AM 3038 / AM 3084/ FM 3036

Continuous Assessment 04

Report 4

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Problem Description

According to the problem, there are 800 people in a village in Anuradhapura. They were introduced to Dialog career in 2000. Initially, the subscription to Dialog is 0 at 2000. But it increases with time. In a census done in 2006, the out of the 600 subscribers to Dialog in the year 2005 shifted to Hutch due to signal issues and other contributing factors. Thus, we model the problem by separating it into 2 time periods. For the period from 2000-2005 and after 2005.

In the year 2000, there are only 2 groups of users who were subscribed into Dialog network and users who were not subscribed Dialog network. From 2005, there are three groups of users as a considerable proportion of users shifted from Dialog to Hutch.

What do we want to model?

Here a mathematical model will be developed for mobile network switch in the rural village in Anuradhapura. The objective is to inspect will the two network carriers will be in continuous fight for the demand in the years to come or will a network monopoly be created due to network issues and popularity of mobile usage in country.

What are we interested in?

We are interested to see whether a network company create a monopoly or not. We can find it by solving the model created below. Then we can discuss what will happen if there will be a monopoly and how to create a market balance because monopolies are considered bad for the industry. We are interested in seeing if the population becomes stable, at what proportion does the population becomes stable? We are also interested in finding whether there are people who does not adopt either or the carriers.

Assumptions

- The (environment) population is 800 homeowners lived in a rural village in Anuradhapura.
- The population is a closed population group.
- Number of homeowners will not be changed when the time passes. The number of people who will arrive at the age of using mobile phones will remain constant.
- There are 3 components of the population. They are the homeowners who subscribed Dialog network, the homeowners who subscribed Hutch network and the homeowners who subscribed neither Dialog network nor Hutch network. (There are only 2 carriers in the village)
- At the year 2005, 600 homeowners have subscribed Dialog network and the rest 200 did not subscribe Dialog before Hutch network was introduced.
- It was assumed that when dialog was initially introduced no one was subscribed to it at the year 2000.
- There are no homeowners who have subscribed both Dialog and Hutch networks at the same time.
- Once a customer is subscribed, they will remain with the carrier or changed to a different career and not move to the none subscribed group
- There are transitions between two networks due to network issues related to signal strength and the popularity of mobile usage of the country. Those two are the reasons which are considered to derive the transition probabilities between two networks. Above two reasons are independent from each other.
- Signal strength of Hutch network is stronger than Dialog network because in census considerable proportion from Dialog network has been switched to Hutch due to network issues.
- Popularity of mobile usage in the country will cause to gain subscribers for both Dialog and Hutch network.
- Transitions, from the group who have not subscribed any networks, to the group who have subscribed either Dialog or Hutch doesn’t affect to the transition probability of Dialog to hutch and the transition probability of Hutch to Dialog.
- It is assumed that the population is interactive and closed

Model Development

Variables

Assume the variables are Defined as follows
$D -$ *Number of people Subscribed to Dialog*
$H -$ *Number of people Subscribed to HutchN*
$- $ *Number of people who have not subscribed any network*
$P_{ND} -$ *Transition probability from none to Dialog*
$P_{NH} -$ *Transition probability from none to Hutch*
$P_{DH} -$ *Transition probability from Dialog to Hutch*
$P_{HD} -$ *Transition probability from Hutch to Dialog*

Model Development

In the year 2000, there are only 2 groups of people. The people who are not subscribed to any carrier and the people who are subscribed to dialog which is initially 0. Therefore, initially, there are 2 groups of people.

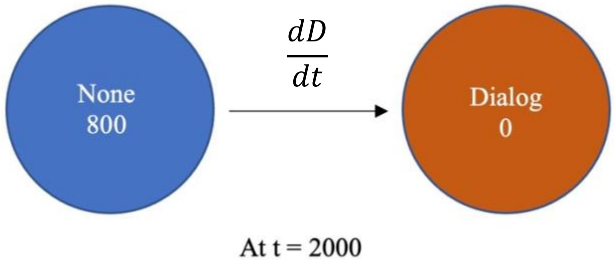


Figure 1:Model 1 figure

Thus, the model for this period can be written as follows.

$$\frac{dD}{dt} = N * P_{ND}$$

$$\frac{dN}{dt} = -(N * P_{ND})$$

$$P_{ND} = \eta D^{\alpha} N^{\beta-1}$$

α *− Difficulty in adopting the new carrier*

β *− Ressistance to loosing their old method*

In the year 2005, another carrier Hutch is introduced to the system thus creating 3 segments inside the population. As per the assumptions, since there aren't any transitions from Hutch to dialog. The model can be developed as follows.

