

Project 3: 2D DFT in Image Filtering

Assigned: March 5, 2024 (Tuesday)

Due 11:59pm: March 24, 2024 (Sunday)

In this project, you are asked to implement the discrete Fourier transform $F(u, v)$ of an input image $f(x, y)$ of size $M \times N$ and then apply the ideal low pass filter $H(u, v)$ to smoothing the image by using the following equation:

$$g(x, y) = \mathfrak{F}^{-1} \{H(u, v)F(u, v)\},$$

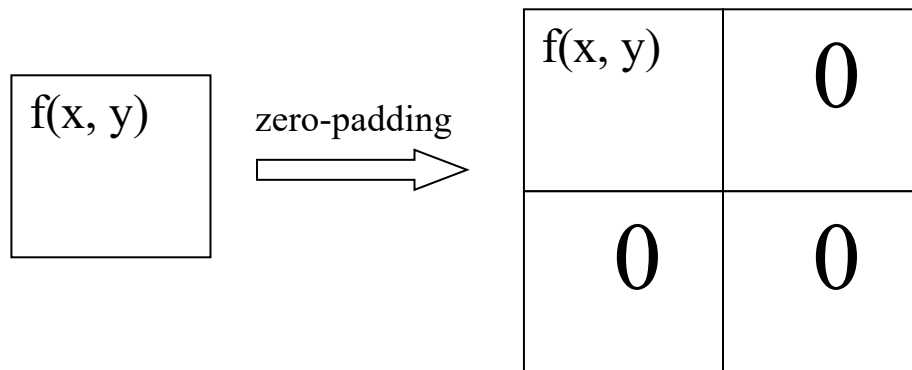
where $g(x, y)$ is the output image (the smoothed image of $f(x, y)$) and the ideal low pass filter is defined as follows:

$$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases},$$

where $D(u, v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$ (see below for the definitions of P and Q) and D_0 is called cutoff frequency.

Notes:

- (1) You need to zero-pad your original image to generate a new image of size $P \times Q$, where $P=2M-1$ and $Q=2N-1$.



- (2) You need to multiply your original image by $(-1)^{(x+y)}$ so that the low frequency of $F(u, v)$ is centered at the center of your domain.

- (3) You are **required** to use the fast 2D DFT algorithm to implement this project.
- (4) Please use the image provided as your input.
- (5) **Two output images** corresponding to two significantly different cutoff frequencies should be submitted.
- (6) Please also submit the source code with your output images.
- (7) You must implement the DFT by following the efficient DFT algorithm in $O(N^3)$ time complexity - calling any built-in functions provided by a 3rd party library is NOT allowed.