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MATH 5710

CH4: 6, 7, 14, 29, 30, 31

HW 9

CH4 6. $f(x) = 2x \quad 0 \le x \le 1$

a. $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^1 x(2x)\,dx$

$= \frac{2}{3}(x^3)\Big|_0^1$

$= \frac{2}{3} - 0 \boxed{= 2/3}$

b. $Y = x^2 \Rightarrow f(y) = 2x^2 \quad 0 \le y \le 1$

$E(Y) = \int_0^1 y(2y^3)\,dy$

$= 2\left[\frac{1}{4}y^4\right]_0^1$

$= \frac{1}{2}(1-0) \boxed{= 1/2}$

c. $\cancel{Y = 0 + \sum_1^1 (x)(x)} \quad g(x) = x^2$

$\cancel{E(Y) = 0 + \sum_1^1 x\left(\frac{2}{3}\right)}$

$E(Y) = \int_{-\infty}^{\infty} g(X) f(X)\,dx$

$\cancel{= \int_0^1 2x^2\,dx} \quad = \int_0^1 (x^2)(2X)\,dX$

$\cancel{= \frac{2}{3}\left[x^3\right]_0^1} \quad = 2\left[\frac{1}{4}x^4\right]_0^1$

$\cancel{= \frac{2}{3}(} \quad = \frac{1}{2}(1-0) \boxed{= \frac{1}{2} = \frac{1}{2}}$

d. $Var(X) = E\left(\left(x - \frac{2}{3}\right)^2\right)$

$= \int_0^1 \left(x - \frac{2}{3}\right)^2 (2x)\,dx$

$= \int_0^1 \left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) 2x\,dx$

$\cancel{= \int_0^1 2x^3 - \frac{8}{3}x^2 + \frac{8}{9}x\,dx} \quad = \int_0^1 2x^3 - \frac{8}{3}x^2 + \frac{8}{9}x$

$\cancel{= \left[\frac{2}{4}x^4 - \frac{8}{9}x^3 + \frac{16}{9 \cdot 18}x^2\right]_0^1} \quad = \left[\frac{2}{4}x^4 - \frac{8}{9}x^3 + \frac{8}{18}x\right]_0^1$

$\cancel{= \frac{2}{4} - \frac{8}{9} + \dots} = \frac{1}{2} + \quad = \frac{1}{2} - \frac{16}{18} + \frac{8}{18} = \frac{1}{2} - \frac{8}{18} = \frac{9}{18} - \frac{8}{18} = \frac{1}{18}$

$\cancel{= \frac{1}{2} - \frac{12}{18} = \frac{1}{2} - \frac{2}{3} = -\frac{1}{3}}$

$Var(X) = E(X^2) - E(X)^2$

$= \frac{1}{2} - \left(\frac{2}{3}\right)^2$

$= \frac{1}{2} - \frac{4}{9}$

$= \frac{9-8}{18} \boxed{= \frac{1}{18} = \frac{1}{18}}$

$E(X^2) = \int_0^1 x^2(2x)\,dx$

$= 2\left[\frac{1}{4}x^4\right]_0^1$

$= \frac{1}{2}(1-0) = \frac{1}{2}$

7. $p_x(0) = \frac{1}{2} \quad p_x(1) = \frac{3}{8} \quad p_x(2) = \frac{1}{8}$

a. $E(X) = \sum_0^2 x\,p(x)$

$= 0(\frac{1}{2}) + 1(\frac{3}{8}) + 2(\frac{1}{8})$

$\boxed{= \frac{5}{8}}$

b. $Y = X^2$

$p_Y(0) = \frac{1}{2} \quad p_Y(1) = \frac{3}{8} \quad p_Y(4) = \frac{1}{8}$

$E(y) = 0 \times \frac{1}{2} + 1(\frac{3}{8}) + 4(\frac{1}{8})$

$\boxed{= \frac{7}{8}}$

c. $g(x) = x^2$

$E(g(x)) = E(Y) = \sum_{i=0}^{4} g(x)p(x)$

$= 0^2(\frac{1}{2}) + 1^2(\frac{3}{8}) + 2^2(\frac{1}{8})$

$\boxed{= \frac{7}{8}} = \frac{7}{8}$

d. $Var(X) = E((x - \frac{7}{8})^2)$

$= \sum (x - \frac{7}{8})^2 p(x)$

$= \sum (x^2 - \frac{14}{8}x + \frac{49}{64}) p(x)$

$= [(\frac{1}{2})(0^2 - \frac{14}{8}(0) + \frac{49}{64}) + (\frac{3}{8})(1^2 - \frac{14}{8} + \frac{49}{64}) + (\frac{1}{8})(2^2 - \frac{14}{8}(2) + \frac{49}{64})]$

$= \frac{7^2}{2 \cdot 8^2} + \frac{3 \cdot (8^2 - 14(8) + 7^2)}{8^3} + \frac{4(8^2) - 28(8) + 7^2}{8^3}$

$= \frac{1}{8^3}[\frac{1}{2}(7^2)(8) + 3(8^2) - 14(3)(8) + 3(7^2) + 4(8^2) - 28(8) + 7^2]$

$= \frac{1}{8^3}[4(\cancel{7^2}) + 3(\cancel{8^2}) - \cancel{42}(8) + 3(\cancel{7^2}) + \cancel{4(8^2)} - \cancel{28}(8) + 1(\cancel{7^2})]$