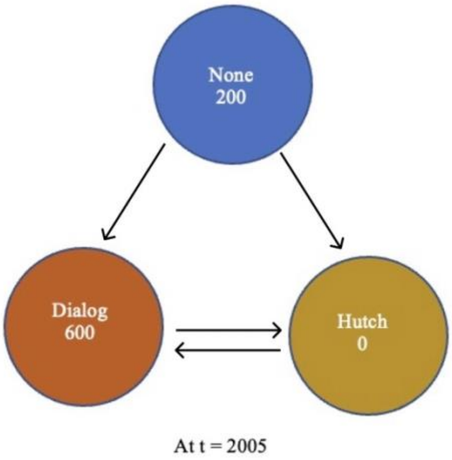


Figure 2: Model 2 Figure

$$\frac{dN}{dt} = -(N * P_{ND} + N * P_{NH})$$

$$\frac{dD}{dt} = (N * P_{ND}) - (D * P_{DH}) + (H * P_{HD})$$

$$\frac{dH}{dt} = (N * P_{NH}) + (D * P_{DH}) - (H * P_{HD})$$

$P_{ND} = \eta D^{\alpha} N^{\beta}$
$\alpha - Difficulty\ in\ adopting\ Dialog\ network$
$\beta - Ressistance\ to\ loosing\ their\ old\ method$
$\eta - amplifying\ factor$

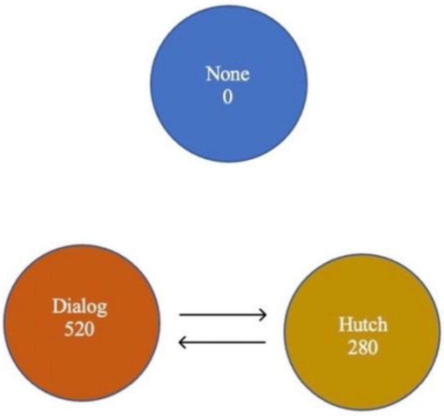
Since it was assumed that the difficulty in adopting the different carriers is different, the factors variables are different for P_{ND} and P_{NH}

$P_{NH} = \eta D^{\lambda} N^{\beta}$
$\alpha - Difficulty\ in\ adopting\ Hutch\ network$
$\beta - Ressistance\ to\ loosing\ their\ old\ method$
$\eta - amplifying\ factor$

$P_{DH} = C * H^{a} * S$
C $- Amplifing\ constant$
a
$- Motivating\ factor\ to\ change\ from\ Dialog\ to\ Hutch\ due\ to\ improved\ signal\ strength$
$S - Relative\ popularity\ of\ Hutch$

$P_{HD} = C * D^{a} * (1 - S)$

At the year 2006 the system looks like follows



At t = 2006

Results

In the following section, we will discuss the results that we obtained. The discussion of these results will be done in the next section. The first graph displays how the dynamic system becomes stable and does not create a monopoly. The hutch career becomes stable with higher number of customers

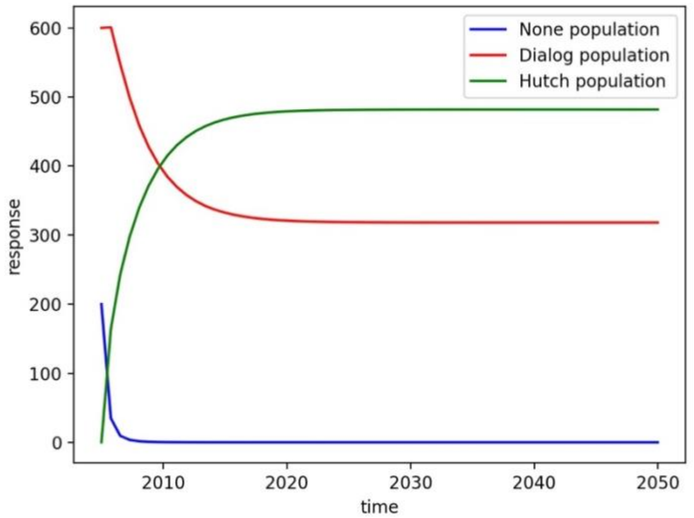


Figure 3: Populations after 2005

The following graph shows how the system behaves before the hutch carrier is introduced. We can observe that the none population is reaching a value close to 200 as t approaches 2005. We can also observe that people are adopting the only carrier Dialog.

