Jonathan Petersen
A01236750

Chapter 6.3

1. Find two λ's and **x**'s so that $u = e^{\lambda t}x$ solves

$$\frac{du}{dt} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} u$$

λ1 = 1
λ2 = 4

λ = 1:

$$\begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}$$

x1 = (1, -1)

λ = 4:

$$\begin{bmatrix} 0 & 3 \\ 0 & -3 \end{bmatrix}$$

x2 = (1, 0)

What combination $u = c_1e^{\lambda_1 t}x_1 + c_2e^{\lambda_2 t}x_2$ starts from u(0) = (5, -2?)

$u(0) = 2x_1 + 3x_2$
C = (2, 3)
$u = 2e^{t}x_1 + 3e^{4t}x_2$

3.

a. If every column of A adds to 0, why is λ = 0 an eigenvalue?

If every column of A adds to 0, then the rows are not linearly independent, therefore A is not invertible and as such λ = 0 must be an eigenvalue.

b. With negative diagonal and positive off-diagonals adding to 0, u' = Au will be a continuous Markov equation. Find the eigenvalues and eigenvectors, and the steady state as t → ∞.

$$\frac{du}{dt} = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} u$$

u(0) = (4, 1)

Trace: -5
Det: 6 - 6 = 0

(-2 - λ)(-3 - λ) - 6
$\lambda^2 - 5\lambda$

λ1 = 0

$\lambda 2 = -5$

<u>$\lambda = 0$:</u>

$$\begin{bmatrix} -2 & 3 \\ 0 & 0 \end{bmatrix}$$

$x1 = (3, 2)$

<u>$\lambda = -5$:</u>

$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$$

$x2 = (3, -2)$

$u(0) = (4, 1)$
$=$

6. A has real eigenvalues but B has complex eigenvalues.

$$A = \begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix}$$

$$B = \begin{bmatrix} b & -1 \\ 1 & b \end{bmatrix}$$

Find the conditions on a and b so that all solutions du/dt = Au and dv/dt = Bv approach 0 as $t \to \infty$.

a: $(-\infty, -1)$
b: $(-\infty, 0)$

10. Find A to change the scalar equation $y'' = 5y' + 4y$ into a vector equation for $u = (y, y')$

$$\frac{du}{dt} = \begin{bmatrix} y' \\ y'' \end{bmatrix} = \begin{bmatrix} A & \\ & \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix} = Au$$

$y' = y'$
$y'' = 5y' + 4y$

$$A = \begin{bmatrix} 0 & 1 \\ 4 & 5 \end{bmatrix}$$

What are the eigenvalues of A?

$(-\lambda)(5 - \lambda) - 4$
$\lambda 2 - 5\lambda - 4$

$5 \pm \sqrt{(25)-4(-4)}\ /\ 2$
$= 5 \pm \sqrt{(9)}\ /\ 2$
$= 5 \pm 3\ /\ 2$

= 1, 4

$\lambda 1 = -1$
$\lambda 2 = -4$

$y'' = 5y' + 4y$ $\quad y = e\lambda t.$
$\lambda^2 e^{\lambda t} - 5\lambda e^{\lambda t} - 4e^{\lambda t} = 0$
$(\lambda^2 - 5\lambda - 4)e^{\lambda t} = 0$
$(\lambda + 1)(\lambda - 4)$

13. Write down two familiar functions that solve d2y / dt2 = -9y. Which one starts with y(0) = 3 and y'(0) = 0?

$y1 = -e^{3t}$
$y2 = -(3/2)y3$

19. The matrix B has $B^2 = 0$. Find e^{Bt} from a (short) infinite series. Check that the derivative of $e^{Bt} = Be^{Bt}$.

$$B = \begin{bmatrix} 0 & -4 \\ 0 & 0 \end{bmatrix}$$

21. Write A in the form $S\Lambda S^{-1}$. Find eAt from $Se^{\Lambda t}S^{-1}$.

$$A = \begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix}$$

22. If $A^2 = A$, show that the infinite series produces $e^{At} = I + (e^t - 1)A$. For the previous problem, this gives e^{At} = ________.

24. Write A as $S\Lambda S^{-1}$. Multiply $Se^{\Lambda t}S^{-1}$ to find e^{At}. Check e^{At} and its derivative when t = 0.