, 9

    l1 = 4
    l2 = 9

    l = 4:
          [  3 -3 | 0 ]
          [ -2  2 | 0 ]

          (x, y) = (1, 1)

    l = 9:
          [ -2 -3 | 0 ]
          [ -2 -3 | 0 ]

          (x, y) = (3, -2)
```

A^2 has the same eigenvectors as A. When A has eigenvectors l1 and l2, A^2 has eigenvectors l1^2 and l2^2. In this example, l1^1 + l2^2 must equal 13 because that is the trace of A^2.

5. Find the evalues of A and B and A + B.

```
    A = [ 3 0 ]
          [ 1 1 ]

    Triangular Matrix!
    l1 = 3
    l2 = 1

    B = [ 1 1 ]
          [ 0 3 ]
```

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Triangular Matrix!
l1 = 1
l2 = 3

A + B = [4 1]
[1 4]

Trace = 8
Det = 15
Factors = 3, 5

0 = (l - 3)(l - 5)
l1 = 3
l2 = 5

Evalues of A + B are not equal to evalues of A + evalues of B.

6. Find the evalues of A and B and AB and BA.

A = [1 0]
[1 1]

Triangular Matrix!
l1 = 1
l2 = 1

B = [1 2]
[0 1]

Triangular Matrix!
l1 = 1
l2 = 1

AB = [1 2]
[1 3]

Trace = 4
Det = 1

0 = (1 - l)(3 - l) - 2
= (3 -4l -l^2 - 2)
= l^2 -4l + 1

(4 +/- sqrt(16 - 4)) / 2
(4 +/- sqrt(3) / 2)
2 +/- sqrt(3)

l1 = 2 + sqrt(3)
l2 = 2 - sqrt(3)

BA = [3 2]
[1 1]

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Trace = 4
Det = 1

l1 = 2 + sqrt(3)
l2 = 2 - sqrt(3)

a. Are the evalues of AB equal to the evalues of A times the evalues of B?
No.

b. Are the evalues of AB equal to the evalues of BA?
Yes.

29. Find the evalues of A, B, and C.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Triangluar Matrix!
l1 = 1
l2 = 4
l3 = 6

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

Trace = 2
Det = 0 + 0 + 0 - 6 - 0 - 0 = -6
Factors = 3 -2 1

l1 = 3
l2 = -2
l1 = 1

$$C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

Trace = 6
Det = 0
Factors = 6, 0, 0

l1 = 6
l2 = 0
l3 = 0