Container Transportation Company

Challenge

- Top Management has challenged regional manager Mr. Thomas to optimize profit and revenues.
  - Find optimal containers of type 20T and 40T to maximize revenue
  - Explore dynamic pricing to enhance revenue
- Concepts and Methods
  - Linear Programming
  - Excel Solver
  - Dynamic Pricing
  - Revenue Management

Decision Variables

- Let X_{ij} be the number of containers of type *i* be loaded at port *j*
- Where *i* is (20T type, 40T type) and
- Where *j* is (Japan, China, Hong Kong, Indonesia, India, Korea, Malaysia, Singapore, Taiwan, Thailand)

Objective Function

- $Max\ z = \sum_{i \in (20T, 40T)} \sum_{j \in (J,\ C,\ HK,\ IDR,\ IND,\ K,\ M,\ S,\ TW,\ TH)} (p_{ij} * X_{ij})$

- *Price per container of each type at each port is multiplied by number of 20T containers and 40T containers respectively.*

- $p_{40Tj} = 2 * p_{20Tj}$

Constraints

- Demand Constraint: Sum of TEU of each container types should not exceed demand at each port.

- Volume Constraint: Sum of the volume of each container type loaded on vessel should not exceed vessel volume capacity (2000 TEU)

- Weight Constraint: Sum of the weight of each container type loaded on vessel should not exceed vessel weight capacity (24000 tons)

- Ratio Constraint: Ratio $N(40T)/N(20T)$ should be between 1.2 and 2

- During high season 95% loading and during low season 90% loading is to be used for volume and weight both

- Standard Non-negativity Constraints.

Formulation Template (High Demand)

Port	$/TEU	HSD	20%	40%	20T Wt	40T Wt		Container Ratio		20T X	40T Y	LHS	D	RHS		LHS	WT	
Japan	940	320	0.3	0.7	20	24		96 >=		96.0	112.0	320 <=		320		4608		
China	878	68	0.4	0.6	18	23		27 >=		27.2	20.4	68 <=		68		959		
Hong Kong	766	737	0.42	0.58	19	22		310 >=		309.5	213.7	737 <=		737		10583		
Indonesia	840	68	0.4	0.6	21	25		27 >=		27.2	15.2	57.633 <=		68		952		
India	790	340	0.4	0.6	22	22		136 >=		99.1	120.4	340 <=		340		4830		
Korea	710	35	0.46	0.54	21	25		16 >=		0.0	0.0	0 <=		35		0		
Malaysia	643	138	0.54	0.46	20	22		75 >=		0.0	0.0	0 <=		138		0		
Singapore	649	43	0.39	0.61	19	25		17 >=		16.8	0.0	16.77 <=		43		319		
Taiwan	702	41	0.41	0.59	19	19		17 >=		16.8	12.1	41 <=		41		549		
Thailand	663	9	0.2	0.8	23	26		2 >=		0.0	0.0	0 <=		9		0		
												1580.4 <=		1900		22800 <=		22800
Max Z =	1281723									592.65	493.876	0 >=		0				
												-395.101 <=		0				

		Objective Function
		Decision variables
		Demand Constraint
		Weight Constraint
		Volume Constraint
		Container Ratio constraint
		1.2 Ratio lower limit
		2.0 Ratio Upper limit

Formulation Template (Low Demand)

Port	$/TEU	HSD	20%	40%	20T Wt	40T Wt		Container Ratio		20T X	40T Y	LHS	D	RHS		LHS	WT	
Japan	940	286	0.3	0.7	20	24		86 >=		85.8	100.1	286 <=	286		4118		
China	878	61	0.4	0.6	18	23		24 >=		24.4	18.3	61 <=	61		860		
Hong Kong	766	660	0.42	0.58	19	22		277 >=		277.2	191.4	660 <=	660		9478		
Indonesia	840	61	0.4	0.6	21	25		24 >=		24.4	18.3	61 <=	61		970		
India	790	304	0.4	0.6	22	22		122 >=		43.3	130.3	304 <=	304		3821		
Korea	710	31	0.46	0.54	21	25		14 >=		14.3	0.0	14.26 <=	31		299		
Malaysia	643	123	0.54	0.46	20	22		66 >=		63.8	0.0	63.8335 <=	123		1277		
Singapore	649	38	0.39	0.61	19	25		15 >=		14.8	0.0	14.82 <=	38		282		
Taiwan	702	37	0.41	0.59	19	19		15 >=		15.2	10.9	37 <=	37		496		
Thailand	663	8	0.2	0.8	23	26		2 >=		0.0	0.0	0 <=	8		0		
												1501.91 <=	1800		21600 <=		21600
Max Z =	1206120									563.22	469.348	0 >=	0				
												-375.478 <=	0				

		Objective Function
		Decision variables
		Demand Constraint
		Weight Constraint
		Volume Constraint
		Container Ratio constraint
		1.2 Ratio lower limit
		2.0 Ratio Upper limit

