UTAH STATE UNIVERSITY

# Homework 2

Jonathan Petersen

September 10, 2014

## CHAPTER 2.1

5. The first of these equations plus the second equals the third:

$$\begin{aligned} x+y+z&=2\\ x+2y+z&=3\\ 2x+3y+2z&=5 \end{aligned}$$

The first two planes meet along a line. The third plane contains the line, because if $x, y, z$ satisfy the first two equations then they also satisfy the third. The equations have infinitely many solutions (The whole line L). Find three solutions on L.

$$x=2 \qquad y=1 \qquad z=-1 \tag{1}$$
$$x=5 \qquad y=1 \qquad z=-4 \tag{2}$$
$$x=10 \qquad y=1 \qquad z=-9 \tag{3}$$

6. Move the third plane in problem 5 to a parallel plane 2x + 3y + 2z = 9. now the three equations have no solution–why not? The first two planes meet along the line L, but the third plane doesn't contain or intersect that line.

7. In problem five the columns are (1, 1, 2) and (1, 2, 3) and (1, 1, 2). This is a "singular case" because the third column is identical to the first. Find two combinations of the columns that give b = (2, 3, 5).

$$2\begin{bmatrix}1\\1\\2\end{bmatrix}+1\begin{bmatrix}1\\2\\3\end{bmatrix}-1\begin{bmatrix}1\\1\\2\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix} \tag{1}$$

$$5\begin{bmatrix}1\\1\\2\end{bmatrix}+1\begin{bmatrix}1\\2\\3\end{bmatrix}-4\begin{bmatrix}1\\1\\2\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix} \tag{2}$$

This is only possible for b = (4, 6, c) if c = 10.

9. Compute each Ax by dot products of the rows with the column vector:

$$\begin{bmatrix}1&2&4\\-2&3&1\\-4&1&2\end{bmatrix}\begin{bmatrix}2\\2\\3\end{bmatrix}$$

$$\begin{aligned}2\times1+2\times2+3\times4&=\\&2+4+12=18\\2\times-2+2\times3+3\times1&=\\&-4+6+3=5\\2\times-4+2\times1+3\times2&=\\&-8+2+6=0\end{aligned}$$

$$Ax=\begin{bmatrix}18\\5\\0\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}2&1&0&0\\1&2&1&0\\0&1&2&1\\0&0&1&2\end{bmatrix}=\begin{bmatrix}1\\1\\1\\2\end{bmatrix}$$

$$
\begin{aligned}
1\times 2+1\times 1+1\times 0+2\times 0 &= \\
2+1+0+0 &= 3 \\
1\times 1+1\times 2+1\times 1+2\times 0 &= \\
1+2+1+0 &= 4 \\
1\times 0+1\times 1+1\times 2+2\times 1 &= \\
0+1+2+2 &= 5 \\
1\times 0+1\times 0+1\times 1+2\times 2 &= \\
0+0+1+4 &= 5
\end{aligned}
$$

$$
Ax = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}
$$

10. Compute each Ax in problem 9 as a combination of the columns:

$$
\begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}
$$

$$
\begin{aligned}
2\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} &= \\
\begin{bmatrix} 2 \\ -4 \\ -8 \end{bmatrix} + \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 12 \\ 3 \\ 6 \end{bmatrix} &= \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}
\end{aligned}
$$

$$
Ax = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix} \tag{1}
$$

$$
\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}
$$

$$1\begin{bmatrix}2\\1\\0\\0\end{bmatrix}+1\begin{bmatrix}1\\2\\1\\0\end{bmatrix}+1\begin{bmatrix}0\\1\\2\\1\end{bmatrix}+2\begin{bmatrix}0\\0\\1\\2\end{bmatrix}=$$

$$\begin{bmatrix}2\\1\\0\\0\end{bmatrix}+\begin{bmatrix}1\\2\\1\\0\end{bmatrix}+\begin{bmatrix}0\\1\\2\\1\end{bmatrix}+\begin{bmatrix}0\\0\\2\\4\end{bmatrix}=\begin{bmatrix}3\\4\\5\\5\end{bmatrix}$$

$$Ax=\begin{bmatrix}3\\4\\5\\5\end{bmatrix} \tag{2}$$

11. Find the two components of Ax by rows or by columns:

$$\begin{bmatrix}2&3\\5&1\end{bmatrix}\begin{bmatrix}4\\2\end{bmatrix}$$

$$4\begin{bmatrix}2\\5\end{bmatrix}+2\begin{bmatrix}3\\1\end{bmatrix}=$$

$$\begin{bmatrix}8+6\\20+2\end{bmatrix}=\begin{bmatrix}14\\22\end{bmatrix}$$

$$Ax=\begin{bmatrix}14\\22\end{bmatrix} \tag{1}$$

and

$$\begin{bmatrix}3&6\\6&12\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}$$

$$2\begin{bmatrix}3\\6\end{bmatrix}-1\begin{bmatrix}6\\12\end{bmatrix}=$$

$$\begin{bmatrix}6-6\\12-12\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$Ax = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2}$$

and

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 4 \\ 1 \end{bmatrix} =$$
$$\begin{bmatrix} 3+2+4 \\ 6+0+1 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 9 \\ 7 \end{bmatrix} \tag{3}$$

