MCMC I
Course Time Plan
July 13-15, 2015

Instructors: Vladimir Minin, Kari Auranen, M. Elizabeth Halloran

Course Description: This module is an introduction to Markov chain Monte Carlo methods with some simple applications in infectious disease studies. The course includes an introduction to Bayesian inference, Monte Carlo, MCMC, some background theory, and convergence diagnostics. Algorithms include Gibbs sampling and Metropolis-Hastings and combinations. Programming is in R. Familiarity with the R statistical package or other computing language is needed.

Course schedule: The course is composed of 10 90-minute sessions, for a total of 15 hours of instruction.

1 Introduction to Bayesian Inference

- Overview of the course.
- Bayesian inference: Likelihood, prior, posterior, normalizing constant
- Conjugate priors; Beta-binomial; Poisson-gamma; normal-normal
- Posterior summaries, mean, mode, posterior intervals
- Motivating examples: Chain binomial model (Reed-Frost), General Epidemic Model, SIS model.
- Lab:
  - Goals: Warm-up with R for simple Bayesian computation
  - Example: Posterior distribution of transmission probability with a binomial sampling distribution using a conjugate beta prior distribution
  - Summarizing posterior inference (mean, median, posterior quantiles and intervals)
  - Varying the amount of prior information
  - Writing an R function

2 Introduction to Gibbs Sampling

- Chain binomial model and data augmentation
- Brief introduction to Gibbs sampling
- Lab
  - Goals: Simple data augmentation using MCMC
  - Example: Gibbs sampler for the chain binomial model.

3 Classical Monte Carlo and Markov chain theory

- Random number generators
- Non-iterative Monte Carlo methods
  - Classical Monte Carlo and importance sampling
- Basic Markov Chain Theory
  - Definitions
  - Stationarity
  - The ergodic theorem
- Lab:
  - Goals: Understanding importance sampling and the ergodic theorem for Markov chains.

4 Metropolis-Hastings algorithm

- Construction
- Proof of detailed balance
- Lab:
  - Goals: elementary missing data imputation on S-I model

5 Gibbs sampling

- Relationship with Metropolis-Hastings
- Revisit simple Gibbs sampler for chain-binomial model

6 Metropolis-Hasting and Gibbs combined

- Example: Hierarchical model
- Lab:
  - Goals: Combining Metropolis and Gibbs in one algorithm
  - Example: Beta-binomial hierarchical model with rat data

7 Chain binomial model revisited

- Hierarchical chain binomial model with hyperparameters
  - Model checking
  - Allowing for heterogeneity
- Lab:
  - Goals: Combined M-H and Gibbs and learning model checking
  - Example: Hierarchical beta-binomial chain binomial model

8 General Epidemic Model

- The general epidemic model and incompletely observed data
- Algorithm
- Lab: General epidemic model
  - Goals: parameter estimation with data augmentation
  - Example: smallpox transmission

9 Diagnostics, etc

- Assessing convergence (more or less), Coda
- Variance reduction, Monte Carlo error
- Poisson process
- Lab: Diagnostics
  - Goals: learn how to do basic diagnostics on chain and output
  - Coda
  - Diagnostics on previous examples

10 SIS model

- Binary Markov process model for a recurrent infection
- Likelihood
- Algorithm
- Lab: Estimating rates in simple SIS model

- Goals: Data simulation and parameter estimation from complete data in a simple SIS model.
- Example: simulate one long chain in one person
- Estimating rates from complete data
- Diagnostics