$= \frac{1}{8^3}[(4+3+1)(7^2) + (3+4)(8^2) - (42-28)(8)]$

$= 1/8^3[8(7^2) + 7(8^2) - (2 \times 8)(8)]$

$= 1/8^3[(7 \times 8)(7+8) - 2 \cdot 8^2]$

$= 1/8^2[7(7+8) - 2 \cdot 8]$

$= 1/8^2[7^2 + 8(7-2)]$

$= 1/8^2[\overset{49}{\cancel{7^2}} + 20]$

$\cancel{= \frac{69}{64}}$

$Var(X) = E(X^2) - E(X)^2$

$= \frac{7}{8} - (\frac{5}{8})^2$

$= \frac{7}{8} - (\frac{25}{64}) = \frac{56}{64} - \frac{25}{64}$

$\boxed{= \frac{31}{64}}$

$E(X^2) = 0(\frac{1}{2}) + 1(\frac{3}{8}) + 4(\frac{1}{8}) = \frac{7}{8}$

14. $f(x) = 2x \qquad 0 \le x \le 1$

From problem #6:

$$\boxed{\begin{aligned} &\text{a.}\quad E(X) = \tfrac{2}{3} \\ &\text{b.}\quad E(X^2) = \tfrac{1}{2} \\ &\qquad Var(X) = \tfrac{1}{2} - \left(\tfrac{2}{3}\right)^2 \\ &\qquad\qquad = \tfrac{1}{18} \end{aligned}}$$

29. Let X, Y be independent continuous R.V.'s, and $g(X)$ and $h(Y)$ be fixed functions.

Then

$$E(g(X)h(Y)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(X)h(Y)\cancel{(xy)\, f_{XY}(x,y)}\, dy\, dx$$

$$\cancel{= \int_{-\infty}^{\infty} x g(X) \int_{-\infty}^{\infty} y h(Y)\, dy\, dx}$$

$$\cancel{= \int_{-\infty}^{\infty} x g(X)\, dx \times \int_{-\infty}^{\infty} y h(Y)}$$

$$\cancel{= E(g(X)}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x)h(Y)\, f_{XY}(x,y)\, dy\, dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(X)h(Y)\, f_X(X) f_Y(y)\, dy\, dx$$

$$= \int_{-\infty}^{\infty} g(x) f_X(X) \int_{-\infty}^{\infty} h(Y) f_Y(y)\, dy\, dx$$

$$= \int_{-\infty}^{\infty} g(X) f_X(X)\, dx \times \int_{-\infty}^{\infty} h(Y) f_Y(y)\, dy$$

$$= E(g(X)) \times E(h(Y)) \quad \blacksquare$$

30. $E\left(\frac{1}{X+1}\right) = \int_{-\infty}^{\infty} \frac{1}{X+1}\left(\frac{\lambda^x e^{-\lambda}}{x!}\right) dx$

$$= \int_{-\infty}^{\infty} \left(\frac{\lambda^x e^{-\lambda}}{(x+1)x!}\right) dx$$

$$= e^{-\lambda} \int_{-\infty}^{\infty} \frac{\lambda^x}{(x+1)x!}$$

$$= e^{-\lambda}\left[(x+1)x!\, \lambda^x / \log(\lambda) - \lambda^x (x+1)(\right. \quad ?$$

31. $E\left(\frac{1}{x}\right) = \int_{-\infty}^{\infty} \frac{1}{X}\, 1\, dx$

$$= \int_{-\infty}^{\infty} x^{-1}\, dx$$

$$= \int_{-\infty}^{\infty} x^{-1}\, dx$$

$$= [\log(x)]_1^2$$

$$\cancel{= [\infty - \infty]}\ ?$$

$$\cancel{= DNE}$$

$$\boxed{= \log(2) - \log(1)}$$

~~WMA~~ $E(X) = \int_{-\infty}^{\infty} x\, 1\, dx$

$$= \tfrac{1}{2}x^2 \Big|_1^2$$

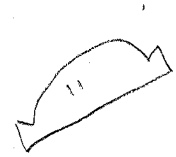
$$= \tfrac{1}{2}(4 - 1)$$

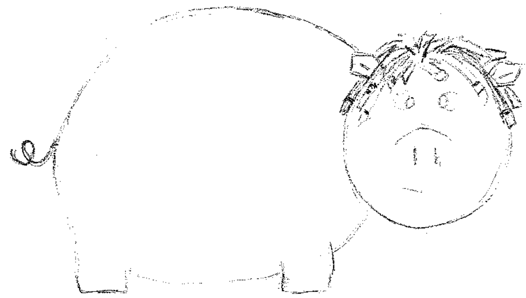
$$= 2 - \tfrac{1}{2}$$

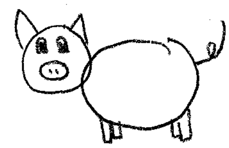
$$\boxed{= 3/2}$$

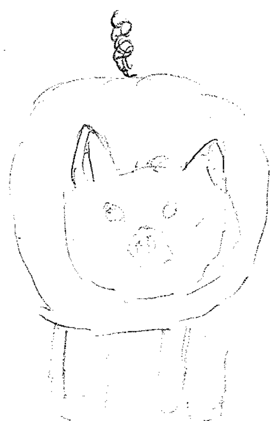
Not the same.

Amelia Josephine Seelhorst









Pregnant Stick Figure