

## Practical: Monte Carlo and Markov chain theory

Instructors: Kari Auranen, Elizabeth Halloran and Vladimir Minin

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### Estimating the tail of the standard normal distribution

Let  $Z \sim \mathcal{N}(0, 1)$ . We would like to estimate the tail probability  $\Pr(Z > c)$ , where  $c$  is large (e.g.  $c = 4.5$ ).

#### Your task

Implement naive and importance sampling Monte Carlo estimates of  $\Pr(Z > 4.5)$ , where  $Z \sim \mathcal{N}(0, 1)$ . Download ‘import\_sampl\_reduced.R’ from the course web page. The code has a couple of things to get you started.

### Ehrenfest model of diffusion

Consider the Ehrenfest model with  $N = 100$  gas molecules. From our derivations we know that the stationary distribution of the chain is  $\text{Bin}(\frac{1}{2}, N)$ . The chain is irreducible and positive recurrent (why?). The stationary variance can be computed analytically as  $N \times \frac{1}{2} \times \frac{1}{2}$ .

#### Your task

Use ergodic theorem to approximate the stationary variance and compare your estimate with the analytical result. Don’t panic! You will not have to write everything from scratch. Download ‘ehrenfest\_diff\_reduced.R’ file from the course web page. Follow comments in this R script to fill gaps in the code.