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## Chapter 1.2

$$u = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix} \qquad v = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad w = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

**2. Compute the lengths || u || and || v || and || w || of those vectors. Check the Schwarz inequalities | u · v | ≤ || u || || v || and | v · w | ≤ || v || || w ||.**

$$\begin{aligned} \|u\| &= \sqrt{(-0.6^2 + 0.8^2)} \\ &= \sqrt{(0.36 + 0.64)} \\ &= \sqrt{(1)} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \|v\| &= \sqrt{(3^2 + 4^2)} \\ &= \sqrt{(9 + 16)} \\ &= \sqrt{(25)} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \|w\| &= \sqrt{(8^2 + 6^2)} \\ &= \sqrt{(64 + 36)} \\ &= \sqrt{(100)} \\ &= 10 \end{aligned}$$

$$\begin{aligned} |u \cdot v| &= (-0.6 * 3) + (0.8 * 4) \\ &= -1.8 + 3.4 \\ &= 1.6 \end{aligned}$$

$$\begin{aligned} |u \cdot v| &\le \|u\| \, \|v\| \\ 1.6 &\le 1 * 5 \\ 1.6 &\le 5 \end{aligned}$$

$$\begin{aligned} |v \cdot w| &= (3 * 8) + (4 * 6) \\ &= 24 + 24 \\ &= 48 \end{aligned}$$

$$\begin{aligned} |v \cdot w| &\le \|v\| \, \|w\| \\ 48 &\le 5 * 10 \\ 48 &\le 50 \end{aligned}$$

**3. Find unit vectors in the directions of v and w in Problem 1, and the cosine of the angle θ. Choose vectors a, b, c, that make 0°, 90°, and 180° angles with w.**

$$\hat{v} = v / \| v \| = 0.2 * \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$$

$$\hat{w} = w / \| w \| = 1/10 * \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix}$$

$\theta = 90° = \pi$

$\cos( \pi ) = 0$

$$a = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$$b = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$c = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$$

**8. True or false:**

**a. If u is perpendicular (in three dimensions) to v and w, those vectors v and w are parallel.**

False. u only needs to be perpendicular to the plane formed by v and w.

**b. If u is perpendicular to v and w, then u is perpendicular to v + 2w.**

True.

**c. If u and v are perpendicular unit vectors, then $\| u - v \| = \sqrt{2}$ .**

True.

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## Chapter 1.3

**1. Find the linear combination $2s_1 + 3s_2 + 4s_3 = b$. Then write b as a matrix vector multiplication S x. Compute the dot products (row of S) · x:**

$$s_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad s_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad s_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$2 * s_1 + 3 * s_2 + 4 * s_3$$

$$= 2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix} = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = b$$

$[\ 1\ 0\ 0\ ] \cdot [\ 2, 3, 4\ ] = 2 + 0 + 0 = 2$

$[\ 1\ 1\ 0\ ] \cdot [\ 2, 3, 4\ ] = 2 + 3 + 0 = 5$

$[\ 1\ 1\ 1\ ] \cdot [\ 2, 3, 4\ ] = 2 + 3 + 4 = 9$

**2. Solve these equations S y = b with s1 · s2 · s3 in the columns of S:**

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1/2 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

**and**

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix}$$

## Chapter 2.1

**9. Compute each Ax by dot products of the rows with the column vector:**

**a.** $$\begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

2 * 1 + 2 * 2 + 3 * 4 = 2 + 4 + 12 = 18
2 * -2 + 2 * 3 + 3 * 1 = -4 + 6 + 3 = 5
2 * -4 + 2 * 1 + 3 * 2 = -8 + 2 + 6 = 0

$$Ax = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$$

**b.** $$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

1 * 2 + 1 * 1 + 1 * 0 + 2 * 0 = 2 + 1 + 0 + 0 = 3
1 * 1 + 1 * 2 + 1 * 1 + 2 * 0 = 1 + 2 + 1 + 0 = 4
1 * 0 + 1 * 1 + 1 * 2 + 2 * 1 = 0 + 1 + 2 + 2 = 5
1 * 0 + 1 * 0 + 1 * 1 + 2 * 2 = 0 + 0 + 1 + 4 = 5

$$Ax = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$$

**11. Find the two components of Ax by rows or by columns:**

$$\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$Ax = 4 * \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 2 * \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 + 6 \\ 20 + 2 \end{bmatrix} = \begin{bmatrix} 14 \\ 22 \end{bmatrix}$$

**and**

$$\begin{bmatrix} 3 & 6 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$Ax = 2 * \begin{bmatrix} 3 \\ 6 \end{bmatrix} + -1 * \begin{bmatrix} 6 \\ 12 \end{bmatrix} = \begin{bmatrix} 6 - 6 \\ 12 - 12 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

**and**

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$Ax = 3 * \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1 * \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 1 * \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 + 2 + 4 \\ 6 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

**12. Multiplay A times x to find three components of Ax:**

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$Ax = \begin{bmatrix} 0 + 0 + z \\ 0 + y + 0 \\ x + 0 + 0 \end{bmatrix} = \begin{bmatrix} z \\ y \\ x \end{bmatrix}$$

**and**

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 3 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$Ax = 1\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 1\begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 + 1 - 3 \\ 1 + 2 - 3 \\ 3 + 3 - 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

**and**

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Ax = 1\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 + 1 \\ 1 + 2 \\ 3 + 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$$