



1.

$T(n) = n^2$ since, $n^2 > n \log(n)$

$T(n) = 2^n$ since $2^n > n^8 > n^3 > n$

2.

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j = 0                ← constant time
for i = 0 to n        ← n times
    while j ≤ 5        ← n x constant time (because the condition of the while loop is nt based on the input size)
        j = j + 1      ← n x constant time (because the condition of the while loop is nt based on the input size)

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Total steps = $1 + n + 7n + 6n = 14n + 1$

Therefore, $T(n) = n$

3.

	INSERTION-SORT(A)	Cost	Times
1	for j = 2 to A.length	c_1	n
2	key = A[j]	c_2	n-1
3	// Insert A[j] into the sorted // sequence A[1..j-1]	0	n-1
4	i = j - 1	c_4	n-1
5	While i > 0 and A[i] > key	c_5	$\sum_{j=2}^n t_j$
6	A[i+1] = A[i]	c_6	$\sum_{j=2}^n (t_j - 1)$
7	i = i-1	c_7	$\sum_{j=2}^n (t_j - 1)$
8	A[i+1] = key	c_8	n-1

Let t_i be the number of times the while loop was executed for that value of i

From the above diagram we can conclude that the running time can be given as

$$T(n) = c_1 n + c_2 (n-1) + c_4 (n-1) + c_5 \sum_{j=2}^n t_j + c_6 \sum_{j=2}^n (t_j - 1) + c_7 \sum_{j=2}^n (t_j - 1) + c_8 (n-1)$$

By simplifying it for the best case and the worse case.

Best Case (Array is in sorted order)

$$- T(n) \leftarrow an+b$$

The time complexity therefore is, $\theta(n)$

•Worst Case (Array is in reverse sorted order)

$$- T(n) \leftarrow cn^2 + dn + e$$

The time complexity therefore is, $\theta(n^2)$

4.

1	2	3	4
3	4	1	2

1) A.length = 4

$$j = 2$$

2) $i = 1$

3) $A[j] > A[i] \leftarrow 4 > 3 \text{ T}$

4) $i = 2$

3) $A[j] > A[i] \leftarrow 4 > 4 \text{ F}$

5) Key = 4

6) $k = 0$ and $j-i-1 = 0$

8) $A[i] = A[2] = 4$

- 1) $j = 3$
- 2) $i = 1$
- 3) $A[3] > A[1] \leftarrow 1 > 3 \text{ F}$
- 5) $\text{key} = A[3] = 1$
- 6) $k = 0 \text{ to } j-i-1 = 1$
- 7) $A[j-k] = A[j-k-1] = A[3] = A[2],$
 $A[3] = 4$
- 6) when $k = 1$ the loop doesn't execute
- 8) $A[1] = 1$

1	2	3	4
1	3	4	2

- 1) $j = 4$
- 2) $i = 1$
- 3) $A[4] > A[1] \leftarrow 2 > 1 \text{ T}$
- 4) $i = 2$
- 3) $A[4] > A[2] \leftarrow 2 > 3 \text{ F}$
- 5) $\text{key} = A[4] = 2$
- 6) $A[4-0] = A[4-0-1]$
- 7) $A[4-1] = A[4-1-1]$
- 8) $A[2] = 2$

1	2	3	4
1	2	3	4