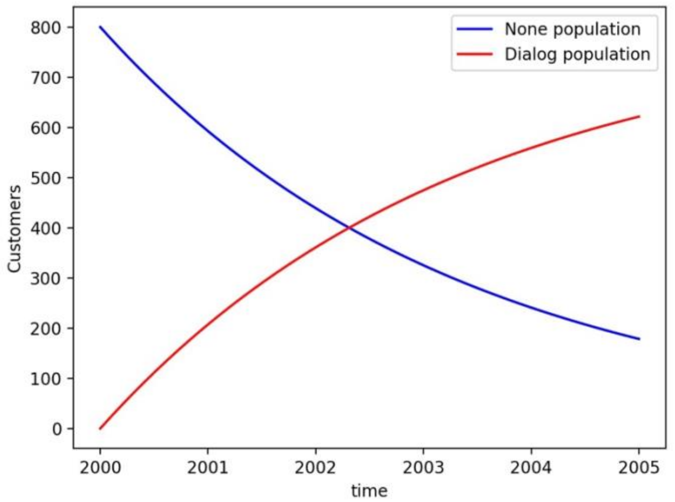


Figure 4: Populations from 2000-2005

The following are the stability points of the dynamic system.

```
Value of N at stable point 2.3825289641549426e-05
Value of D at stable point 318.4126820511599
2.3825289641549426e-05
Value of H at stable point 481.58729412354967
```

Discussion

According to the model that has been developed, initially the problem deals only with two groups of people and at the moment of census this was expanded into three groups.

We can identify one-way transition, from the group who didn't subscribe any package, to the group who subscribed Dialog network. Similarly, another one-way transition occurs from the group who didn't subscribe any package, to the group who subscribed Hutch network. Two-way transition occurs in between the groups who subscribed Hutch network and Dialog network.

Model carries out four transition probabilities that have been affected by some external factors such as adopting to the new technology, popularity, resistance to change old method etc. that have been included into the mathematical models.

Graphically, we can notice that, from year 2000 to 2005 Dialog population has been increased while none population has been decreased according to the graph 1. In graph 2 we have obtained the establishment of both networks and as the result Hutch has established with higher number of customers than the Dialog. So, roughly after the year 2010 the number of customers who would have subscribed both networks, doesn't change with time and they remain as same while the demand of Hutch becoming higher than Dialog. Also, at that time period the customers who would not use either Dialog or Hutch is zero.

Conclusions

Since the transition probabilities are in the range of [0,1], we consider the transition probabilities as in the implemented python code in appendix.

The figu is plotted according to the model which considered the time period of year 2000 to 2005. It shows that the Dialog network users will increasing from 0 to 600 and non-subscribers will decrease from 800 to 200. Therefore, we can conclude that there are 200 non-subscribers and 600 dialog subscribers at 2005 before the introduction of Hutch network.

Then we have the results as in the figure 1. According to that we can see there will be no monopoly occurs. The number of Dialog network subscribers will be stable, after it get 319 subscribers (because its stable point is 318.41) and the number of Hutch network subscribers will be stable, after it get 482 subscribers (because its stable point is 318.41). All houseowners will subscribe either Dialog or Hutch network and there will be no house owner who will not using any mobile network.

References

Hindawi, et al. “Dynamic Analysis of a Two-Language Competitive Model With Control Strategies.” *Dynamic Analysis of a Two-Language Competitive Model With Control Strategies*, 3 Dec. 2013, www.hindawi.com/journals/mpe/2013/654619.

Appendix

Solution code for the system of ODE

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.integrate import odeint

def odes(x, t):
    al = 0.1
    beta = 0.2
    neeta = 0.25

    lam = 0.1

    a = 0.02
    c = 0.3
    s = 0.6

    N = x[0]
    D = x[1]
    H = x[2]

    # PND = neeta*(D**al)*(N**(beta -1))
    # PNH = neeta*(D**lam)*(N**(beta -1))
    PND = neeta * (D ** al) * (N ** (beta ))
    PNH = neeta * (D ** lam) * (N ** (beta ))
    PDH = c*(H**a)*s
    PHD = c*(D**a)*(1 - s)




    dNdt = -N*PND - N*PNH
```

```python
    dDdt = N*PND - D*PDH + H*PHD
    dHdt = N*PNH + D*PDH - H*PHD

    return [dNdt, dDdt, dHdt]

#initial conditions
x0 = [200, 600, 0]

t = np.linspace(2005,2050,60)
x = odeint(odes,x0,t)

N = x[:, 0]
D = x[:, 1]
H = x[:, 2]
# plot results
plt.plot(t,N,'b-',label=r'None population')
print("Value of N at stable point",N[-1])
plt.plot(t,D,'r-',label=r'Dialog population')
print("Value of D at stable point",D[-1])
print(N[-1])
plt.plot(t,H,'g-',label=r'Hutch population')
print("Value of H at stable point",H[-1])
plt.ylabel('Customers')
plt.xlabel('time')
plt.legend(loc='best')
plt.show()
```