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Homework 7

Chapter 3.5

21. Suppose that the columns of a 5 x 5 matrix A are a basis for R^5.
a. The equation Ax = 0 has only the solution x = 0 because ________.

Since the columns of A form a basis for R^5, there must be five linearly independent columns in A. Because A only has 5 columns, this means that there are no free variables in A, and every column has a pivot. Because of those conditions, the null space of A must contain only the zero vector, or, in other words, x = 0 is the only solution to Ax = 0.

b. If b is in R^5 then Ax = b is solvable because the basis vectors _______ R^5.

The basis vectors span R^5.

24. True or false (justify your answer):
a. If the columns of a matrix are dependent, so are the rows.

True. In order for the columns to be dependent, there must be at least one free variable. In order for there to be a free variable, there must be at least one column without a pivot. Because of the nature of pivots, if there is at least one column without a pivot, there must be at least one row without a pivot. Therefore, at least one row must be a linear combination of the others, and is thusly dependent.

b. The column space of a 2 x 2 matrix is the same as its row space.

False. Consider

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

The row space of this matrix is [1, 1] and [0, 0], but the column space is simply (1, 0).

c. The column space of a 2 x 2 matrix has the same dimension as its row space.

False. See the above example. The dimension of the row space is 2 while the dimension of the column space is 1.

d. The columns of a matrix are a basis for the column space.

False. This is only true if the columns are linearly independent.

26. Find the basis (and dimension) for each of these subspaces of 3 x 3 matrices:
a. All diagonal matrices.

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All diagonal matrices are formed by linearly independent columns, therefore the basis of the matrix is simply the columns of the matrix, and the dimension of a 3 x 3 diagonal matrix is 3.

b. The rank one matrices.

Rank one matrices have only one linearly independent column, therefore the basis of the matrix is the column with the solitary pivot and the dimension is one.

c. The identity matrix.

The identity matrix is a special diagonal matrix, therefore as proven in part a its basis is the set of columns of I and its dimension is three.

28. Find a basis for the space of all 2 x 3 matrices whose columns add to 0.

```
A = [  1,  2, -3 ]
    [  2,  3, -5 ]

A = [  1,  2, -3 ]
    [  0,  1, -1 ]

A = [  1,  0, -1 ]
    [  0,  1, -1 ]
```

A basis for this problem is the set of vectors (1, 0), (0, 1), and (-1, -1).

Find a basis for the subspace of the above where the rows also add to zero.

```
A = [  1, -1, 0 ]
    [ -1,  1, 0 ]
```

A basis for this problem is the set of vectors (1, -1), (-1, 1), (0, 0).

Chapter 3.6

1.
a. If a 7 x 9 matrix has rank 5, what are the dimensions of the four subspaces?

The row space has dimension 5.
The column space has dimension 5.
The null space has dimension 9 - 5 = 4.

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The left null space has dimension 7 - 5 = 2.

What is the sum of all four dimensions?

16.

b. If a 3 x 4 matrix has rank 3, what are its column space and left null space?

It's column space is formed by the three pivot columns, and its left null space only contains the zero vector, as it must have dimension 3 - 3 = 0.

2. Find the bases and dimensions for the four subspaces associated with A and B:

```
A = [  1,  2,  4 ]
    [  2,  4,  8 ]

A = [  1,  2,  4 ]
    [  0,  0,  0 ]
```

The row space has dimension 1, a basis for this subspace is [1, 1, 1].
The column space has dimension 1, a basis for this subspace is (1, 2).

The pivot is x1, free variables are x2, x3.
Let x2 = 1, x3 = 0. Then x1 = -2.
Let x2 = 0, x3 = 1. Then x1 = -4.

Tne null space has dimension 2, a basis for this subspace is (-2, 1, 0), (-4, 0, 1).

```
A^T =     [  1,  2 ]
          [  0,  0 ]
          [  0,  0 ]
```

The pivot is x1, free variable is x2.
Let x2 = 1, then x1 = -2.

The left null space has dimension 1, a basis for this subspace is [-2, 1].

```
B = [  1,  2,  4 ]
    [  2,  5,  8 ]

B = [  1,  2,  4 ]
    [  0,  1,  0 ]
```

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```
B = [  1,  0,  4 ]
    [  0,  1,  0 ]
```

The row space has dimension 2, a basis for this subspace is [1, 0, 4], [0, 1, 0].
the column space has dimension 2, a basis for this subspace is (1, 0), (0, 1).
The null space has dimension 1, a basis for this subspace is (-4, 0, 1).
the left null space has dimension 0, a basis for this subspace is [0, 0, 0].

4. Construct a matrix with the required property or explain why this is impossible:

a. Column space contains (1, 1, 0), (0, 0, 1), row space contains [1, 2], [2, 5].

```
A = [  1,  2 ]
    [  3,  4 ]
    [  2,  5 ]
```

b. Column space has basis (1, 1, 3), null space has basis (3, 1, 1).

c. Dimension of null space = 1 + dimension of left null space.

Any m by m + 1 matrix.

d. Left null space contains [1, 3], row space contains [3, 1].

e. Row Space = Column Space, but Null Space != Left Null Space.

Chapter 4.1

3. Construct a matrix with the required property or say why that is impossible:

a.
b.
c.
d.
e. Columns add up to a column of zeros, rows add up to a row of ones.

4. If AB = 0 then the columns of B are in the null space of A. The rows of A are in the left null space of B. Why can't A and B be 3 x 3 matrices of rank 2?

They cannot be 3 x 3 matrices of rank 2 because then the columns of B would have dimension 2 while the null space of A would have only dimension 1. Since the columns of B must be in the null space of A, the dimensions of the null space would have to be equal to the dimension of the columns of B. This is not true.

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9. If A^T(A)x = 0, then Ax = 0. Reason, Ax is in the null space of A^T and also in the null space of A, and those spaces are identical.

20. Suppose V is the whole space R^4. Then V^Perp contains only the vector (0, 0, 0, 0). Then (V^Perp)^Perp is any vector in R^4. So (V^Perp)^Perp is the same as R^4.