# CHAPTER 2.2

4. What multiple $l$ of equation 1 should be subtracted from equation 2 to remove $c$?

$$ax + by = f \tag{1}$$
$$cx + dy = g \tag{2}$$

Equation 1 should be divided by $a$ and multiplied by $c$, or, in other words $\frac{c}{a}$.

6. Choose a coefficient $b$ that makes this system singular. Then choose a right side $g$ that makes it solvable.

$$2x + by = 16$$
$$4x + 8y = g$$

If $b = 4$ then this system is singluar, as that value of $b$ would make the line describing the first equation parallel to the line describing the second.

If $g = 32$ then this singular system is solvable, as the first and second equations describe the same line, so every point on the line is a solution.
Find two solutions in that singular case.

$$x = 2 \qquad y = 3 \tag{1}$$
$$x = 4 \qquad y = 2 \tag{2}$$

13. Apply elimination (circle the pivots) and back substitution to solve:

$$\begin{aligned} 2x - 3y &= 3 \\ 4x - 5y + z &= 7 \\ 2x - y - 3z &= 5 \end{aligned}$$

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 4 & -5 & 1 \\ 2 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & |3 \\ 4 & -5 & 1 & |7 \\ 2 & -1 & -3 & |5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 & |3 \\ 2 & -1 & -3 & |5 \\ 4 & -5 & 1 & |7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & |3 \\ 2 & -1 & -3 & |5 \\ 4 & -5 & 1 & |7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 & |3 \\ 0 & 2 & -3 & |2 \\ 4 & -5 & 1 & |7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & |3 \\ 0 & 2 & -3 & |2 \\ 4 & -5 & 1 & |7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 & |3 \\ 0 & 2 & -3 & |2 \\ 0 & 1 & 1 & |1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & |3 \\ 0 & 2 & -3 & |2 \\ 0 & 1 & 1 & |1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 & |3 \\ 0 & 1 & 1 & |1 \\ 0 & 2 & -3 & |2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 & |3 \\ 0 & 1 & 1 & |1 \\ 0 & 2 & -3 & |2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 & |3 \\ 0 & 1 & 1 & |1 \\ 0 & 0 & -5 & |-4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$$

Therefore,

$$A = \begin{matrix} 2x & -3y & & = 3 \\ & y & z & = 1 \\ & & -5z & = -4 \end{matrix}$$

$$z = \frac{4}{5}$$

$$\begin{aligned} y + z &= 1 \\ y + \frac{4}{5} &= 1 \\ y &= \frac{5}{4} \end{aligned}$$

$$\begin{aligned} 2x - 3y &= 3 \\ 2x - 3 \times \frac{5}{4} &= 3 \\ 2x - \frac{15}{4} &= 3 \\ 2x &= \frac{12}{4} + \frac{15}{4} \\ x &= 2 \times \frac{27}{4} \\ x &= \frac{27}{2} \end{aligned}$$

List three row operations:
Subtract 1 times row 1 from row 2.
Subtract 2 times row 1 from row 3.
Subtract 2 times row 2 from row 3.

14. Which number b leads later to a row exchange? Which b leads to a missing pivot? In that singular case find a nonzero solution x, y, z.

$$\begin{matrix} x & +by & & = 0 \\ x & -2y & -z & = 0 \\ & y & +z & = 0 \end{matrix}$$

$b = -1$ later leads to a row exchange, since row 2 cannot be used to remove the y in row three.
$b = -2$ leads to a missing pivot, as any change to remove x makes y in row 2 zero.

Suppose $b = -2$, the matrix is then

$$\begin{bmatrix} 1 & -2 & 0 & |0 \\ 1 & -2 & -1 & |0 \\ 0 & 1 & 1 & |0 \end{bmatrix}$$

Therefore:

$$\begin{aligned} x - 2y &= 0 \\ -z &= 0 \\ y + z &= 0 \end{aligned}$$

In this case, (0, 0, 0) is the only solution.