Solve using Solver

- Microsoft Excel can solve this linear program using Solver package which is available in the Excel Add-ons
- The snapshot of potential solution is shown.
- You can schedule a call for constraint formulation if you face difficulty

Revenue Management

- Currently, Price and Demand is given as constants
- Now we will introduce the price volume flexibility in formulation.
- Step 1:
  - Estimate Demand curve.
  - Slope = 0.05 * Demand / -0.03 * Price (for High Demand Season)
  - Slope = 0.10 * Demand / -0.05 * Price (for Low Demand Season)
  - Intercept = Demand – Slope * Price
  - Get the equation in y = m*x + c format
  - i.e. Demand = Slope * Price + Intercept

Revenue Management

- Step 2:
  - Estimate demand curve for each port.
  - i.e. 10 for high season and 10 for low season
- Step 3:
  - Assume that demand curve is linear at the point it is currently estimated.
  - Using the estimated slope make the demand a function of price.
  - Now price can be introduced as new variable.
- Step 4:
  - Run the solver program again including new constraint

New Optimum (High Demand)

Port	$/TEU	HSD	20%	40%	20T Wt	40T Wt		20	40	LHS	D	RHS	LHS	WT	Container Ratio	Slope	Intercept
Japan	1059.9	320	0.3	0.7	20	24		75.6	88.2	251.951	<=	252	3628		75.6	-0.57	853.3
China	1013.0	68	0.4	0.6	18	23		23.4	17.6	58.5724	<=	58.6	826		23.4	-0.13	189.3
Hong Kong	980.3	737	0.42	0.58	19	22		282.4	195.0	672.455	<=	672	9656		282.4	-1.60	2244.4
Indonesia	1038.1	68	0.4	0.6	21	25		21.9	16.4	54.7691	<=	54.8	871		21.9	-0.13	194.8
India	1012.9	340	0.4	0.6	22	22		68.6	109.5	287.701	<=	288	3920		115.1	-0.72	1014.3
Korea	996.0	35	0.46	0.54	21	25		14.0	8.2	30.3961	<=	30.4	499		14.0	-0.08	112.2
Malaysia	947.1	138	0.54	0.46	20	22		73.2	31.2	135.477	<=	135	2149		73.2	-0.36	474.2
Singapore	965.6	43	0.39	0.61	19	25		15.7	12.3	40.1764	<=	40.2	604		15.7	-0.11	146.8
Taiwan	943.9	41	0.41	0.59	19	19		16.7	12.0	40.6241	<=	40.6	544		16.7	-0.10	132.5
Thailand	1014.7	9	0.2	0.8	23	26		0.8	3.2	7.31046	<=	7.31	103		1.5	-0.02	30.3
								592.29	493.573	1579.43	<=	1900	22800	<=	22800		
	1576966.14									2E-07	<=	0			0		

New Optimum (Low Demand)

Port	$/TEU	HSD	20%	40%	20T Wt	40T Wt		20	40	LHS	D	RHS		LHS	WT		Slope	Intercept
Japan	1082.8	286	0.3	0.7	20	24		71.7	83.6	238.955	<=	239	0	3441	72		-0.57	853.3
China	1033.2	61	0.4	0.6	18	23		22.4	16.8	55.9646	<=	56	0	789	22		-0.13	189.3
Hong Kong	1001.1	660	0.42	0.58	19	22		268.4	185.3	639.068	<=	639	0	9177	268		-1.60	2244.4
Indonesia	1062.3	61	0.4	0.6	21	25		20.6	15.4	51.4976	<=	51.5	0	819	21		-0.13	194.8
India	1035.7	304	0.4	0.6	22	22		64.5	103.5	271.388	<=	271	0	3694	109		-0.72	1014.3
Korea	1020.2	31	0.46	0.54	21	25		13.1	7.7	28.4077	<=	28.4	0	466	13		-0.08	112.2
Malaysia	968.0	123	0.54	0.46	20	22		69.1	29.4	127.97	<=	128	0	2030	69		-0.36	474.2
Singapore	987.0	38	0.39	0.61	19	25		14.7	11.5	37.813	<=	37.8	0	569	15		-0.11	146.8
Taiwan	962.9	37	0.41	0.59	19	19		15.9	11.4	38.7724	<=	38.8	0	519	16		-0.10	132.5
Thailand	1040.2	8	0.2	0.8	23	26		0.8	2.9	6.73341	<=	6.73	0	96	1		-0.02	30.3
								561.21	467.678	1496.57	<=	1800	303	21600	<=	21600		
	1526619.08									4.5E-07	<=	0	0	0				0

Points to Note

- Observe how the demand is met completely now after RM.

- Study the Sensitivity and Limit Reports to understand the outcome

- Important concepts to understand here.
    - Shadow Price / Dual / Lagrangian Multiplier.
    - Writing constraints in excel for solver.